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Report on the Thesis
Mathematical Modelling of Investment Portfolio Management Strategies. Theory and Applications within Heston Market Model
submitted for a PhD degree by
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The doctoral thesis under review consists of three parts, each dealing with different aspects of portfolio management theory and practice.

The first part of the Thesis, Chapter 2, develops a coherent mathematical framework for defining, describing and analysing portfolio management strategies within a market of assets. Within this framework, a number of known strategies for portfolio management are rigorously defined, both with and without transaction costs, and then their properties are established. The framework also allows to rigorously define sound portfolio performance measures that are robust with respect to differences in portfolio duration and invariant with respect to initial wealth. It is in this chapter that the main goal of the thesis becomes apparent - different portfolio management startegies are introduced and heuristically motivated in various communities, yet their performance and contexts within which they prove efficient are not well understood. Consequently, it is difficult to make recommendations regarding a suitable investment strategy in practice. The thesis aims to address this problem by executing the following strategy:

- Assume a specific continuous time stochastic parametric model for the market of assets;
- By the means of stochastic simulation investigate the performance (and relative performance) of different portfolio management strategies for combinations of parameters of the assumed stochastic market model;
- Use these results to identify the preferred investment strategy in different regions of the parameter space of the assumed market model;
- Develop an estimation procedure that given the real market data would allow to estimate the parameters of the assumed market model and, consequently, identify the preferred investment strategy.

To this end, Chapter 3, focuses on the Heston stochastic volatility model without jumps and the Heston stochastic volatility model with jumps in the price process. These are continuous time stochastic models whose evolution, and in particular instantaneous volatility, is fairly complex for the purpose of mimicking the real life dynamics of phenomena such as stock prices. These models are commonly used in stochastic finance literature and the standard interpretation of their use is that the actual stock prices are discrete time observations of the underlying continuous time process. There are also further attempts in literature, but not considered here, to more accurately model the observed instantaneous volatility, for example by introducing jumps to the hidden state variable.

Next, an Euler-Maruyama discretisation is used to simulate an approximation of these two models on a discrete time grid. This allows to implement the portfolio management strategies on this data, simulated from the discretised model, and investigate their performance by Monte Carlo repeated simulation. Simulations are performed for the periodically and/or partially balancing investment strategies, and the trading indicator strategies: MACD (moving average convergence divergence) and RSI (relative strength index). For periodically and/or partially balancing investment strategies the focus is on the impact of strategy/trading parameters such as exchange fees, investment horizon, rebalance period and rebalance coefficient on strategy performance. For the trading indicator strategies the focus is on the impact of buy and sell strategy indicators and the Heston model parameters, in particular drift and jump intensity, on strategy performance. This results in a heat map indicating drift and jump parameter regions where the active MACD strategy is preferable to a passive strategy (buy-and-hold).

Chapter 4 of the Thesis moves on to parameter estimation of the earlier specified Heston model and Heston model with jumps. This is done in the Bayesian inference framework where parameters are modelled as random variables that have posterior probability distributions derived from their prior probability distributions and observed data through the model likelihood. The Bayesian inference framework is recognised for its robustness, interpretability and the feasibility of uncertainty quantification that it provides. This is particularly relevant in the context of complex models such as stochastic differential equations with stochastic volatility considered here.

The crucial step of performing Bayesian inference in practice is obtaining samples from the posterior distribution of model parameters. This is usually done by designing an MCMC algorithm, that is an ergodic Markov chain with the posterior as its stationary and limiting distribution. For complex models, especially those involving intractable likelihood functions, this task typically requires substantial insight into the assumed data generating model and its probabilistic properties.

Designing such an MCMC algorithm for the Heston model and the Heston model with jumps is presented in Chapter 4. The key simplification made by the author is discretisation of the model in Chapter 3. This discretisation is also the basis for defining the posterior distribution and developing the MCMC algorithm in Chapter 4. It must be noted that while the effect of discretisation has not been discussed in the present work, the inference problem for the Heston model without discretisation is widely considered infeasible.

For the discretised approximation of the posterior distribution a Markov chain Monte Carlo procedure is developed that utilises a Gibbs sampler on the posterior target whose conditional distributions are conjugate to the priors set on transformed parameters. This Gibbs sampler is combined with a particle filter for sampling the unobserved volatility process. It must be stressed that developing this MCMC procedure is a nontrivial task and is a significant contribution to the literature.

In conclusion, the PhD Thesis is a significant contribution to the theory and practice of mathematical modelling for portfolio management. It is a complete piece of work that builds on developing a formal and rigorous framework for market and portfolio description, a thorough simulation study of portfolio management strategies under the Heston model (without and with jumps) and concludes with Bayesian inference methodology for the model by developing an advanced MCMC algorithm. These steps can be put together in the applied context making this contribution valuable both scientifically and practically.

Hence, the PhD Thesis fully satisfies both the formal requirements and customary expectations of the scientific community. I recommend the Thesis be accepted for proceeding to the PhD viva.

For the work presented in this interesting Thesis to be recognised by the rigorous computational finance and computational statistics research communities and for it to be publishable in the leading journals in these areas, several aspects could be revised, further considered or discussed. The reminder of this report focuses on detailing how this could be addressed.

- In the simulation Chapter 3:
  - The theoretical validity of the discretisation of the Heston model (and the Heston model with jumps) should be discussed. In what sense does the discretised model approximate the continuous time model? Are there known results about the strong or weak convergence for this approximation and possibly about its order?
  - When considering the investment strategies, what is the impact of discretisation on the results? Are the conclusions robust with respect to refining the discretisation?
  - In particular, the discretisation changes the stationary distribution of each of the models and changes its dynamics. However, some of the conclusions about the investment strategies concern very long time horizons. Are these conclusions at all valid? The discretisation errors will accrue in these scenarios and so will their impact on the conclusions, such as the long term evolution of the portfolio wealth.
  - How is the discretisation grid chosen? In simulation studies it does not

seem to be chosen in a systematic way ($\Delta t = 0.1$ p. 45 then 0.03 on p. 48; 0.02 on p 50; 2^{-8} on p. 52, etc.) and these choices are never justified nor is their impact considered.

- Discretisation implies that simulated price process or the volatility process can become negative. How has this been addressed?
- The choice of some of the Heston model parameters in the simulation seems arbitrary. How were they chosen? Do the conclusions hold for different combinations of these parameters?

- In the inference Chapter 4:
  - This chapter starts with a (very brief) discussion of the Bayesian methodology, including the prior and posterior distributions. The presentation would benefit from a more in depth discussion of the nature of Bayesian inference, the use of posterior distribution, and how it conceptually differers from the frequentist setting.
  - There are several statements indicating that the Bayesian principles of posterior estimation and sampling should be considered more carefully. The Thesis repeatedly talks about "estimating parameters" but it should really talk about sampling from the posterior distributions of these parameters. Another example is a statement from page 92: "A well designed MCMC estimation algorithm should bring us closer to the true values of parameters with each new round of samplings." However, in the Bayesian context there are no true values of parameters, only their posterior distributions, so this statement is confusing. Similarly, on p. 78 we read "By averaging out all of those particles we will get an estimate of the true volatility $v(t)$", however, again, there is no true volatility in this setting, instead the volatility process has a distribution which is conditional on the observed data and results from the model definition and prior specification.
  - It is worth specifying the posterior more carefully. Currently it is only specified through conjugate priors and posteriors of individual parameters conditionally on the volatility process. It would be more coherent to first specify the posterior distribution (on an augmented state space if necessary) and only then devise the MCMC algorithm. It would then make sense to check if the Markov chain in question is recurrent and ergodic and hence if the procedure is valid at least in the asymptotic regime, which presently is not discussed at all.
  - Some of the steps in the MCMC procedure are enigmatic while others are questionable:
    * Why is resampling of the V_j particles needed?
    * What is the purpose of introducing the Connected CDF on p. 84 for the resampling step?
    * The SMC weights should be reported in simulations as deterioration of weights (when a single weight is close to 1 and the rest close to 0) implies poor quality of the sampling procedure.

    * On p. 86 there is an "estimator" of the mean of the volatility process (equation 4.67). What is this used for?
    * Note that the mean of the volatility process is not a good estimate of the typical realisation of the volatility process in the same way as the mean of the Brownian Motion is not a typical realisation of the Brownian Motion.
    * If the mean volatility process is used in the sampling algorithms 1 and 2 in order to sample other parameters conditionally on the mean volatility process then this results in biased sampling.
    * The sampling algorithms 1 and 2 conclude with computing the mean of the Heston model parameters. However, if the purpose is to evaluate which investment strategy performs better, this should be done for the realisations of parameters from the posterior, not for their mean.
    * There are some natural questions about the algorithmic efficiency in different regimes: what happens with efficiency and convergence of the algorithm if (i) discretisation goes to 0? (ii) time horizon goes to ∞?
    * Studying *Particle Markov chain Monte Carlo methods* by Andrieu, Doucet, Holenstein, JRSSB 2010 could help address some of these points above.

- Integration with literature (introduction, discussion and also research chapters 3 and 4):
  - The Thesis would benefit from integration with existing literature. In what ways is this work different from the referenced works of Polson and coauthors? Polson seems to address very similar models, including more general version of the jump stochastic volatility model where also jumps in the stochastic volatility process are allowed. This should be discussed carefully in terms of applicability, differences and similarities in methodology and a comparison through simulation should be demonstrated where appropriate.
  - Other stochastic volatility estimation works should be included in literature review and discussed. Some computatinally focused references to stochastic volatility models are included in section 3.2 of Andrieu, Doucet, Holenstein, JRSSB 2010.
  - Finally, how does the proposed methodolog compare to classical methods based on maximum likelihood, etc?

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