Prof Marek Rutkowski
School of Mathematics and Statistics
University of Sydney

Report on the doctoral dissertation

Mathematical Modelling of Investment Portfolio Management Strategies. Theory and Applications within Heston Market Model.

by Jarosław Gruszka

submitted for the degree of Doctor of Philosophy

The doctoral dissertation by Jarosław Gruszka is devoted to the classical problem of optimal investment strategies. It should be stressed that the existing literature pertaining to portfolio management problems is abundant – both in the discrete and continuous time framework – and thus it was undoubtedly a very challenging task to deliver original contributions to the area, which was previously studied in plethora of research papers. Needless to say, the list of references containing 66 publications is merely a relatively small sample of the existing literature on portfolio management. However, it is also fair to say that the author is fully aware that his studies cover only a limited range of investment strategies proposed by other researchers. It is, of course, a completely different question if methods and results from these papers are of great practical value since real-world financial markets are known to be very difficult to model. The author makes a serious attempt to revisit and reassess some instances of trading strategies. Although some basic properties of these strategies are analyzed, it is clear that he is mainly concerned with their practical implementations. To this end, he carefully studies numerical experiments based on simulations of Heston's model and presents also examples relying on the actual market data for stock indices.

Chapter 1 contains a brief introduction to the area of portfolio management and an overview of the thesis. In Chapter 2, the author introduces the notation and definitions of self-financing investment strategies both without and with transaction fees. A particular attention is paid to a *balanced strategy* for which fractions of wealth invested in each stock is constant over time (as opposed to a *buy-and-hold strategy* where the number of shares of each stock is kept constant). Although the author attempts to provide fully rigorous definitions and mathematical results, it should be noted that Definition 11 of a balanced strategy under transaction fees is not fully correct since equation (2.18) for the total value of fees uses an artificial concept of "desired quantity of each kind of asset", rather that the actual number of shares of each asset after all transactions at a given time are completed. Thanks to this simplification, the author obtains the straightforward expression (2.15) for the updated quantities, whereas the actual formula would rely on a nonlinear equation, which can be obtained by replacing $q'(k\Delta t)$ by $q(k\Delta t)$ in the right-hand side of (2.18). The same comment applies to Definitions 12, 13 and 14 where three additional variants of a balanced strategy with transaction fees are introduced. Needless to say, one can still use Definitions 11-14 to study the impact of transaction fees on profitability of a balanced strategy, although they will yield only approximated results.

Let me now make more comments od Definition 13 (and hence also Definition 14). Suppose that we denote the right-hand side of equation (2.25) by $q_i^D(k\Delta t)$. Then we obtain the equality

$$q_i^D(k\Delta t) = (1 - D)q_i^{temp}(k\Delta t) + Dq_i(k\Delta t)$$

where $q_i(k\Delta t)$ is given by (2.15). This makes is clear that a partially balanced strategy with the inclusion of transaction fees can be represented as a linear (in fact, a convex) combination of the passive strategy and a fully balanced strategy with the inclusion of transaction fees. Therefore, Facts 1 and 2 on page 20 are obvious and it is also clear that, for any $D \in [0, 1]$, the strategy introduced in Definition 13 is manifestly self-financing with the inclusion of transaction fees.

Further specifications of strategies are based on classical indicators (MACD and RSI). In Theorems 1 and 2 it is formally shown that the MACD-based strategy is self-financing and all q_is are positive. This is surely correct and, in fact, a formal algebraic proof is not really warranted since these properties are readily seen from Definition 19. It should be noted, however, that it is assumed in Definition 19 (as well as in Definition 25) that selling of assets at time t is done first so that the proceeds from the sale can be used at the same time to increase holdings in some other assets. A more natural option would be to assume that selling and buying of assets at any date t is done concurrently and thus only the cash amount inherited from the previous date can be used to purchase additional shares of risky assets. It would even more practical to assume that decisions about buying and selling are done at time t but all transactions are executed at the next date but it is likely that such postulate would lead to similar results for simulations. Therefore, I contend that the fact that Definitions 19 and 25 are somewhat unrealistic should not be seen as a major issue. The expected value is not present in Definition 26 but it is clear from simulations that the author uses the expected value of $g_{\mathcal{P}}(t)$ as the chosen performance measure for the sake of comparison of various trading strategies through numerical experiments.

Original classes of trading strategies are first appropriately formulated in Chapter 2 and later compared with a buy-and-hold strategy and variants of balanced strategies within the setup of Heston's model and its jump-diffusion extension. Results of numerical experiments based on simulations for Heston's model are presented in Chapter 3, which can be seen as a major contribution of the thesis. Although all experiments are presented in detail and with pertinent comments, it is not completely clear whether the prices of individual assets are modelled using independent Heston's models and, if so, why the author decided not to cover the case of correlated assets. Needless to say, the prices of assets observed in practice can hardly be modeled by independent stochastic processes.

Chapter 4 is entirely devoted to the issue of estimation of parameters of Heston's model and its jump-diffusion extension. The proposed method of estimation and presentation of steps is sufficiently detailed except for an important step in the proof of Theorem 3. At the bottom of page 85 it is stated that "it becomes clear that as long as the function given in Eq. (4.56) is bounded, increasing number of particles leads to diminishing the value of the ratio denoted as $\check{W}_j$" and thus $\lim_{G\to\infty} \check{W}_j = 0$. In my opinion, the stated convergence requires a more formal proof so, in my opinion, it was conjectured, rather than established. Finally, in Chapter 5, the author presents an attempt to estimate the parameters of Heston's model using real-world data for market indices and, in Chapter 6, he summarises the thesis and outlines areas for future research.

Conclusions. The author has shown that he is familiar with the existing literature on classical approaches to portfolio optimisation. He has presented his original research, especially in Chapters 3 and 4, which have already been published in journals. In general, I have found the research by Jarosław Gruszka interesting and well presented, although it should be observed that some parts of the thesis could be further improved. It is clear that the author's expertise is much closer to numerical methods than to mathematical theory, which explains why no essential new mathematical results have been obtained. Nevertheless, the research of Jarosław Gruszka presented in his doctoral dissertation brings nontrivial contributions to practical issues arising in the context of portfolio optimisation problems. In my opinion, the work satisfies the requirements of a doctoral dissertation and thus I recommend award to Jarosław Gruszka the degree of Doctor of Philosophy in the Discipline of Science Mathematics.

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M. Rutkowski