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Recommendation and evaluation of the PHD dissertation of Mgr. Lukasz T. Bielak titled Application of Stochastic Processes for Modeling Market Risk Factors in a Mining Company.

The PHD dissertation submitted by mgr. Lukasz Bielak focuses on modeling market risk factors of a mining company. His work is applicable to many mining enterprises, but his focus is on KGHM Polska Miedz S.A. The dissertation presents several modeling options for metal prices, foreign exchange rates, market risk in a univariate and multivariate settings. It is important to note that all discussed mathematical models are implemented numerically using algorithms developed in the dissertation and Mr. Bielak's publications. The implementation work is particularly important and valuable from the industry stand point.

The dissertation comprises of eight chapters. I will comment on those chapters one by one below.

Chapter 1 is an introduction to the problem and the research area. It also contains a description of the other chapters. Chapter 2 contains the formulation of the research problem addressed in the dissertation. Indeed, the research problem is the analysis and selection of stochastic processes for modeling of market data. The processes of interest include commodity prices, foreign exchange rates, and market risk. The main challenges stemming from the market itself include non-Gaussian distributions of many market prices/products, changes of the processes driving the prices (price regimes), and prices responding to a continuously evolving macroeconomic situation of the global markets. Statistical challenges include modeling functions of several assets and selecting appropriate time interval of the records used to fit the models. These challenges are addressed by the models developed and presented in the dissertation.

Chapter 3 is devoted to the description of the state of the art in modeling and forecasting market risk and the initial description and motivation for the models proposed in the dissertation. In particular, it contains a thorough literature review of the stochastic processes used in this area, including the econometric, statistical, and applied mathematics works. Next, this chapter lists approaches proposed to tackle the challenges of market modeling. First, the issue of the non-Gaussian distribution of the increments of the price process is proposed to be solved by using a skewed generalized Student's t distribution (SGT) in place of the typical Gaussian distribution. Next, the issue of the changing economic situation impacting the financial processes is proposed to be handled by allowing the model coefficients to vary in time. The changes of the underlying processes driving the market prices (regime changes) are proposed to be modelled with processes which include such changes in their construction. Further, the multidimensional issues are addresses with a Vector Autoregressive (VAR) type model. Finally, the question of optimal time interval for model calibration is addresses by studying various techniques involving averaging of forecasts produced by several different models or forecasts produced by one model calibrated on data from different time periods. The last section of chapter 3 contains a listing of the author's contributions in the field of the dissertation. In particular, the contributions include seven research papers already published in peer revied journals and one more publication submitted for publication.

Chapter 4 is devoted to modeling of metal prices. Section 4.1 discusses modeling metal prices over medium to long time horizons. The proposed approach involves a family of skewed generalized Student t distributions, which includes many standard distributions used in this area such as Gaussian, Laplace or t. The stochastic process X(t) considered for price modeling satisfies the following stochastic differential equation:

$$(1) \qquad dX_t = \alpha(X_t, t)dt + \beta(X_t, t)dS_t,$$

where $\alpha()$ and $\beta()$ are appropriate functions and the process S(t) has stationary independent increments having an SGT distribution with mean zero and finite variance. This is a generalization of a standard model with S(t) replacing the standard Brownian motion. The coefficients are allowed to vary in time, and have the following form:

$\alpha(t) = \alpha_1(t) + \alpha_2(t)X(t)$ and $\beta(t) = \beta_1(t) + \beta_2(t)X(t)$.

There is a detailed discussion of the quite complex process of parameters' estimation. The discussion is augmented with a flow-chart of the estimation steps, which is very clear and useful for the user. The estimation method's performance is studied via a Monte Carlo simulation, and the results show that the estimates are very close to the true parameters and the estimators have reasonably small variance. The simulation study supports the practical usefulness of the model. The section ends with an application of this new model to the copper price data.

Section 4.2 proposes a non-Gaussian regime switching model for the data produced by processes driven by phenomena changing in time. The model is a modification of the one presented in equation (1), so that we have two different SGT processes S_t^1 and S_t^2 driving the price process at different time intervals. Again, the estimation is presented thoroughly and the presentation includes an algorithm summarizing all the steps. A simulation study is performed and shows that the estimation produces reasonable estimates and the estimators have relatively small variability, which makes the estimation reasonably accurate, which is necessary for any practical application. Finally, a data set of daily copper prices is fitted with the model and the results show a very good fit.

Chapter 4 ends with a short section summarizing the new models and their benefits together with limitations.

Chapter 5 is devoted to modeling the foreign exchange rates. The model proposed for this task is a special case of the model described in equation (1). It is based on the SGT increments and is not homogeneous in time. The model is a generalization of the CKLS model introduced in 1992 for the short term interest rates. A thorough discussion of the complex estimation process is included and followed by a simulation study. Accuracy of the estimation is assessed using the mean absolute percentage error. In addition, a novel idea of a "validation" factor is presented and applied in the data analysis section. The idea of the validation factor is similar to that on cross-validation in machine learning. An application of the proposed model to exchange rates between Euro and USD, and between USD and PLN is also presented. The analysis includes comparison with the Brownian motion based model and the model selection utilizes the validation factor. The results show that the new model has a better forecasting ability than the classical Brownian motion based approach.

The second problem addressed in Chapter 5 is selecting an optimal calibration period when working with data which is not homogeneous in time, that is the data generating process changes in time. The method proposed and analyzed is based on averaging the calibration window. The process analyzed is a time inhomogeneous Vasicek process. The proposed method is shown to produce very good results on simulated data. It is also applied to the exchange rates data (Euro vs. USD, and USD vs. PLN) with very good results confirming that averaging several models is a very good practical solution for modeling foreign exchange rates.

In Chapter 6 the author discusses a bivariate model for asset returns with focus on the association between the price returns of copper (Cu) in USD and the returns of the foreign exchange rate between USD and PLN. This issue is fundamental to the profits of the mining company KGHM. Two time series are considered: weekly returns of the Cu price in USD, and weekly exchange rates between USD and PLN. The time period is from January 2000 to October 2020. It is important to note that this time period includes the period of the financial crisis around 2008. The measures of association considered are the Pearson, Spearman and Kendal correlation coefficients. Inclusion of the rank-based Spearman and Kendall correlation measures allows for non-Gaussian distribution of the data. The main model considered in this chapter is the VAR model with α-stable distribution of the "noise" vector. To account for the (possibly) infinite second moment of the α-stable distribution the author uses covariation in place of the standard covariance. The model used for the data is actually VAR(1), with the possibility of a regime change. The change is reflected in the main parameters of the α-stable distribution. The analysis shows two regimes during the time period of record, with the switch around 2006. The authors consider two scenarios: models with and without dependence between the asset prices. Both approaches show non-Gaussian distribution of the returns. A particularly useful in practice is the inclusion of the change point of the structure of the model. I believe this methodology can be quite useful in climate change analysis.

Chapter 7 comes back to univariate analysis, but discusses a difficult problem of modeling a function, a product of two market components. The author is motivated by the estimation of the cost of electricity used for production which is the product of the electricity price and the electricity load/demand. The main result is the autocovariance function of the product of the two components of a 2-dimensional VAR(1) time series with finite variance. To aid the understanding of the behavior of the trajectories of the product process, the author presents simulation results. Finally, the model is applied to the modeling of the cost of errors in the electricity load prediction. The data are one-day-ahead electricity predictions of the Danish market between 2016 and 2021. The analysis yields a very good fit of the model to the market data. This model may have many other practical applications.

The last chapter, chapter 8 provides a summary of the work presented in the dissertation and ideas for continuation of this line of research.

The topics addressed in Mr. Bielak's dissertation are not only important mathematically, but have a big practical impact for industry and the sciences. The dissertation shows Mr. Bielak's great command of the techniques and theory of the modern mathematical and applied statistics. The computational methods applied in this work show mastery of the statistical computing techniques, which are in the top research areas of the modern statistics. The research has already been published in several journals and the publication record provides additional evidence of the impact of this research for the sciences and industry in addition to new mathematical results. I believe that the methodology developed by Mr. Bielak will be useful for problems in climate, hydrology and ecology, as it provides tools for work with complex and changing systems. The overall work shows a mature and accomplished researcher in mathematics with a keen interest and deep knowledge of the important problems of the current times.

The dissertation is well thought out and well written in English. The minor language errors do not impact my high evaluation of the work.

In summary, the author's theoretical contributions are original, substantial, and significantly advance the area of probability and statistics. I recommend awarding of the PHD degree to Mr. Bielak most strongly.

Sincerely,

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