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Title of the doctoral dissertation

A more-generalized multiple suppliers, single manufacturer inventory control model with stochastic demand, variable production rate and differing supplier delivery times

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Dedication

I dedicate this work, first and foremost, to God Almighty, the source of my strength, wisdom, and success. Without His divine help and unending favor, this achievement would not have been possible. He has been the mystery behind every step of my journey, guiding and sustaining me even in moments when my own strength failed.

Humanly, I dedicate this dissertation to the cherished memory of my late father, Mr. Henry Taiwo Adegbola. Though you were my father, you were also my friend. You related with me not only as a parent but as a confidant and mentor. Your teachings continue to echo in my heart, your generosity toward those in need remains an inspiration, and your unwavering love for your family is a legacy I will forever carry with pride.

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This work is also a testament to your influence and the foundation you laid for me. It is my honor to fulfill this dream, not only for myself but also in honor of you.

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DISSERTATION ABSTRACT

In the wake of the COVID-19 pandemic and ongoing geopolitical conflicts, small and medium-sized enterprises (SMEs) face heightened financial and operational pressures. Vertical collaboration with financially stronger manufacturers has emerged as a strategy to integrate SMEs into large-scale production systems and enhance supply chain resilience. This study develops computational optimization models for a multi-supplier, single-manufacturer supply chain operating under stochastic demand, employing a (Q, R) inventory policy to determine optimal replenishment strategies. The models consider both homogeneous and heterogeneous suppliers delivering raw materials in batch shipments at varying intervals. Two optimization strategies are examined: an integrated model, which jointly minimizes total supply chain costs, and a Stackelberg leader-follower model, where the manufacturer optimizes its costs independently while suppliers adjust production rates competitively. Pseudocode-driven algorithms are developed to implement both approaches, with key decision variables including production rates, batch sizes, reorder levels, lot sizes, and the number of delivery trips per cycle.

Sensitivity analysis is conducted primarily with the analytical model to evaluate the impact of parameters such as mean demand, demand variability, supplier production-to-manufacturer consumption ratios, raw material requirements per finished product, and holding costs. Building on these results, discrete-event simulation optimization using Arena and OptQuest relaxes the analytical assumptions by allowing asynchronous dispatch, equal batch sizes across suppliers, and the introduction of a dynamic reorder timing buffer as a decision variable. This flexibility enables the supply chain to respond effectively to variations in supplier production rates, delivery capabilities, and operational constraints, achieving greater cost savings compared with the synchronized, lead-time-driven analytical approach.

This research advances computational optimization in supply chain management by providing quantitative decision-support models that improve cost efficiency and operational resilience in uncertain multi-echelon systems. The findings demonstrate that flexible dispatch policies, equal batch sizes, and dynamic timing can enhance collaboration between manufacturers and SME suppliers while reducing overall supply chain costs through data-driven decision-making.

STRESZCZENIE ROZPRAWY DOKTORSKIEJ

W obliczu pandemii COVID-19 oraz trwających konfliktów geopolitycznych małe i średnie przedsiębiorstwa (MŚP) borykają się z rosnącymi wyzwaniami finansowymi i operacyjnymi. Współpraca wertykalna z finansowo silniejszymi producentami stała się strategią umożliwiającą integrację MŚP w systemy produkcji na dużą skalę i zwiększenie odporności łańcucha dostaw. W niniejszym badaniu opracowano modele optymalizacji obliczeniowej dla łańcucha dostaw z wieloma dostawcami i jednym producentem działającego w warunkach popytu stochastycznego, wykorzystujące politykę zapasów (Q,R) w celu określenia optymalnych strategii uzupełniania zapasów. Modele uwzględniają zarówno jednorodnych, jak i zróżnicowanych dostawców, którzy dostarczają surowce w partiach o różnym interwale czasowym. Zbadano dwie strategie optymalizacji: model zintegrowany, który minimalizuje całkowite koszty łańcucha dostaw, oraz model Stackelberga (lider-podążający), w którym producent optymalizuje własne koszty, a dostawcy dostosowują tempo produkcji w sposób konkurencyjny. Opracowano algorytmy pseudokodowe do realizacji obu podejść, przy czym kluczowymi zmiennymi decyzyjnymi są tempo produkcji, wielkość partii, poziomy uzupełniania zapasów, wielkość partii produkcyjnej oraz liczba dostaw na cykl.

Analizę wrażliwości przeprowadzono głównie na modelu analitycznym w celu oceny wpływu takich parametrów, jak średni popyt, zmienność popytu, stosunek tempa produkcji dostawców do tempa zużycia przez producenta, ilość surowca na jednostkę produktu gotowego oraz koszty magazynowania. Na podstawie tych wyników zastosowano optymalizację w oparciu o symulację zdarzeń dyskretnych przy użyciu narzędzi Arena i OptQuest, co pozwoliło zrezygnować z założeń modelu analitycznego, w tym synchronizacji wysyłek, nierównych partii oraz powiązania uzupełniania z czasem dostawy najszybszego dostawcy. Model symulacyjny umożliwia asynchroniczne wysyłki, równe wielkości partii oraz wprowadzenie dynamicznego bufora czasowego jako zmiennej decyzyjnej, co pozwala lepiej reagować na różnice w tempie produkcji, zdolnościach dostawczych i ograniczeniach operacyjnych, zapewniając wyższe oszczędności kosztowe w porównaniu z sztywnym, zsynchronizowanym podejściem analitycznym.

Badanie przyczynia się do rozwoju optymalizacji obliczeniowej w zarządzaniu łańcuchami dostaw, dostarczając ilościowe modele wspomagające podejmowanie decyzji, które zwiększają efektywność kosztową i odporność operacyjną w złożonych systemach wielopoziomowych. Wyniki pokazują, że elastyczne polityki wysyłek, równe partie oraz dynamiczne sterowanie

czasem uzupełniania mogą poprawić współpracę między producentami a MŚP oraz obniżyć całkowite koszty łańcucha dostaw dzięki podejmowaniu decyzji opartych na danych.

CHAPTER 1

INTRODUCTION

1.0 Supply Chain and Global Supply Chain Management

The term "supply chain" describes an intricate logistical network made up of establishments that transform raw materials into completed goods and then ship those goods to final users. Supply chain management, on the other hand monitors, controls and optimizes the activities and events of the supply chain process such that it optimally benefits all participants/stakeholders.

For governments and businesses alike, several factors are crucial in guaranteeing accountability and responsibility in supply chain management. Government rules provide legal frameworks that require businesses to follow norms and laws, enforcing compliance through sanctions and monitoring systems. In addition, corporations are under pressure to match market expectations by implementing transparent and ethical supply chain policies due to the growing customer demand for ethical and sustainable products.

The Internet of Things (IoT) has revolutionized the speed and power of data analysis and dissemination. Therefore, making the business world more interconnected. For example, the European Union's (EU) financial crises are having a profound effect on North America and Asian economies. A situation which compelled supply chain specialists to consider the uncertainty in EU supply chains, given the level of direct investment and trading relationships that exist between EU and the two largest economies (USA and China). Otherwise, the world could creep into economic crisis, volatile exchange rates, workers' strike, and social unrest. Individual firms, on the other hand, had to adjust rapidly to burgeoning technological changes to meet the increasing competition with rival firms, and to cater for the unending demand of clients/customers.

Traditionally, the management of materials and component parts was the most neglected element in the

production process. Management only seeks alternative approaches to the planning and control of the acquisition and transformation functions in an organization when the costs of materials and subassemblies surge. Instead, most firms focused more on reducing labor and capital costs, which makes sense given that the industrial revolution produced many labor-intensive enterprises. Producing large, standardized batches represented the norm for some manufacturers. Some invested in technology driven manufacturing systems, but they are being run according to a traditional model which pays little or no attention to managing the inventory of raw materials, and thus the primary purpose for setting up such technology was defeated. The reality is that both technological advancement and inventory management must be intertwined to obtain the full benefits from such a process, which could easily translate into large financial savings.

1.1 Supply Chain Optimization and Analytical Modeling

Supply chain optimization and analytical modelling have a symbiotic relationship. This is because analytical modeling serves as a building block for optimizing a supply chain process. Analytical models use mathematical and statistical techniques to analyze data and gain possible insights into various aspects of the supply chain. These models provide the quantitative basis for making viable decisions, while optimization is carried out to find the best possible solutions to complex supply chain problems from many viable solutions. Some applications are enumerated below.

- ***Analytical Modeling as a Basis for Optimization:*** Forecasting models, inventory management models, and transportation models provide a representation of the current state of the supply chain system, using different diagrammatic profiles, networks, historical data, and relevant parameters to make viable decisions. These models provide explicit insight into the connections and dependencies that exist between different components of a supply chain system. They also facilitate a more precise evaluation of performance and pinpoint areas in need of improvement.

- ***Data-Driven Decision Making:*** Analytical models use both historical and current data to give a quantitative understanding of the functioning of the supply chain. Optimization techniques apply these data to find optimal solutions, considering factors such as costs, time, resources, and constraints.
- ***Identifying Opportunities for Improvements:*** Analytical modeling helps in finding areas of subpar performance, bottlenecks, and inefficiencies in the supply chain. Applying the insights drawn from these models, optimization procedures suggest changes to processes, resource allocation, and decision-making to improve overall efficiency and effectiveness.
- ***Optimization Algorithms and Techniques:*** Optimization algorithms like linear programming, dynamic programming, integer programming, non-linear programming, and simulation, are applied to analytical models to determine the best possible solutions to complex supply chain problems. These approaches are used in solving several problems that are related to economic order quantity, production schedules, reorder level, shipment size, safety stocks and other critical decisions that could impact the supply chain system adversely if not well taken.
- ***Continuous Refinement:*** The relationship is iterative and dynamic. Analytical models are updated to reflect the shifting environment as the supply chain changes, and new data become available. Organizations can therefore adjust to shifting market conditions and operational demands using appropriate optimization techniques, which continuously hone strategies based on the most recent analytical data.
- ***Scenario Analysis:*** Decision-makers can perform sensitivity analysis through analytical modeling to study the impact of different parameters on the supply chain settings. Results from such analyses are used in developing strategies that are robust and adaptable to similar scenarios.
- ***Improved Decision Making:*** Analytical modeling provides a systematic approach to decision-making by quantifying the impact of different factors on supply chain performance. Optimization

enhances decision-making through a systematic and objective procedure that is orchestrated to find the best solutions among various alternatives.

- ***Technology Integration:*** Advanced technologies like artificial intelligence and machine learning are frequently integrated with analytical modelling to provide a more complex knowledge of supply chain dynamics. Optimization methods make use of these sophisticated models to improve the accuracy and precision of decision-making.

In brief, supply chain optimization and analytical modelling combine to form a potent framework that facilitates strategic decision-making and high-quality operations. Analytical insights direct optimization efforts, which over time improve and hone the efficacy of analytical models.

1.2 Management and Mathematical Modeling Techniques

Mathematical modelling and management have an interesting history that illustrates how both disciplines have evolved together over time. The intersection of mathematics and management has significantly added to the development and improvement of organizational processes, decision-making, and overall efficiency. Frederick Taylor pioneered the scientific management movement in the early 1900s, which placed a strong emphasis on using scientific techniques to improve industrial operations. Although not specifically mathematical, this signaled the start of a change in management strategy towards one that was more methodical, and data driven. On the other hand, the use of mathematical modelling in management became more popular around the middle of the 20th century.

The development of management science and operations research during World War II was a major influence in how mathematics and management became related. Planning for logistics, making strategic decisions, and allocating resources effectively were all necessary for military operations. To address these issues, statisticians and mathematicians played a key role in creating models and methods that laid the groundwork for their later use in the business sector.

The development of linear programming, a mathematical optimization technique, occurred in the 1950s and 1960s. This became a landmark in operations research, as it allows managers to make optimal decisions in different areas of resource allocation, production planning, logistics and supply chain management. The field of operations research was later advanced to incorporate different mathematical modeling techniques, such as game theory, simulation, and queuing theory.

The application of mathematical modelling in management further gained momentum in the second half of the 20th century through the introduction of computers. Complex computations and the handling of enormous datasets made it possible to develop more advanced modelling strategies. Decision support systems and management information systems emerged, integrating mathematical models to help managers in decision-making processes.

As organizations face increasing competition in a very complex dynamic human and business environment, mathematical modeling has become an important tool for risk management and strategic planning. For instance, financial modeling plays a significant role in managing financial risk and influencing investment decisions.

With the development of machine learning and artificial intelligence in recent decades, the interdependent relationship between mathematical modelling and management has continued to grow. These technologies have made it possible for businesses to automate decision-making procedures, forecast trends, and analyze enormous volumes of data. Across many industries, data-driven decision-making has emerged as a critical component of successful management.

In a nutshell, there has been a development in the historical relationship between mathematical modelling and management, from the basic ideas of scientific management to the more complex, technologically advanced methods used today. With its ability to provide insights, optimize workflows, and improve overall organizational performance, mathematical modelling has emerged as a crucial tool

for managers. The ongoing synergy between mathematics and management guarantees that innovative modeling techniques will always shape or provide insight into the future of strategic decision-making and organizational efficiency.

1.3 General Statement of the Problem

In this section, a general statement of the problem addressed in this research work is presented.

1.3.1 An inventory model with multiple-suppliers and a single-manufacturer

The outbreak of conflict in various parts of the world has disrupted the global corporate landscape, compounding the challenges faced by businesses as they recover from the pandemic. For many SMEs, these difficulties have only intensified, as they now contend with even harsher conditions that further weaken their commercial stability. In 2021 and 2022, global GDP declined by more than 3.4%, leaving countless SMEs uncertain about the state of their liquidity reserves (OECD, 2022). This situation necessitates either government intervention or strategic alliances, such as partnerships with larger corporations, to ensure survival.

Against this backdrop, this research examines collaborations between SMEs and larger organizations within a multiple-supplier, single-manufacturer supply chain system. The suppliers—assumed to be SMEs—source and produce the primary raw materials required by the manufacturer, who places orders with appropriate batch size. The manufacturer operates under a make-to-stock policy, while the suppliers produce solely on demand. Since the manufacturer’s finished goods are subject to random consumer demand, both raw materials and finished products are stored in dedicated warehouses.

Each supplier operates independently, with varying delivery timelines. The manufacturer’s primary objective is to maintain minimal inventory while ensuring an uninterrupted supply of raw materials, preventing production delays as suppliers deliver on schedule. Furthermore, no single supplier has the capacity to meet the manufacturer’s total demand for raw materials. As a result, the manufacturer acts

as the principal partner, determining the optimal number of suppliers required for the supply chain.

Figure 1.1 illustrates the inventory profile of the proposed multiple-supplier, single-manufacturer supply chain model.

1.3.2 Description of the model

The supply chain profile shown in Figure 1.1 represents a large-scale manufacturer that sources its primary raw material, corn starch, from a network of independent suppliers, which are small and medium-sized enterprises (SMEs). These small and medium-sized enterprises (SMEs) are characterized by their limited scale of operations, resource constraints, and greater flexibility, operating under a make-to-order production system. The suppliers produce corn starch in bulk, packaging it into 3 kg units, which are then delivered in batches. The batch size is determined by the manufacturer and is assumed to be uniform for homogeneous suppliers but may vary for heterogeneous suppliers.

The manufacturer follows a make-to-stock production strategy, using the supplied corn starch to produce pre-packaged corn chips, where three units of 3 kg corn starch are required to manufacture one unit of 10 kg chips. Each 100 kg batch of corn chips (10 units of 10 kg each) requires exactly 30 units of 3 kg corn starch, along with other ingredients. The finished chips are stored in warehouses to ensure product availability and meet customer demand, which is inherently stochastic (random). The production rate should be adapted to variation in demand (large enough to deal with peaks in demand, but so large that excessive inventory builds up).

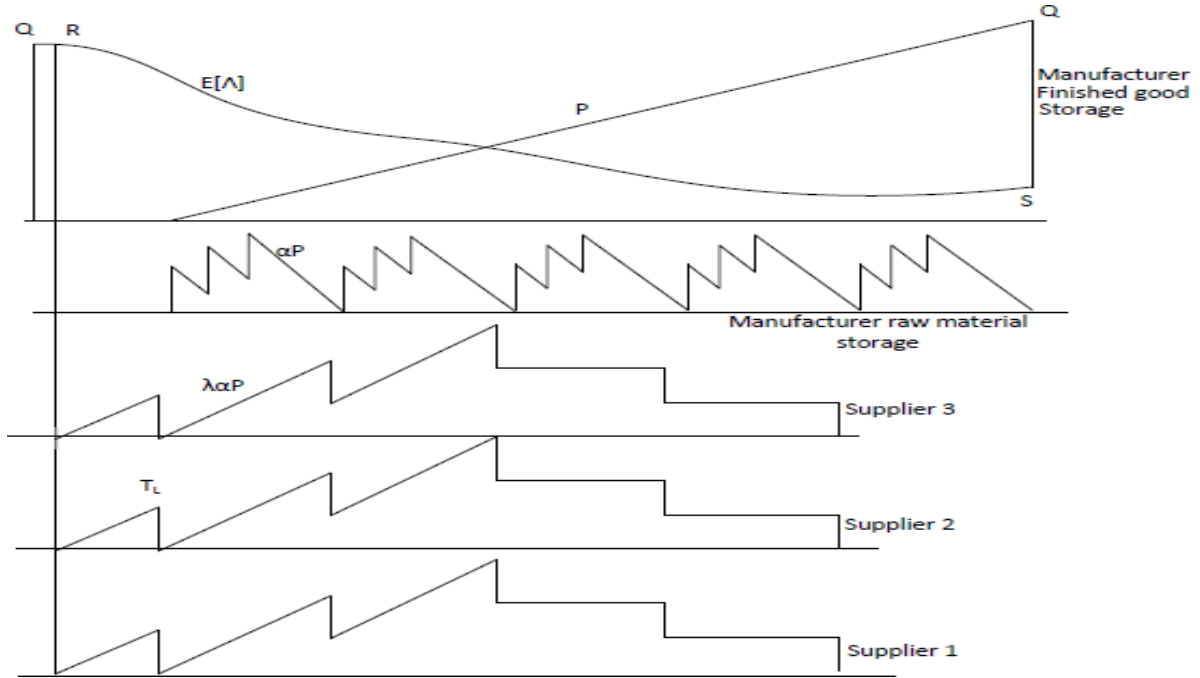


Figure 1.1: Inventory profile of the Multiple Suppliers, Single Manufacturer Supply Chain System.

Figure 1.1: Inventory profile of the Multiple Suppliers, Single Manufacturer Supply Chain System.

Importantly, no single SME supplier can independently meet the manufacturer's total demand for raw materials within the required timeframe. As a result, multiple suppliers are engaged, with the number of suppliers determined in advance by the manufacturer. The manufacturer, as the larger partner, retains the authority to terminate or modify supplier partnerships as needed. Each supplier produces corn starch at a rate of $\lambda_i \alpha P$, while the manufacturer's consumption rate is αP , where P represents the rate at which the final good is produced.

To ensure seamless operations, the manufacturer employs a continuous review system (Q, R) for inventory management. Production begins as soon as the first shipment arrives from the supplier with the shortest lead time (the time required to produce and deliver the raw materials), minimizing delays and optimizing material flow. Given the unpredictable demand for finished chips, maintaining a steady and responsive supply chain is crucial. To sustain this collaborative model, the manufacturer assumes responsibility for all direct and indirect costs, ensuring timely procurement, efficient delivery schedules,

and streamlined logistics. Coordinating the packing and delivery of corn starch in scheduled batches of a chosen number of truckloads helps optimize raw material usage, reduce storage inefficiencies, and improve overall supply chain resilience. The objective of the research is to determine:

- The optimal production rate for both the manufacturer and the suppliers during a production cycle.
- The most efficient batch size for the raw material shipments.
- The number of deliveries required by each supplier per cycle.
- The economic manufacturing quantity that minimizes costs.
- The optimal reorder point that balances inventory levels while reducing the risk of raw material stockouts/ production shut down.

This integrated approach enhances supply chain efficiency, ensuring cost-effectiveness, reliability, and sustained collaboration between the manufacturer and its SME partners, ultimately benefiting all of the stakeholders involved.

1.3.3 Motivation

This research work is inspired by the following points.

1. To explore and optimize the benefits of partnership/collaboration in a supply chain system.
2. To the best of our knowledge, flexibility in the manufacturing rate has only been considered within the context of a single supplier, single manufacturer setting, where demand is assumed to be stochastic. Therefore, it is pertinent to investigate how flexibility in the production rate on the side of multiple suppliers can further improve the financial savings of the supply chain system.
3. In practice, some suppliers limit shipment sizes to integer multiples of, e.g., a truckload (T_l) or containers, as in Aldurgam et al., (2017). However, in some cases batch sizes can be assumed to be continuously adjustable. The choice of batch size is based on customer requirements and influenced by other production parameters.

4. Most multiple-vendor, single-buyer models in the literature have been studied under the assumption of deterministic demand for the final product. Hence, there is a need to study multiple-supplier, single-manufacturer models under stochastic demand.
5. Most models assume that the various suppliers deliver raw materials to the producer rotationally or simultaneously for ease of modeling. Therefore, it is crucial to study supply chains where the suppliers deliver at different independent times.
6. Adegbola (2023) employed simulation optimization to analyze two inventory policies, Joint Reorder Point and Vendor Managed Inventory, in a single-vendor, multiple-manufacturer supply chain. Building on this methodology, the present study applies simulation to a more complex and realistic scenario involving collaboration between multiple SME suppliers and a single manufacturer. This research seeks to provide valuable insights into collaborative inventory management and distribution planning in multi-supplier, single-manufacturer supply chain networks.

1.4 Research Objectives

1. To develop an integrated mathematical model for the multiple-supplier, single-manufacturer supply chain problem using the inventory profile shown in Figure 1.1. Initially, the model will be analyzed under the assumption of homogeneous suppliers. This assumption will later be relaxed to examine heterogeneous suppliers under synchronized dispatch assumption, considering variations in production rates and the times at which raw materials are dispatched.

2. To develop an algorithm (pseudocode) for solving the proposed mathematical model.

3. To conduct a detailed sensitivity analysis on key parameters to assess the robustness of both the algorithm and the assumption of the mathematical model.

4. To develop a discrete-event simulation model that extends the problem framework by relaxing the assumption of synchronized dispatch among heterogeneous suppliers described in Point (1).

Unlike the analytical model, which assumes that raw material replenishment is governed by the

lead time of the fastest supplier, the simulation model allows replenishment timing (d) to be endogenously determined by the system dynamics. This approach enables the evaluation of alternative supply chain configurations under more realistic, non-synchronized supplier dispatch conditions.

1.5 Research Contribution

- **Mathematical modeling of a multi-supplier, single-manufacturer system with stochastic demand, synchronized dispatch, and unsynchronized raw-material arrivals.**

The study first establishes a mathematical modelling framework for a multi-supplier, single-manufacturer system operating under stochastic demand. Although suppliers are assumed to dispatch raw materials synchronously through the manufacturer, arrivals at the manufacturer's warehouse are allowed to be unsynchronized. This asynchronization arises endogenously from differences in suppliers' proximity to the manufacturer and associated delivery times. By explicitly distinguishing dispatch decisions from arrival realizations, the model captures a realistic operational feature that is commonly overlooked in classical inventory and production models.

- **Extension from homogeneous to heterogeneous suppliers under synchronized dispatch**
Maintaining the assumptions of synchronized dispatch and unsynchronized arrivals, the model is extended by relaxing supplier homogeneity. Suppliers are allowed to differ in production capacity, reflecting realistic supply chains composed of small and medium-sized enterprises with varying operational capabilities. This extension demonstrates how supplier heterogeneity affects batch-size decisions, production rates, and replenishment timing, while preserving analytical tractability within the synchronized-dispatch framework.
- **Integrated versus Stackelberg solution strategies**

The mathematical model is solved under two alternative coordination structures. The integrated solution jointly determines all decision variables across the supply chain and serves as a centralized benchmark. In contrast, the Stackelberg framework captures decentralized decision-making, where the manufacturer acts as a leader and suppliers adjust their production strategies in response to manufacturer-imposed endogenous parameters. Comparing these solutions reveals the efficiency loss associated with decentralized control and highlights the role of power asymmetry in shaping supplier behavior and system performance.

- **Relaxation of synchronized dispatch through DES-based optimization**

To overcome the structural rigidity imposed by synchronized dispatch in analytical modelling, a discrete-event simulation (DES) optimization framework is developed. This approach relaxes the synchronized dispatch assumption while preserving unsynchronized raw-material arrivals from different suppliers. The simulation model allows dispatch decisions to occur independently and dynamically, enabling a more realistic representation of operational variability and system interactions that cannot be fully captured analytically.

- **Comparative evaluation of heterogeneous supply policies**

Two heterogeneous supply policies are evaluated. The first policy, derived from the mathematical model, features unequal batch sizes with synchronized dispatch and replenishment determined by the lead time of the fastest supplier. The second policy, obtained through simulation, imposes equal batch sizes but allows unsynchronized dispatch, with replenishment governed by a dynamic reorder-timing buffer d determined endogenously by the system. Numerical results show that the second policy frequently outperforms the first, despite its batch-size rigidity, due to its greater flexibility in replenishment timing.

From a managerial perspective, the combined contributions of this study indicate that effective supply chain performance depends not only on coordination among suppliers, but also on how replenishment decisions are timed and managed within the system. Centralized, integrated planning offers the advantage of jointly optimizing decisions across the manufacturer and suppliers, providing a structured framework for alignment. At the same time, the study shows that synchronized dispatch policies can limit flexibility when suppliers differ in location or production capacity. Extending the analysis through simulation-based optimization illustrates that allowing dispatch decisions to adjust dynamically while maintaining unsynchronized raw material arrivals provides managers with greater flexibility to accommodate variability in supply and demand. The approach of using system determined reorder timing, rather than relying solely on the fastest supplier's lead time, suggests that managerial attention should focus on adaptive replenishment strategies that support responsiveness and efficiency. Overall, these contributions provide managers with guidance on balancing coordinated planning with operational flexibility to enhance resilience, reliability, and cost effectiveness in multi-supplier manufacturing systems.

1.6 Structure of the Dissertation

The remainder of this thesis is organized as follows:

Chapter 2 presents a comprehensive review of the relevant literature, covering:

1. Supply chain models with variable production rates.
2. Systems with multiple suppliers and a single manufacturer.
3. Partnership/Collaboration in a supply chain
4. Game theoretical approaches in supply chain optimization
5. Simulation of supply chain problems.

Chapter 3 formally defines the problem and introduces a mathematical model for a system with homogeneous suppliers under the assumption that the time required to deliver raw materials may vary according to supplier.

The chapter also extends the model by relaxing this assumption of homogeneity, while retaining synchronized dispatch of orders for batches of raw materials and allowing for unsynchronized arrivals of batches at the manufacturer. Two algorithms for solving this problem are presented.

Chapter 4 presents base-case examples under both homogeneous and heterogeneous suppliers, and further carries out a detailed sensitivity analysis on key model parameters, including:

- Mean demand.
- Variability in demand
- The production rates of raw materials, relative to the manufacturer's consumption rate.
- The units of raw materials required per unit of the finished product.
- Manufacturer's unit inventory holding costs for the final product.

Chapter 5 further expands the mathematical model using DES simulation optimization to relax the assumption of homogeneous and heterogeneous suppliers discussed in Chapter 3, allowing equal batch sizes with unsynchronized dispatch and delivery.

Chapter 6 concludes the thesis by summarizing key findings and proposing directions for future research.

CHAPTER 2

LITERATURE REVIEW

Inventory modeling plays a crucial role in optimizing supply chain systems. Recently, scholars have focused on the importance of flexibility in manufacturing, particularly through adjustments in production rates, as this helps manage costs associated with inventory and lost sales. Flexibility also enhances product quality by ensuring the efficient use of production tools and equipment, while maintaining shorter, and more accurate lead times. Partnership on the other hand emphasizes strategic collaboration between stakeholders to achieve mutual benefits and long-term success. It involves shared obligations, aligned objectives, and joint investments in technology and processes to enhance efficiency, productivity, innovation, and agility across the supply chain. While flexibility and partnership are generally studied separately in the literature, this research aims to integrate both concepts. Specifically, it seeks to identify the best operational policy/strategy for stakeholders engaged in a partnership (vertical collaboration) within a flexible production system. In this context, the partners include raw material suppliers (SMEs), and the principal partner is a manufacturer that consumes the raw materials in proportion to the demand for a product. The partnership requires the SMEs to ensure timely production and delivery of raw materials according to the schedule, while the manufacturer who operates a flexible production system due to volatility in market demand, sustains the SMEs by incorporating them into its production cycle. As illustrated in Figure 2.1, the manufacturer functions as the central node in a coordinated network of SME suppliers, facilitating synchronized material flows. This configuration strengthens supply chain resilience by reducing vulnerability to production disruptions and promotes sustainability through minimized inventory waste and the sustained economic participation of SME partners.

This section provides a review of the literature related to this topic under the following categories.

I. Variable production rate

II. Multiple suppliers, single manufacturer

III. Partnership/Collaboration in a supply chain

VI. Game theoretical approaches in supply chain optimization

V. Simulation of Supply Chain Problems

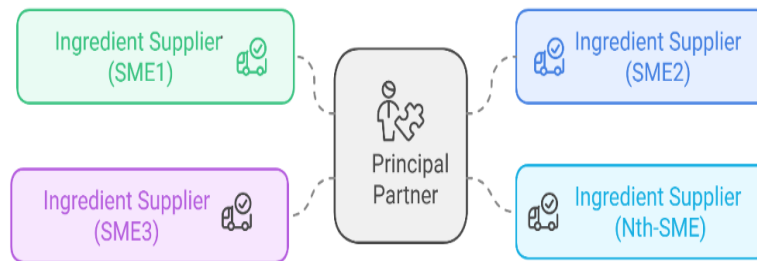


Figure 2.1: A schematic diagram portraying the vertical collaboration between the principal partner (manufacturer) and set of suppliers (SMEs).

2.1 Variable Production Rate

Buzacott and Ozkaran (1983) studied the problem of production scheduling on machines through a variable production rate that can be adjusted when the system is idle. The authors proved that optimality is achieved when the selected production rate necessitates full utilization of the machines. Arizono et al. (1989) used minimum lattice paths to investigate the effects of varying production rates on inventory control. Silver (1990) extended the work of Buzacott and Ozkaran (1983) by studying the production of a group of items in a single facility. They assumed that production occurs in cycles, and that the production rate can be varied within defined limits. Sarker and Babu (1993) showed how the production rate could affect the shelf life of perishable items. The authors suggested that it would be beneficial to reduce either the length of the production cycle or the production rate, to limit the time spent by goods on the shelf.

Khouja (1994) relaxed the assumption of a fixed production rate in the classical economic production quantity (EPQ) model. A variable production rate enhances the flexibility of a

manufacturing system under managerial control. Khouja and Mehrez (1994) extended the work of Khouja (1994) by linking the quality of the production process with the production rate. The authors developed a model that was solved under the assumption that the function describing the costs of producing a unit is convex in the production rate. Their results showed that in situations where an increase in the production rate causes a significant reduction in quality, the optimal production rate may be lower than the production rate that minimizes the unit production cost. Similarly, when quality is independent of the production rate, the optimal production rate may be larger than the rate that minimizes the unit production cost.

Khouja (1999) relaxes the assumption of perfect production adopted in Khouja (1994) by making the production process shift out of control with a probability that depends on the production rate. This reduces the optimal cycle time and lot size. Also, Eiamkanchanalai and Barnerjee (1999) developed a model that determines concurrently the optimal length of a production run and production rate for a single item. Giri et al. (2005) studied a manufacturing problem in which the production rate affects the stress level and failure rate of the production facility. The unit production cost was expressed as a function of the production rate, and the classical model of the economic manufacturing quantity was extended by introducing general distributions for failure and repair times. Larsen (2005) proposed an EPQ model that considered the production rate and corresponding run time as decision variables. Moon (2005) used a continuous inventory model to consider lead time reduction by allowing the manufacturer to alter the production rate at an additional cost. Quyang et al. (2008) proposed an integrated inventory model with a price sensitive demand rate and the production rate as a decision variable. The authors increased profit through trade credit and freight rate policies according to the quantities ordered. Glock (2010) studied the case of equal and unequal batch shipments in a single vendor, single buyer system. The author assumed that each batch is consumed at discrete time intervals and the production rate was varied to investigate its impact on costs and inventory build-up. Aldurgam and Duffuaa (2013)

considered maximizing the overall effectiveness of a system by choosing the optimal process rate and maintenance schedule.

Singh and Sharma (2013) defined the production rate as a function of the demand rate to examine the impact of inflation and the current value of money on a supply chain system. The authors reduce supply chain costs through reducing the holding costs of the manufacturer who receives raw materials in small quantities and supplies the product in small lots to the buyer. Bouslah et al. (2013) used the production rate as a control variable in an imperfect production system where the production rate does not influence the rate of defects or the probability of a breakdown of the facility. Zanoni et al. (2014) reduced the cost of energy consumption through a production rate that may be varied within specified limits in a multistage manufacturing system. Huang et al. (2015) developed a model that chooses and assigns a customer's order to one of two available facilities, while determining the production rate that minimizes the expected total cost.

La Fata and Passannanti (2016) proposed a model for the combined optimization of production and preventive maintenance policies. This model assumes a continuous production system with random deterioration and the availability of buffers ensures a continuous supply of products, while preventive maintenance is being performed. Like Bouslah et al. (2013), Cheng et al. (2016) developed a model to minimize the expected cost per unit time through the numbers of tools replaced, production rate and a maintenance strategy under a known demand. The production rate is assumed to impact the lifetime of a tool and the reliability of the system. Marchi *et al.* (2016) assumed that the production rate of the supplier may be increased through investment by the buyer, which may or may not be successful. Intuitively, the higher the level of investment risk, the lower the effort put in place by the buyer to improve the production process of its supplier.

Aldurgam et al. (2017), considered production rate as a decision variable for adjusting to randomness in demand for a single vendor, single manufacturer supply chain system, where the

manufacturer receives raw materials in multiples of full truck loads. Jauhari (2018) concluded that the rate of defects and total cost can be lowered through capital investment by the vendor and an adjustable production rate. Kim et al. (2018) incorporated the choice of a production machine through a variable production rate when production capacity outweighs demand for the product. El-Baz et al. (2019) formulated a coordination policy for an instantaneously adaptable production rate that changes with continuous variation in product prices in procurement, production, and delivery settings, such that the expected total cost is minimized. Sarkar et al. (2019) addressed the higher cost incurred for a flexible manufacturing system through discrete investment. Total cost is reduced through an optimal flexible production rate that shortens the lead time and thus reduces the expected demand over this period. Chiu et al. (2019) proposed a model that determines the optimal common cycle time through an adjustable production rate for a single multi-item system, to meet a continuously increasing demand. Ben Daya et al. (2019) considered the remanufacturing of economically viable products recovered from end customers. The authors showed that producing larger lots per set up gives economies of scale when the vendor's set up costs increase relative to the buyer's ordering cost. Ashgar et al. (2020) developed a model to improve manufacturing quality and reduce the unit production cost for an automated smart manufacturing system with a controllable production rate. Alkathani et al. (2021) considered uncertainties created by the covid-19 pandemic through a production plan developed from the optimal utilization of resources and a controllable production rate. Aldurgam et al. (2023) control production rate via manpower planning with a flexible workforce in a single vendor, single manufacturer supplier chain system.

2.2 Multiple Suppliers, Single Manufacturer

We now further explore the research work on inventory models with multiple suppliers and a single manufacturer. It can be observed that some work has been done in this area under the assumption of a known demand rate. Kim et al. (2009) studied the impact of transportation costs on the joint lot-sizing

policy in a supply chain with numerous suppliers and a single buyer. Successive coordination where deliveries are from many heterogeneous suppliers was considered by Glock (2011). Braglia et al. (2011) improved Glock's (2011) algorithm. Kamali et al. (2011) studied quantity discounts for vendors to attract buyers. Kamali et al. (2016) increase the purchase value by considering both recently delivered and defective goods. Two different shipment policies to coordinate the consumption and production cycle of both the suppliers and manufacturers were investigated by Beck et al. (2017). Kumar (2017) studied the impact of product shortages on the manufacturer's demand. Zahran and Jaber (2017) studied four possibilities (consignment stock, traditional policy, and two combinations of these approaches) for coordination in a three-level supply chain system with numerous suppliers, a manufacturer, and numerous buyers. Supplier selection and order allocation were jointly investigated by Hedian et al. (2018). Huynh and Pisal (2020) proposed a vendor managed inventory (VMI) model that allows suppliers to take decisions independently of manufacturers.

2.3 Collaboration in Supply Chain Systems

Yu et al. (2002) investigated the advantages of information sharing in a partnership based decentralized supply chain system. Khaji and Shafaei (2011) proposed a system dynamic simulation model to link both supply chain coordination and strategic partnership. The impact of collaborative supply chain practices on firms' achievements was investigated through empirical analysis by Cao and Zhang (2011). Baihaqi and Sohal (2013) used structural equation modelling to examine the relationship between organization performance and the extent of information sharing in a partnership system. A smart approach for selecting partners in agile supply chain system using fuzzy set theory and an artificial neural network was proposed by Wu and Barnes (2014). A two-way integrated model that aids partner selection was proposed by Rezaei (2015). Mandal (2016) studied the influence of trust and commitment on the capacity for logistics integration. Mofokeng and Chinomona (2019) studied the extent to which integration, partnership and collaboration jointly influence the performance of a supply chain system. A four-stage optimization

model for setting up criteria for selecting partners in a sustainable supply chain was proposed by Wu et al. (2020). Mahmud et al. (2021) developed a framework to analyze barriers affecting supply chain collaboration among SMEs in Bangladesh.

2.4 Game Theoretical Approaches in Supply Chain Optimization

Game theoretic techniques like the standard Nash equilibrium strategies (wherein players do not form a hierarchical structure) and Stackelberg solutions (players form a hierarchical chain) have been regularly used in the literature to reduce costs in a supply chain system, particularly in decentralized systems where multiple partners with conflicting objectives interact. These methods have aided the formulation and implementation of strategies that consider both the competitive and cooperative behaviors of participants within such a supply chain.

Xiao (2003) considered a supply chain where the seller takes the lead and determines the minimum replenishment period and the quantity discount policy. The buyer, however, regulates the optimal stock policies. Leng and Parlar (2005) performed a comprehensive review of existing literature on the use of game theory in supply chain management. Incentives to entice suppliers in a Stackelberg leader-follower game were considered by Jia (2009). Zhang and Liu (2013) showed the advantages of cooperative decision making in sustainable supply chains for both the system as a whole and the individual stakeholders. The performance of a decentralized supply chain with a vendor managed inventory and the need for an appropriate contract prior to implementation was investigated by Golmohammadi (2016). Hjalila et al. (2017) showed that integrating the Nash equilibrium approach and Stackelberg games in a model of a multi-enterprise supply chain reduces uncertainty and achieves higher payoffs. Feng et al. (2018) suggested a new optimization approach for improving the global equilibrium in an integrated production-distribution-construction system using the theory of Stackelberg games. Kaviyani et al. (2020) proposed a dual-level Stackelberg model designed to optimize production, inventory levels, and pricing in supply chains, regardless of the agile capabilities of the participants. Costa and Lourenco

(2022) combined Stackelberg's approach and factor scaling to improve the characteristics of product and process quality with minimal computational requirements and limited subjective information. A supply chain financing model to minimize the risk incurred by SMEs at project level was proposed by Zhang et al. (2023) based on the Stackelberg approach and deep learning.

2.5 Simulation of Supply Chain Problems

Chang and Makatsoris (2001) gave an in-depth discussion about potential difficulties that could make supply chain integration necessary. The authors further discussed discrete event simulation as a tool for supply chain modelling, and they provided suggestions for the techniques and data needed to model supply chain systems. A two-echelon supply chain with a single warehouse and several retailers was studied by Trkman and Groznik (2006). Chan and Chan (2006) considered quantity flexibility in a distributed constraint satisfaction problem. Trkman and Groznik (2006) evaluated a petroleum company's performance and procurement process using a simulation model. Different costs were evaluated, including those related to process execution, inventory management, changes in the lead time, and process quality. Jie and Cong (2009) developed a simulation model that optimizes a multi-echelon supply chain under the assumption that customer demand and lead times were both unknown. The model particularly focused on how frequently retailers restocked their inventory and the average inventory level maintained in the supply chain. The simulation model gave better results compared with the analytical solution based on an analogous mathematical model. The effect of different capacity levels on supply chain costs and performance was studied by Cannella et al. (2008). Benko (2010) used Arena software to create a simulation model that mimics customer-oriented problems in a supply chain system. It is assumed that a specific product is delivered via a facility to the manufacturer's warehouse. The stock of raw materials feeds the production process, and customers consume the completed product. It is further assumed that customer demand is distributed uniformly, and to guarantee that the production process is not idle, it is assumed that raw materials are constantly available in storage. Using Arena 7.0, Patil et al.

(2011) created a simulation model for a multi-echelon supply chain system. This model considers several retailers, a distribution hub, and customers that represent the ultimate consumers. The goal of this model is to increase sales by leveraging both mutual and contingent relationships between retailers to improve customer service. The impact of disruption caused by stockout at the retailer in a supply chain network was studied by Wu et al. (2012). Jaegler et al. (2012) studied the effect of CO₂ emissions in a supply chain system. Mishra et al. (2012) considered a model of a supply chain that can accommodate multiple supply chain initiatives concurrently. The benefits of an IT enabled supply chain to operational performance and the level of customer service were studied by Cannella (2014). Hoshyar et al. (2014) used simulation optimization to evaluate two inventory management policies (based on a traditional inventory model and a vendor managed inventory model). The results from the study show that the vendor managed inventory strategy had a 5% advantage according to system efficiency and a 50% advantage according to inventory or stock level. Bottani et al. (2014) optimizes different inventory policies for perishable food products. Grewal et al. (2015) use discrete-event simulation to develop a dynamic reorder point policy, demonstrating that adaptive replenishment timing outperforms static rules in a capacitated supply chain. Sahin et al. (2017) study the use of cloud computing and a radio frequency identification system in improving the operational efficiency of a construction supply chain. Ivanov (2018) studied the propagation of disruptions in a supply chain. Wiśniewski (2018) developed a simulation model to manage the inventory of online stores. Siswanto et al. (2018) study possible disruptions and recovery methods in maritime transportation. The importance of collaborative logistics through information sharing in a supply chain was studied by Yuan et al. (2019). Ivanov (2019) showed the benefits of contingent recovery policies during a period of disruption and of a revival policy in the post-disruption period. Lohmer et al. (2020) studied the propagation of disruptions in a block chain coordinated supply chain. The usefulness of the Internet of Things (IoT) when assessing risk in a supply chain was studied by Gao et al. (2020). Aljanabi (2021) suggested an approach to integrate supply chain and market

responsiveness parameters. Yin (2021) studied the connection between market competition and the bullwhip effect (a phenomenon in supply chains where small changes in consumer demand cause exaggerated or opposite adjustments in orders and inventory upstream) experienced when multiple retailers are supplied by a single supplier. The impact of additive manufacturing on various supply chain settings was studied by Rinaldi et al. (2021). Huang et al. (2021) studied the effect of the covid-19 pandemic on food disruption and sustainability. Tsiamas and Rahimfard (2021) proposed a framework and simulation-based decision support to improve the resilience of a food supply chain against several unexpected disruptions. Binsfeld and Gerlach (2022) offered a method to quantify the benefits of digital supply chain twins (DSCT). Kumar et al. (2022) study the carbon footprint due to supply chain uncertainties through emissions from warehousing and transportation. Wen and Diao (2022) used a system dynamics model to simulate different carbon emission scenarios. Sarfaraz (2023) proposed a point-of-authority consensus algorithm to improve trust among stakeholders in a decentralized block chain coordinated supply chain. Adegbola (2023) compared a VMI policy and a joint reorder policy (JRP) for a single vendor, multiple manufacturer supply chain. The effect of the volume of returns among various reverse flows in a closed-loop supply chain, where each member may have its own reverse flow, was studied by Fussone et al. (2023).

2.6 Chapter Summary

So far, the reviewed literature has highlighted significant advancements in models of an integrated supply chain, particularly in the areas of variable production rates, supplier-manufacturer coordination, and game-theoretic approaches. However, despite extensive research, most studies assume fixed values for key parameters such as the demand rate, production rate, and batch size. Additionally, deliveries from suppliers are often modeled as intermittent or cyclic (rotational), without considering real-time flexibility in response to fluctuations in demand. A notable gap exists in the analysis of multiple independent suppliers collaboratively adjusting their production rates to meet the dynamic demand of a single manufacturer.

Previous studies, such as Aldurgam et al. (2017), which is the work most closely related to the development of the model presented here, primarily focused on the manufacturer's production flexibility, while assuming that suppliers operated under predetermined production rates and fixed batch sizes. This simplification limits the practical applicability of such supply chain models in real-world scenarios where demand is inherently stochastic.

To address this gap, the present research develops an integrated supply chain model involving multiple suppliers (SMEs) and a single manufacturer under stochastic demand. The study emphasizes vertical collaboration, wherein SMEs provide essential products and services to a larger manufacturing company. In return, the manufacturer integrates these SMEs into its production cycle, fostering a mutually beneficial relationship that enhances supply chain resilience, optimizes inventory management, and mitigates risks such as stockouts and excess inventory. A key contribution of this research is the incorporation of flexible production rates for both manufacturers and suppliers. This flexibility enables adapting the production rate to the variability of demand, ensuring better coordination and efficiency across the supply chain. Additionally, batch size is treated as a variable to be optimized rather than a fixed parameter, and delivery times depend on the distance from each supplier to the manufacturer's warehouse. This refined approach enables a more realistic analysis of supplier-manufacturer interactions, offering insights into strategies that enhance the overall performance of a supply chain in unpredictable market conditions.

CHAPTER 3

FORMULATION OF THE MATHEMATICAL MODEL

3.1 Development of the Analytical Model

This chapter aims to develop a mathematical model for the inventory profile depicted in Figure 1.1, focusing on two distinct supply chain scenarios: one with homogeneous suppliers and the other with heterogeneous suppliers. In the first scenario, where suppliers are homogeneous, it is assumed that all of the suppliers have identical production capacities and set-up requirements. As a result, their batches are ready for dispatch at the same time. However, despite this simultaneous or synchronized dispatch, the delivery times to the manufacturer's warehouse differ. This is because, although the suppliers are homogeneous in terms of production capabilities, they are located at different distances from the manufacturer in terms of the time required to deliver.

The second scenario involves heterogeneous suppliers, each differing in production capacity, setup time, and other operational characteristics. As a result, synchronizing the dispatch of equal batch sizes of raw materials from independent suppliers may be difficult to achieve without disrupting the manufacturer's production plan and schedule. Therefore, given that suppliers are assumed to operate independently of one another while the manufacturer's ordering of raw materials is synchronized, each supplier is expected to deliver batches of a size that is feasible at the agreed (synchronized) dispatch times coordinated by the manufacturer. However, just like in the homogeneous scenario, the delivery times will vary because the suppliers are located at different distances from the warehouse.

In both cases, homogeneous or heterogeneous, the varying delivery times resulting from differences in the proximity of the suppliers to the manufacturer introduce challenges in managing inventory **levels** and production schedules. These differences must be carefully considered in the mathematical model to ensure effective inventory management, reduce the risk of stock outs, and improve the overall efficiency of the

supply chain. By incorporating these variations into the model, we aim to create a more accurate and optimized solution for supply chain operations.

3.1.1 Definition of the problem

A large-scale manufacturer sources its primary raw material, corn starch, from a group of n suppliers (SMEs), each operating under a make-to-order production policy. These suppliers produce corn starch in bulk, packing it into 3 kg units, which are then delivered in (integer) multiples of a standardized load, e.g. truckloads. The manufacturer, in turn, follows a make-to-stock production strategy, producing pre-packaged corn chips, where 3 units of 3 kg corn starch are required to produce 1 unit of 10 kg chips. The finished chips are stored in warehouses to ensure the product's availability and meet customer demand. Given the unpredictable nature of demand, the manufacturer relies on a collaborative model with these suppliers to maintain a steady and responsive supply chain. For example, each 100 kg batch of corn chips (10 units of 10 kg each) requires exactly 30 units of 3 kg corn starch (3 units per 10 kg chips), along with other ingredients, and production is adjusted accordingly based on fluctuations in demand. The suppliers, though homogeneous, cannot individually fulfill the manufacturer's demand for raw materials within a given time frame. However, when combined, they can meet this demand.

To preserve this collaboration, the manufacturer bears all direct and indirect costs, ensuring timely procurement and efficient delivery schedules. By coordinating the packing and delivery of corn starch in scheduled batches, the suppliers help streamline production, optimize raw material usage, and reduce storage inefficiencies. The overall goal is to determine the rates of production of both the manufacturer and the suppliers during a production cycle, the batch size, the number of deliveries from each supplier per cycle, and the reorder point that minimize the total costs of the supply chain per unit of time. This integrated approach not only enhances the supply chain's resilience but also guarantees consistent availability of the final product, benefiting both the manufacturer and its SME partners.

3.1.2 Definition of terms

Before moving forward with the development of the model, it is essential to first define several key terms that are crucial for formulating the total cost function in the supply chain problem being discussed.

- **Holding/Inventory Carrying Costs:** These are the expenses associated with maintaining inventory or assets over a certain period. They include various costs incurred while holding or storing inventory or assets, such as warehousing expenses, insurance premiums, depreciation, obsolescence, and opportunity costs related to tying up capital in inventory rather than investing it elsewhere.
- **Transportation Costs:** The costs of transportation pertain to the monetary outlay of a business to move goods, materials, or people from one location to another. They cover a range of costs related to the transportation process, including fuel costs, vehicle maintenance and depreciation, labor costs for drivers and support staff, insurance premiums, taxes, tolls, and fees charged by carriers or transportation service providers.
- **Set-up Costs:** These refer to the expenses incurred when setting up equipment in readiness to process a new batch. They include costs associated with cleaning equipment, reconfiguring machinery, changing tools or dies, adjusting settings, testing, and calibrating processes.
- **Shortage Costs:** These are otherwise called stockout costs or stock shortage costs and refer to the expenses and negative consequences incurred when a business experiences a shortage or absence of inventory or stock to meet customer demand. Shortage costs typically result from lost sales opportunities, dissatisfied customers, and damage to the company's reputation.
- **Costs of a Purchase Order:** These are related to the authorizing documents required for the provision of goods or services from a manufacturer/client. Orders become a legally binding contract on acceptance by the supplier.
- **Order Lead Time:** The period between placing an order and receiving the item from a supplier.
- **Re-order point:** This signifies a predefined inventory level of the finished good at which a new order is

placed to replenish the consumed stocks of the finished goods.

- Back Order: The part of an order that cannot be delivered at the scheduled date, but will be delivered later.
- Safety Stock: The (variable) surplus inventory held to protect against randomness in demand. It is needed to cover the demand during the replenishment lead time in case the actual demand is greater than the expected demand.
- Production rate is the amount of goods or services produced in a specified unit of time, such as per hour, per day, per week, or per month.

3.2 Notation for the Basic Model with Homogeneous Suppliers

The following tables summarize the key components of the decision-making framework used in this study. Table 3.1 lists the exogenous parameters, which represent fixed inputs and environmental conditions beyond the control of the decision maker. Table 3.2 presents the functions associated with the decision problem, defining how various elements interact mathematically or operationally. Table 3.3 outlines the decision variables, or endogenous parameters, which are the controllable elements that drive the outcomes of the system. Finally, Table 3.4 illustrates the endogenous parameters resulting from the interaction between the fixed parameters and the decision variables, capturing the system's behavior and response to different decision scenarios. Collectively, these tables provide a structured overview of the model's components and their interrelationships, forming the foundation for subsequent analysis and optimization.

Fixed Parameters

As_i	<i>Supplier i's set up cost</i>
A_m	<i>Manufacturer's set up cost for production</i>
d_i	<i>Time required for a delivery from supplier i</i>
Z	<i>Constant cost of each order made to a supplier by the manufacturer</i>
K	<i>Fixed cost component related to transportation: clearance, licence, and other documentation</i>
K_r	<i>Fixed cost component linked to a unit of raw material: loading, offloading</i>
V	<i>Transport cost per unit of raw material per unit time</i>
β	<i>Power term in the production cost function for the finished goods</i>

b	<i>Coefficient of the power term in the production cost function for the finished goods</i>
g	<i>Direct production cost parameter for the finished goods per unit of time</i>
β_1	<i>Power term in the production cost function for raw materials</i>
b_1	<i>Coefficient of the power term in the production cost function for raw materials</i>
g_1	<i>Direct production cost parameter for raw materials per unit of time</i>
I	<i>Capacity of the finished goods warehouse</i>
T_{smin}	<i>Minimum batch size</i>
T_{smax}	<i>Maximum batch size</i>
$E[\Lambda]$	<i>Mean demand per unit time</i>
σ_A	<i>Variability in demand per unit time</i>
$h_{r,m}$	<i>Inventory holding cost for a unit of the raw material at the manufacturer's warehouse</i>
h_s	<i>Inventory holding cost for a unit of the raw material in the supplier's warehouse</i>
h_m	<i>Inventory holding cost for a unit of the finished product at the manufacturer's finished goods store</i>
n	<i>Number of suppliers</i>
P_m	<i>Maximum production rate of the manufacturer</i>
P_{ms}	<i>Maximum production rate of a supplier (homogeneous suppliers)</i>
α	<i>Raw material usage per unit of final product</i>
$\lambda_m = \frac{P_{ms}}{\alpha P_m}$	<i>Maximum production rate of a supplier relative to the maximum raw material consumption rate of the manufacturer (homogeneous suppliers)</i>
P_{msi}	<i>Maximum production rate of supplier i (heterogeneous suppliers)</i>
$\lambda_{mi} = \frac{P_{msi}}{\alpha P_m}$	<i>Maximum production rate of supplier i relative to the maximum raw material consumption rate of the manufacturer (heterogeneous suppliers)</i>
π	<i>Manufacturer's penalty per unit shortage</i>

Table 3.1: Exogenous Parameters of the Model

Functions associated with the decision problem

V_{ns}	<i>Average total set up cost spent by the n suppliers per unit time</i>
S_c	<i>The average cost incurred due to shortages or lost sales per unit time</i>
O_r	<i>Manufacturer's ordering cost per unit time</i>
$f(y), F(y)$	<i>Density function and cumulative distribution function of the lead time demand</i>
S_{PC}	<i>Supplier's production cost per unit time</i>
TC_{SC}	<i>Long-run average cost of the integrated supply chain system</i>
TC_s	<i>The average cost incurred by the suppliers per unit time</i>
TC_m	<i>Long-run average cost incurred by the manufacturer per unit time</i>

TC_r	<i>The average cost of ordering, transporting, and holding raw materials incurred by the manufacturer per unit time</i>
T_{co}	<i>Manufacturer's transportation cost per unit time</i>
M_{SC}	<i>Manufacturer's set up cost for production per unit time</i>
S_{PC}	<i>Supplier's production cost per unit time</i>
DPC	<i>Manufacturer's direct production cost per unit time</i>
H_{ns}	<i>Average holding costs incurred by the n suppliers per unit time</i>
H_{rm}	<i>Average cost of holding the raw material in the manufacturer's warehouse per unit time</i>
H_{fp}	<i>Average cost of holding the final products by the manufacturer per unit time</i>

Table 3.2: Functions associated with the decision problem

Decision Variables

R	<i>Manufacturer re-order level to trigger the production and delivery of raw materials</i>
P	<i>Manufacturer's production rate per unit of time</i>
T_l	<i>Size of a delivery batch from the suppliers</i>
m	<i>Number of deliveries made by the suppliers in a production cycle</i>
P_s	<i>Supplier's production rate (homogeneous suppliers)</i>
P_{si}	<i>Supplier i's production rate (heterogeneous suppliers)</i>

Table 3.3: Decision Variables (Endogenous parameters)

Variables resulting from the relation between the decision variables and fixed parameters

Q	<i>Manufacturer's production lot size</i>
$E[Y]$	<i>Average demand over the lead time</i>
σ_Y	<i>Variability in lead time demand</i>
S	<i>A random variable denoting safety stock</i>
s	$s = E[S]$
τ	<i>Manufacturer's lead time for a production cycle</i>
T	<i>Length of production cycle</i>
λ	<i>Production rate of a supplier relative to the consumption rate of the manufacturer $\lambda = \frac{P_s}{\alpha P}$ (homogeneous suppliers)</i>

λ_i	Production rate of a supplier relative to the consumption rate of the manufacturer $\lambda_i = \frac{P_{si}}{\alpha P}$
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Table 3.4: Endogenous parameters resulting from the interaction between the fixed parameters and the decision variables

3.2.1 Assumptions of the basic model with homogeneous suppliers

The following assumptions were adopted in formulating the model.

1. The rate at which a manufacturer consumes raw materials is directly proportional to the rate at which the final good is produced i.e., the manufacturer's consumption rate = αP (Aldurgam et al. 2017).
2. Each supplier's maximum production rate is $\alpha \lambda_m$ times the manufacturer's maximum production rate (P_m) for the final product, i.e., λ_m times the manufacturer's maximum consumption rate of the raw material (αP_m). It is assumed that $n \lambda_m \geq 1$, i.e. acting together, the suppliers can produce raw materials at the maximum consumption rate of the manufacturer. In addition, the manufacturer's maximum demand rate for the raw material (αP_m) exceeds the production capacity of each individual supplier ($\lambda_m \alpha P_m$) when operating at full capacity.
3. To avoid production shut down by the manufacturer due to shortages in the raw material shortages, the total actual production rate of all the suppliers exceeds the manufacturer's actual consumption rate, i.e. $n \alpha P_s \geq P$, or equivalently $n \lambda \geq 1$. It should be noted that since λ is measured relative to the actual production rate of the manufacturer, it is possible that $\lambda > \lambda_m$ as long as the condition $P_s \leq P_{ms}$ is satisfied.
4. Once the quantities of finished goods on the shelf reach the reorder level, the manufacturer starts a new production cycle by placing orders for the primary raw material. The rate of producing the raw materials is adapted to the consumption rate of raw materials in the production process. Production begins as soon as the first shipment from the nearest supplier is received.

5. In each production cycle there are m delivery cycles, where m is a decision variable. Each supplier makes one delivery in each delivery cycle. Each shipment of the raw material made by a supplier is of size T_l , which is a decision variable and assumed to be an integer number of units of the raw material.
6. Suppliers are completely homogeneous except for the time required to make a delivery. The time required for the delivery of a batch from supplier i is d_i . The suppliers are ordered so that $d_1 \leq d_2 \leq \dots \leq d_n$. The n deliveries in a cycle are all dispatched from the suppliers at the same time. Hence, the delivery from supplier i arrives $(d_i - d_1)$ units of time after the corresponding delivery from Supplier 1. When a production cycle is initiated, each supplier immediately begins production and dispatches the first batch as soon as the required amount is produced.
7. The production rate and delivery schedule of each supplier is set to ensure that a) the time between deliveries from a given supplier within a production cycle is constant. The first delivery is made $\frac{T_l}{\lambda \alpha P} + d_i$ units of time after the manufacturer initiates a cycle. b) Each batch of raw material from the supplier nearest to the manufacturer is delivered in a manner which ensures that the manufacturer does not experience a stockout (see Fig. 1).
8. All shortages are backordered.
9. Only one production run is outstanding at any given time, and the average demand rate remains constant over an infinite time horizon (Darwish et al., 2013).
10. Demand is assumed to follow a stochastic process with independent increments, expected rate $E[\Lambda]$ and variance per unit time σ_A^2 . Hence, given that for reasonably large t , the demand over a period of length t has approximately a normal distribution with mean $tE[\Lambda]$ and variance $t\sigma_A^2$.
11. The manufacturer's and suppliers' rates of production are determined before the production run, see Silver (1990), Buzacott and Ozkarahan (1980), Saka and Babu (1993), Goyal (1994), Viswanathan (1995), Silver (1995) and Aldurgam et al. (2017).

12. Suppliers are autonomous to each other in the production of raw materials and the time of delivery to the manufacturer's warehouse.
13. The costs of production per unit of the final product are formulated as a production rate function using $F(P) = \frac{g}{p} + bP^\beta$, where $g, b, \beta > 0$. See Khouja (1994) and Aldurgam et al. (2017). The corresponding costs per unit of time are $g + bP^{1+\beta}$.
14. The costs of production per unit of the raw material are formulated as a production rate function using $F(P) = \frac{g_1}{p} + b_1P^{\beta_1}$, where $g_1, b_1, \beta_1 > 0$. The corresponding costs per unit of time are $g_1 + b_1P^{1+\beta_1}$.
15. The aim is to minimize the costs per unit time of such an integrated supply chain.
16. The manufacturer is assumed to bear the transportation costs.

3.2.2 Derivation of the average cost per unit time for the basic model with homogeneous suppliers

We now derive the mean cost per unit time for the supply chain system described using a continuous review (Q, R) inventory policy. The conventional continuous-review (Q, R) inventory policy employed in this study is formulated strictly within a non-preemptive operating environment, in which replenishment cycles do not overlap and all orders are assumed to arrive within the same production cycle in which they are placed. This assumption aligns with the classical premises of continuous-review inventory control, where the timing and execution of orders remain sequential and non-interfering.

However, a pre-emptive setting may arise when the lead time exceeds the cycle time, resulting in overlapping cycles and the possibility that orders may be placed while a production cycle is still active, but arrive in the following cycle. Under such conditions, the system departs from the structural assumptions of the conventional (Q, R) framework. Pre-emptive control introduces interdependencies across cycles that fundamentally alter the timing, feasibility, and cost structure of inventory decisions (Stolyar & Wang, 2018). Consequently, the traditional (Q, R) policy cannot adequately represent or solve such dynamics

without substantial modifications to the underlying modelling architecture. For this reason, pre-emptive scenarios are excluded from the present study.

Re-establishing the analysis within the non-preemptive domain, this research investigates the operational behavior of a vertically collaborative supply chain under stochastic market demand. The overarching objective is to determine the optimal means of coordinating a partnership featuring a flexible production system driven by random fluctuations in demand. In this configuration, the manufacturer serves as the principal decision-maker and is engaged in vertical collaboration with n suppliers responsible for producing a primary raw material required for the manufacturer's production of finished-goods. The suppliers' principal objective is to remain operationally viable despite challenging economic constraints, while the manufacturer, who bears all direct and indirect costs, benefits from timely delivery of materials and the cost savings associated with avoiding an excessive raw-material inventory before, during, or after production. The various costs that are associated with the model are presented below. Note that the long-run average production rate of the manufacturer under such a policy is $E[\lambda]$. Hence, the mean length of a production cycle is given by $\frac{Q}{E[\lambda]}$. Since the amount of raw materials delivered in a cycle is mnT_l and α units of the raw material are required to produce one unit of the finished good, we have $Q = \frac{mnT_l}{\alpha}$.

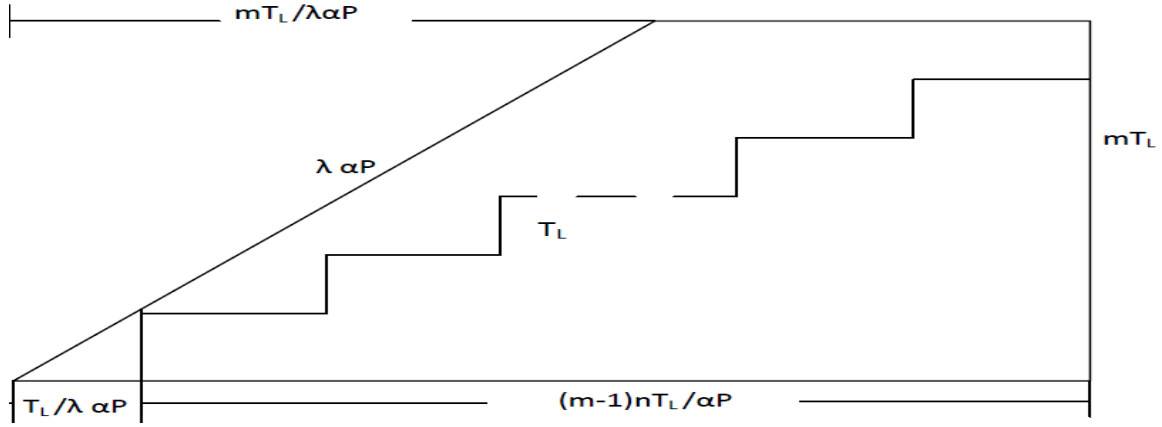


Figure 3.1: The profile of supplier i 's inventory when m shipments are made to the manufacturer's warehouse per production cycle.

1. Mean supplier set up cost per unit time: This is the long run average cost per unit time incurred by the suppliers in getting the production equipment ready for raw material production. It can be calculated for a supplier as $\frac{A_s}{T}$ and thus the total average cost for n independent and identical suppliers is given by

$$V_{ns} = \frac{nA_s E[\lambda]}{Q} \quad (3.1)$$

2. Supplier's holding cost per unit time: This is the long-run average cost of holding raw materials in stock for a given period. It varies directly with both the amount held and the period of holding the stock. The inventory profiles of the suppliers are similar because the suppliers are assumed to be homogeneous. However, they have different delivery times to the manufacturer's warehouse as shown in Figure 3.2. The accumulated inventory in each supplier's warehouse is computed by deducting the amount of raw materials dispatched to the manufacturer over time (stairs shape) from the total raw material produced and initially held in the supplier's warehouse. For one supplier, the holding cost per unit time is determined by calculating the area of the trapezium shown in Figure 3.1 as follows.

Length of the lower boundary of the trapezium

$$= \frac{T_l}{\lambda \alpha P} + \frac{mnT_l}{\alpha P} - \frac{nT_l}{\alpha P}$$

Length of the upper boundary of the trapezium

$$= \frac{T_l}{\lambda\alpha P} + \frac{mnT_l}{\alpha P} - \frac{nT_l}{\alpha P} - \frac{mT_l}{\lambda\alpha P} = \frac{T_l}{\alpha P} \left(\frac{1}{\lambda} - \frac{m}{\lambda} + nm - n \right)$$

Since the height of the trapezium is mT_l , its area is given by

$$\begin{aligned} &= \frac{1}{2} \left(\left(\frac{T_l}{\lambda\alpha P} + \frac{mnT_l}{\alpha P} - \frac{nT_l}{\alpha P} \right) + \frac{T_l}{\alpha P} \left(\frac{1}{\lambda} - \frac{m}{\lambda} + nm - n \right) \right) mT_l \\ &= \frac{mT_l^2}{\lambda\alpha P} + \frac{m^2nT_l^2}{\alpha P} - \frac{mnT_l^2}{\alpha P} - \frac{m^2T_l^2}{2\lambda\alpha P} \end{aligned}$$

For the staircase region in Figure 3.1, the mean height is $\frac{T_l + (m-1)T_l}{2}$ and the length is $\frac{(m-1)nT_l}{\alpha P}$. The area is the product of the mean height and the length

$$= \frac{m^2nT_l^2}{2\alpha P} - \frac{mnT_l^2}{2\alpha P}$$

The holding costs are proportional to the area left after removing the stair shape from the trapezium, i.e.

$$= \frac{mT_l^2}{2\alpha P} \left(\frac{2-m}{\lambda} + n(m-1) \right)$$

Thus, the holding cost per unit time for one supplier is

$$\frac{mT_l^2 h_s E[\Lambda]}{2\alpha P Q} \left[\frac{2-m}{\lambda} + n(m-1) \right]$$

and for n suppliers,

$$H_{ns} = \frac{mnT_l^2 h_s E[\Lambda]}{2\alpha P Q} \left[\frac{2-m}{\lambda} + n(m-1) \right] \quad (3.2)$$

3. Mean production cost incurred by the suppliers per unit time: This is the direct cost incurred by each supplier in producing the raw material per unit time. The cost of production per unit time is composed of a fixed cost plus a variable cost, which is both increasing and convex in the production rate.

The supplier production cost function per unit of the raw material is $\left[\frac{g_1}{\lambda\alpha P} + b_1(\lambda\alpha P)^{\beta_1} \right]$, Khouja (1994), Aldurgam et al. (2017). For each supplier, the production cost per unit time for the m batches shipped to the manufacturer is

$$\left[\frac{g_1}{\lambda \alpha P} + b_1 (\lambda \alpha P)^{\beta_1} \right] \frac{\alpha E[\Lambda]}{n}.$$

Therefore, for n independent, identical suppliers, the production cost per unit time is

$$S_{PC} = \left[\frac{g_1}{\lambda \alpha P} + b_1 (\lambda \alpha P)^{\beta_1} \right] \alpha E[\Lambda]. \quad (3.3)$$

The total cost incurred by the n homogeneous suppliers per unit time is

$$TC_s = V_{ns} + H_{ns} + S_{PC}$$

$$TC_s(Q, P, T_l) = \frac{nA_s E[\Lambda]}{Q} + \frac{mnT_l^2 h_s E[\Lambda]}{2\alpha PQ} \left[\frac{2-m}{\lambda} + n(m-1) \right] + \left[\frac{g_1}{\lambda \alpha P} + b_1 (\lambda \alpha P)^{\beta_1} \right] \alpha E[\Lambda] \quad (3.4)$$

4. Mean costs of ordering raw materials per unit time – These costs are the expenses incurred by the manufacturer in ordering from each supplier. This is assumed to be a one-time cost for ordering from each supplier in each production cycle. Hence,

$$O_r = \frac{nZE[\Lambda]}{Q}. \quad (3.5)$$

5. Transportation Cost– The transportation costs incurred by each supplier are split into three components: i) a fixed cost, K , for organizing each delivery, ii) loading costs, assumed to be proportional to the load, and iii) transport costs, assumed to be proportional to both the load size and the journey time. Summing the costs incurred in transporting from each supplier and dividing by the average length of a cycle, these mean costs are given by

$$T_c = \frac{mE[\Lambda]}{Q} \left((K + K_r T_l)n + VT_l \sum_{i=1}^n d_i \right) \quad (3.6)$$

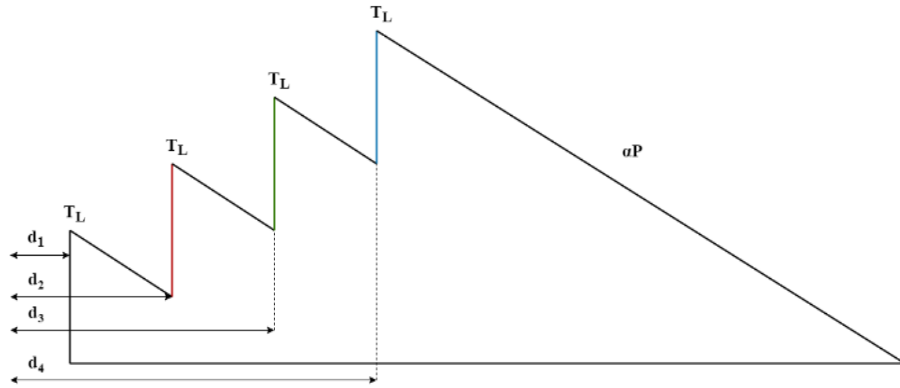


Figure 3.2: Raw material inventory from four independent suppliers building up at the manufacturer's warehouse.

6. Costs for holding the raw materials in the manufacturer's warehouse-These costs represent the expense of holding raw materials in the manufacturer's warehouse during the production cycle. Within the manufacturer production cycle, multiple deliveries are received from identical suppliers, and the corresponding inventory profile for each of the m delivery cycles is illustrated in Figure 3.2. This cost can be determined by computing the area between the supply and consumption curve of the manufacturer as shown in Figure 3.3

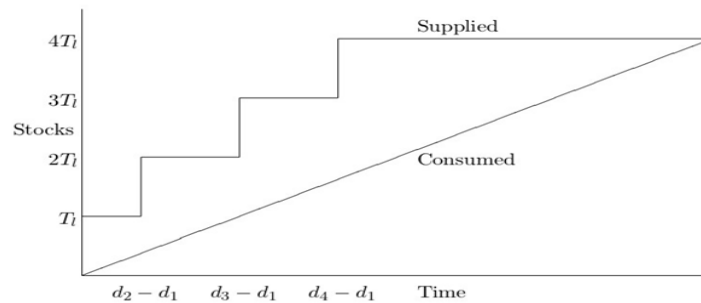


Figure 3.3: Inventory of raw materials from four (4) suppliers at the manufacturer's warehouse during a delivery cycle

In figure 3.3, it is assumed that the first arrival of raw materials in a delivery cycle occurs at time 0. This profile is repeated m times per production cycle and the holding costs are proportional to the area between the supply and consumption profile. In each delivery cycle, nT_l units of raw materials are delivered. The length of this cycle is the time taken for the manufacturer to consume these materials. i.e. $\frac{nT_l}{\alpha P}$.

The area under the supply profile is made up of n rectangles, each of height T_l . The length of the i -th rectangle is $\frac{nT_l}{\alpha P} + d_1 - d_i$. It follows that the area under the supply profile is

$$\sum_{i=1}^n T_l \left(\frac{nT_l}{\alpha P} + d_1 - d_i \right) = \frac{n^2 T_l^2}{\alpha P} + nT_l d_1 - T_l \sum_{i=1}^n d_i.$$

The area under the consumption curve is given by a triangle of height nT_l and length $\frac{nT_l}{\alpha P}$. Thus, the area between the supply curve and the consumption curve is

$$= \frac{n^2 T_l^2}{2\alpha P} + nT_l d_1 - T_l \sum_{i=1}^n d_i.$$

These costs are incurred m times during the manufacturer's production run. Hence, the mean holding cost for raw materials per unit time incurred by the manufacturer is given by

$$H_{rm} = \frac{mE[\Lambda]h_{rm}}{Q} \left(\frac{n^2 T_l^2}{2\alpha P} + nT_l d_1 - T_l \sum_{i=1}^n d_i \right). \quad (3.7)$$

The average costs per unit time of ordering, transporting, and holding the raw material in the manufacturer's warehouse is given by

$$TC_{rm} = O_r + T_c + H_{rm}$$

$$TC_{rm} = \frac{E[\Lambda]}{Q} \left(nZ + m \left((K + K_r T_l) n + V T_l \sum_{i=1}^n d_i \right) + m h_{rm} \left(\frac{n^2 T_l^2}{2\alpha P} + n T_l d_1 - T_l \sum_{i=1}^n d_i \right) \right) \quad (3.8)$$

7. Manufacturer's holding costs for the inventory of finished goods- The manufacturer operates a make-to-stock strategy, i.e., the goods produced in the current cycle are meant for consumption in the next cycle and this continues over an infinite horizon. Hence, we consider both the inventory of finished goods on the shelf awaiting demand and the inventory still in production that will be transferred to storage for the next cycle. The anticipated net inventory at the beginning of a cycle is the safety stock plus the lot size produced from the previous cycle, which falls to the anticipated safety stock at the end of the cycle i.e. with a constant expected demand rate, the on-hand inventory decreases linearly from $Q + S$ to S when the expected number of back orders is neglected. Consequently, the average level of the finished goods inventory is $\frac{Q}{2} + E[S]$, which can also be expressed as $\frac{Q}{2} + s$, where s is the expected safety stock.

The average holding costs per unit time for the finished goods that have been dispatched are

$$[s + \frac{Q}{2}] h_{mf}. \quad (3.9)$$

During the manufacturer's production runs, the average level of finished stock waiting for dispatch is

$$\frac{Q}{2} = \frac{mnT_l}{2\alpha}.$$

Since the length of a production run is $\frac{Q}{P}$ and the average length of a production cycle is $\frac{Q}{E[\Lambda]}$, the mean holding cost per unit time for the finished product waiting for dispatch is

$$\frac{mnT_l E[\Lambda] h_{mf}}{2\alpha P} \quad (3.10)$$

The long run average holding cost of holding the finished product is the sum of equations 3.9 and 3.10.

$$H_{fp} = \left[s + \frac{Q}{2} \right] h_{mf} + \frac{mnT_l E[\Lambda] h_{mf}}{2\alpha P}$$

$$H_{fp} = h_m \left[\frac{1}{2} \left[Q + \frac{mnT_l E[\Lambda]}{\alpha P} \right] + s \right]. \quad (3.11)$$

It is important to note that the expected safety stock, s , is influenced by the assumptions regarding shortages, specifically whether they are backlogged or lost. When shortages are backordered, the safety stock becomes a random variable and may take on both positive and negative values. In this case, it is computed as follows:

We have $s = E[S] = R - E[Y]$, see Darwish et al. (2013), Aldurgam et al. (2017).

$$\begin{aligned} \text{Since } E[Y] &= E[\Lambda] \left[\frac{mnT_l}{\alpha P} + \delta_1 + \frac{T_L}{\lambda \alpha P} \right], E[S] = R - E[\Lambda] \left[\frac{mnT_l}{\alpha P} + d_1 + \frac{T_l}{\lambda \alpha P} \right] \\ H_{fp} &= h_m \left[\frac{1}{2} \left[Q - \frac{mnT_l E[\Lambda]}{\alpha P} \right] + R - E[\Lambda] \left[d_1 + \frac{T_l}{\lambda \alpha P} \right] \right] \end{aligned} \quad (3.12)$$

8. Shortage Costs-These are the penalties incurred when the manufacturer has no finished products in stock to satisfy demand. They are incurred when the lead time demand surpasses the quantity available at the re-order point. The shortage quantity is a random variable and mathematically defined as:

$$\text{Shortage} = \begin{cases} Y - R & Y > R \\ 0 & Y \leq R \end{cases}$$

The expected shortage cost per unit time is thus given as

$$S_C = \frac{\pi E[\Lambda]}{Q} \int_R^\infty (Y - R) f(y) dy \quad (3.13)$$

9. Direct production costs of the manufacturer- These are the direct expenses incurred by the manufacturer in producing the final product. According to Assumption 13, the production cost per unit time is assumed to be $g + bP^{1+\beta}$, where $g, b, \beta > 0$. It follows that the mean production cost for a unit of the finished good is $\left(\frac{g}{P} + bP^\beta \right)$. Since the long run average rate of production must be $E[\Lambda]$, the mean cost of producing the finished good per unit time must be

$$DPC = E[\Lambda] \left(\frac{g}{P} + bP^\beta \right). \quad (3.14)$$

10. The manufacturer's set up costs for production- These are the manufacturer's expenses incurred for setting up each production cycle. Since these costs are incurred once per cycle, the mean set up costs per unit time are

$$M_{sc} = \frac{A_m E[\Lambda]}{Q}. \quad (3.15)$$

11. Manufacturer's raw material sourcing costs- This represents the additional amount a manufacturer pays over the supplier's direct production costs when procuring raw materials. It includes the supplier's profit margin, business overhead, and any procurement-related markups embedded in the acquisition price. These are assumed to be proportional to the amount of raw materials consumed.

$$M_{sp} = \frac{c T_l m n E[\Lambda]}{Q} \quad (3.16)$$

The right-hand side of equation (3.16) is, however, constant (specifically equal to $c E[\Lambda] \alpha$) because it has already been established that $Q = \frac{m n T_l}{\alpha}$, and α is fixed, representing the fixed quantity of the principal raw material required to produce one final product. Consequently, this sourcing cost does not influence the optimization of the overall supply chain cost. The only managerial insight that can be drawn is that for a fixed cost of a unit of the raw material, a higher value of α increases the overall costs of the manufacturer, as more raw-material units are consumed per unit of finished output.

The long run average cost incurred by the manufacturer per unit time is thus

$$TC_m = H_{fp} + S_c + M_{sc} + DPC + T.C_{rm} + M_{sp}$$

$$T.C_m(Q, m, T_l, P, R)$$

$$\begin{aligned}
&= \frac{A_m E[\Lambda]}{Q} + h_m \left[\frac{1}{2} \left[Q - \frac{mnT_l E[\Lambda]}{\alpha P} \right] + R - E[\Lambda] \left[d_1 + \frac{T_l}{\lambda \alpha P} \right] \right] + \frac{cT_l mnE[\Lambda]}{Q} \\
&+ E[\Lambda] \left(\frac{g}{P} + bP^\beta \right) + \frac{\pi E[\Lambda]}{Q} \int_R^\infty (Y - R) f(y) dy + \frac{nZE[\Lambda]}{Q} \\
&+ \frac{mE[\Lambda]}{Q} \left((K + K_r T_l) n + VT_l \sum_{i=1}^n d_i \right) \\
&+ \frac{mE[\Lambda] h_{rm}}{Q} \left(\frac{n^2 T_l^2}{2\alpha P} + nT_l d_1 \right. \\
&\left. - T_l \sum_{i=1}^n d_i \right)
\end{aligned} \tag{3.17}$$

Centralized System

If it is assumed that the supply chain system above is optimized centrally, then the total cost of the supply chain system is

$$T.C_{sc} = TC_s + TC_m$$

$$\begin{aligned}
T.C_{sc}(Q, m, T_l, P, R) &= \frac{nA_s E[\Lambda]}{Q} + \frac{mnT_l^2 h_s E[\Lambda]}{2\alpha PQ} \left[\frac{2-m}{\lambda} + n(m-1) \right] + \left[\frac{g_1}{\lambda \alpha P} + b_1 (\lambda \alpha P)^{\beta_1} \right] \alpha E[\Lambda] + \\
&\frac{A_m E[\Lambda]}{Q} + h_m \left[\frac{1}{2} \left[Q - \frac{mnT_l E[\Lambda]}{\alpha P} \right] + R - E[\Lambda] \left[d_1 + \frac{T_l}{\lambda \alpha P} \right] \right] + \frac{cT_l mnE[\Lambda]}{Q} + E[\Lambda] \left(\frac{g_2}{P} + b_2 P^\beta \right) + \\
&\frac{\pi E[\Lambda]}{Q} \int_R^\infty (Y - R) f(y) dy + \frac{nZE[\Lambda]}{Q} + \frac{mE[\Lambda]}{Q} \left((K + K_r T_l) n + VT_l \sum_{i=1}^n d_i \right) + \frac{mE[\Lambda] h_{rm}}{Q} \left(\frac{n^2 T_l^2}{2\alpha P} + nT_l d_1 - \right. \\
&\left. T_l \sum_{i=1}^n d_i \right)
\end{aligned} \tag{3.18}$$

To ensure the feasibility of this model of a supply chain, the following constraints are imposed

$$d_j - d_1 \leq \frac{(j-1)T_l}{\alpha P} \quad \forall j = 2, 3, 4, \dots, n$$

$$T_{smin} \leq T_l \leq T_{smax}$$

$$\begin{aligned}
m &< \frac{I \alpha}{n T_l} & m \in \text{Integers} \\
\frac{Q}{E[\Lambda]} &\geq \frac{T_l}{\lambda \alpha P} + d_1 + \frac{Q}{P}
\end{aligned}
\tag{3.19}$$

The first constraint ensures that raw materials from the j^{th} supplier arrive before previously delivered batches are fully consumed. The second constraint enforces that the selected batch size is feasible. The third constraint restricts the manufacturer's lot size to the warehouse capacity. The fourth constraint ensures that the expected supply chain cycle time is greater than or equal to the finished goods replenishment lead time, thereby maintaining synchronization between overall supply chain operations and market demand (i.e. the policy is non-preemptive).

3.2.3 Derivation of the average cost per unit time for the basic model with heterogeneous suppliers, single manufacturer

Next, we relax the assumption of homogeneous suppliers by studying a scenario with heterogeneous suppliers. The suppliers are assumed to differ (Figure 3.4) in terms of their production capacities, a factor that directly affects their batch sizes. The manufacturer is assumed to place a one-time joint order with all suppliers at the reorder level, and the suppliers are assumed to dispatch their batches at the same time, i.e. the size of a batch from a supplier is assumed to be proportional to the supplier's production rate. Such a schedule is designed to align with the manufacturer's production plan, which is operated in a single setup without interruptions caused by raw material shortages. Under these relaxed assumptions, the production rate of supplier i in a production schedule relative to the consumption rate of the manufacturer is represented by λ_i (instead of λ). Accordingly, the total quantities of raw materials delivered within a delivery cycle becomes $\sum_{i=1}^n T_{li}$, replacing $n T_l$. Furthermore, it is assumed that $\sum_{i=1}^n \lambda_i \geq 1$, ensuring that the aggregate production capacity of all suppliers is at least sufficient to meet, and preferably exceed, the manufacturer's consumption rate. From the above assumptions, T_{li} is proportional to λ_i , i.e. $\frac{T_{li}}{\lambda_i}$ is a constant independent

of i . It should be noted that such a regime can be achieved in the two following ways:

- a) Each of the suppliers produces at a rate proportional to its capacity. In such a case, the batch size will also be proportional to a supplier's capacity. This approach has the advantage of not increasing the dimension of the solution space. Given that the batch size and production rate for supplier i are T_{li} and P_{si} , respectively, then the batch size and production rate for supplier i are $\frac{P_{msi}T_{li}}{P_{ms1}}$ and $\frac{P_{msi}P_{s1}}{P_{ms1}}$, respectively. If the production rate of supplier 1 is feasible, then by definition the production rate of supplier i will also be feasible. Note that it is necessary to check the feasibility of the batch sizes.
- b) The batch sizes of the suppliers may be allowed to vary independently. In this case, given T_{l1}, T_{li} and P_{s1} the production rate of supplier i is given by $\frac{T_{li}P_{s1}}{T_{l1}}$. While this approach has the disadvantage of increasing the dimension of the solution space, it has the advantage of being able to take advantage of asymmetries in the costs incurred by the suppliers to find a better solution than the one found using approach a).

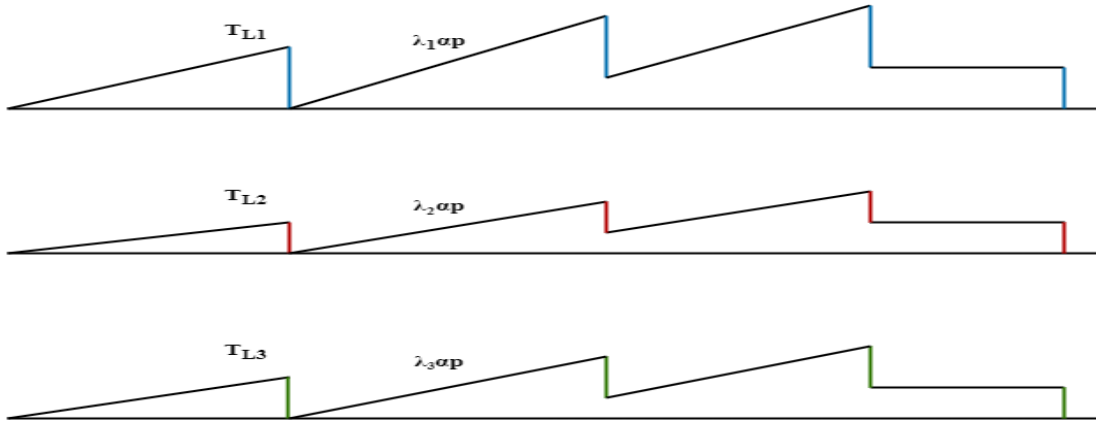


Figure 3.4: Heterogeneous suppliers dispatching different batch sizes to the manufacturer's warehouse.

With this modification, the reformulated costs are presented below. A detailed derivation is provided in Appendix A.

1. Suppliers' set up per unit time

$$V_{sn} = \frac{E[\Lambda] \sum_{i=1}^n A_{si}}{Q} \quad (3.20)$$

2. Suppliers' holding cost per unit time

$$H_{ns} = \frac{h_s E[\Lambda]}{2P} \sum_{i=1}^n T_{li} \left[m - 1 + \frac{T_{li}(2-m)}{\lambda_i \sum_{j=1}^n T_{lj}} \right] \quad (3.21)$$

3. Mean production cost per unit time

$$S_{PC} = \sum_{i=1}^n \left[\left[\frac{g_{1i}}{\lambda_i \alpha P} + b_1 [\lambda_i \alpha P]^{\beta_1} \right] \frac{\alpha E[\Lambda] T_{li}}{\sum_{j=1}^n T_{lj}} \right] \quad (3.22)$$

4. Average transportation cost per unit time

$$T_c = \frac{m E[\Lambda]}{Q} \left(\left(nK + K_r \sum_{i=1}^n T_{li} \right) + V \sum_{i=1}^n T_{li} d_i \right) \quad (3.23)$$

5. Manufacturer's holding cost per unit time for finished goods inventory

$$H_{fp} = h_m \left[\frac{1}{2} \left[Q - \frac{m \sum_{i=1}^n T_{li} E[\Lambda]}{\alpha P} \right] + R - E[\Lambda] \left[d_1 + \frac{T_{l1}}{\lambda_1 \alpha P} \right] \right] \quad (3.24)$$

6. Manufacturer's raw material sourcing premium

$$\frac{cm E[\Lambda] \sum_{i=1}^n T_{li}}{Q} \quad (3.25)$$

7. Raw material holding cost in the manufacturer's warehouse per unit time

$$H_{rm} = \frac{E[\Lambda] \alpha h_m}{\sum_{i=1}^n T_{li}} \left[\frac{(\sum_{i=1}^n T_{li})^2}{2\alpha P} - \sum_{i=1}^n T_{li} d_i + d_1 \sum_{i=1}^n T_{li} \right] \quad (3.26)$$

The long-run average cost of the integrated heterogeneous supply chain, under the assumption of a centralized system, is presented in Equation (3.27).

$$T.C_{SC}(Q, m, T_{li}, P, R)$$

$$\begin{aligned}
&= \frac{E[\Lambda] \sum_{i=1}^n A_{si}}{Q} + \frac{h_s T_{li} E[\Lambda]}{2P} \left[m - 1 + \frac{T_{li}(2-m)}{\lambda_i \sum_{j=1}^n T_{lj}} \right] + \sum_{i=1}^n \left[\left[\frac{g_{1i}}{\lambda_i \alpha P} + b_1 [\lambda_i \alpha P]^{\beta_1} \right] \frac{\alpha E[\Lambda] T_{li}}{\sum_{j=1}^n T_{lj}} \right] \\
&+ \frac{A_m E[\Lambda]}{Q} + \frac{cm E[\Lambda] \sum_{i=1}^n T_{li}}{Q} + h_m \left[\frac{1}{2} \left[Q - \frac{m \sum_{i=1}^n T_{li} E[\Lambda]}{\alpha P} \right] + R - E[\Lambda] \left[d_1 + \frac{T_{l1}}{\lambda_1 \alpha P} \right] \right] \\
&+ E[\Lambda] \left(\frac{g_2}{P} + b_2 P^{\beta_2} \right) + \frac{\pi E[\Lambda]}{Q} \int_R^\infty (y - R) f(y) dy + \frac{n Z E[\Lambda]}{Q} \\
&+ \frac{m E[\Lambda]}{Q} \left(\left(nK + K_r \sum_{i=1}^n T_{li} \right) + V \sum_{i=1}^n T_{li} d_i \right) \\
&+ \frac{E[\Lambda] \alpha h_m}{\sum_{i=1}^n T_{li}} \left[\frac{(\sum_{i=1}^n T_{li})^2}{2\alpha P} - \sum_{i=1}^n T_{li} d_i + d_1 \sum_{i=1}^n T_{li} \right]
\end{aligned}$$

$$\text{Subject to} \quad d_j - d_1 \leq \frac{\sum_{i=1}^{j-1} T_{li}}{\alpha P} \quad j = 2, 3, \dots, n$$

$$T_{smin} \leq T_{li} \leq T_{smax} \quad i = 1, 2, 3, \dots, n$$

$$m < \frac{I\alpha}{\sum_{i=1}^n T_{li}} \quad m \in \text{Integers}$$

$$\frac{Q}{E[\Lambda]} \geq \frac{T_{l1}}{\lambda_1 \alpha P} + d_1 + \frac{Q}{P}$$

(3.27)

3.2.4 Solution Methodology for the problem with homogeneous suppliers

To develop an approach that efficiently finds an optimal (or near-optimal) solution while remaining adaptable to broader scenarios, we establish the following practical assumptions for setting limits on decision variables:

1. **Batch Size Constraints:** The sizes of batches of raw materials sent to the manufacturer's warehouse must fall within appropriately given constraints, according to the form of transport.
2. **Production Rate Adjustments:** The manufacturer's production rate is assumed to take discrete values within a defined range. In the Stackelberg framework, the manufacturer, acting as the leader, sets

the production schedule. Suppliers are required to deliver the first batch of raw materials as quickly as possible, since production of final goods begins immediately upon receipt of these raw materials from the nearest supplier. Once the initial batch is delivered, the manufacturer can adjust the production rate to optimize cost savings. The cost of changing production rates is considered negligible, as raw materials are produced in a single uninterrupted production run.

3. **Supplier Participation and Sustainability:** One of the main goals of partnership is to ensure that all suppliers remain financially sustainable. Therefore, each supplier is required to make at least one delivery per production cycle, and the total number of deliveries must be the same across all suppliers to maintain economic viability and fairness. The maximum number of deliveries a supplier can make is given by $\frac{I\alpha}{nT_l}$, where:

- I represents the warehouse capacity for finished goods,
- α is the units of raw material per unit of the final product,
- n is the number of suppliers, and
- T_l represents the batch size.

These assumptions create a structured, yet flexible, framework for optimizing the supply chain while balancing efficiency, cost savings, and supplier sustainability.

3.2.4.1 Integrated solution

The first solution proposed assumes that the logistics chain is an integrated system in which the suppliers and the manufacturer cooperate to jointly determine the production rate that minimizes the costs incurred by the whole supply chain system. It is worth remembering that the supplier's production rate is assumed to be proportional to the manufacturer's production rate. Since the problem is constrained, taking direct derivatives with respect to the decision variables may not give a feasible solution in all the decision variables, as some of the constraints may be violated. Suppose that m, P and T_l are fixed. From Equation

(3.18), the only costs that depend on the re-order point R are given by $g(R)$, where

$$g(R) = h_m R + \frac{\alpha \pi E[\Lambda]}{mnT_l} \int_R^\infty (y - R) f(y) dy.$$

Differentiating with respect to R , we obtain

$$g'(R) = h_m - \frac{\alpha \pi E[\Lambda] P(Y > R)}{mnT_l}.$$

Assume that either demand in each unit of time (assumed to be a day) has approximately a normal distribution or the length of the production cycle is reasonably large (one month or more). From the central

limit theorem Y has approximately a normal distribution with mean $E[\Lambda] \left[\frac{mnT_l}{\alpha P} + d_1 + \frac{nT_l}{\alpha P} \right]$ and variance

$\sigma^2 \left[\frac{mnT_l}{\alpha P} + d_1 + \frac{nT_l}{\alpha P} \right]$. Note that $g'(0) = h_m - \frac{\alpha \pi E[\Lambda]}{mnT_l}$, $\lim_{R \rightarrow \infty} g'(R) = h_m > 0$ and $g'(R)$ is increasing in R .

It follows that when $h_m - \frac{\alpha \pi E[\Lambda]}{mnT_l} < 0$, for fixed m, P and T_l there exists a unique R that minimizes the total

costs of the supply chain. This condition is equivalent to $h_m < \frac{\pi E[\Lambda]}{Q}$, i.e. the penalty costs per unit for not

satisfying demand are sufficiently high in relation to the storage costs. The unique value of R that minimize

these costs satisfies $g'(R) = 0$, i.e.

$$P(Y > R) = \frac{h_m mnT_l}{\alpha \pi E[\Lambda]} \Rightarrow P(Y < R) = 1 - \frac{h_m mnT_l}{\alpha \pi E[\Lambda]}. \quad (3.28)$$

Thus, the optimal value of R is given by the $\left(1 - \frac{h_m mnT_l}{\alpha \pi E[\Lambda]}\right)$ -quantile of the distribution of Y .

Next, the optimal values of m , T_l and λ are determined so that the value P , the manufacturer's production

rate, is optimized through a local search over a permissible range of production rates, under the assumption

that the corresponding optimal reorder point is used. The values of m, P, T_l, λ and R that give the minimum

average total cost per unit time over the search loop is the optimal solution. A description of this procedure

is presented through the pseudocode in Figure 3.5

```

 $T.C_{sc}^{opt} = \text{big } W$ 
For  $T_l = T_{smin} : \text{step size} : T_{smin}$ 
  For  $m = m_{min} : \text{step size} : \frac{I\alpha}{nT_l}$ 
    For  $P = P_{min} : \text{step size} : P_{max}$ 
      For  $P_s = P_{smin} : \text{step size} : \lambda_m \alpha P_{max}$ 
         $\mu_L = \left( \frac{mnT_l}{\alpha P} + d_1 + \frac{T_l}{\lambda \alpha P} \right)$ 
         $\sigma_L = \sigma \sqrt{\left( \frac{mnT_l}{\alpha P} + d_1 + \frac{T_l}{\lambda \alpha P} \right)}$ 

         $\lambda = \frac{P_s}{\alpha P}$ 
        Determine  $Q = \frac{mnT_l}{\alpha}$ 
         $R = F^{-1} \left( 1 - \frac{h_m mnT_l}{\alpha \pi E[\Lambda]} \right)$ 
        Compute  $T.C_{sc}(Q, T_l, P, m, \lambda, R)$ 

        If  $T.C_{sc} \leq T.C_{sc}^{opt} \&\& d_j - d_1 \leq \frac{(j-1)T_l}{\alpha P} \forall j \&\& T_{smin} \leq T_l \leq T_{smax} \&\& d_j - d_{j-1} \leq \frac{T_l}{\alpha P} \&\& \lambda \leq 1 \&\& n\lambda \geq 1$ 
           $T.C_{sc}^{opt} = T.C_{sc}(Q, T_l, P, m, \lambda, R)$ 
           $Q^{opt} = Q, m^{opt} = m, T_l^{opt} = T_l, P^{opt} = P, R^{opt} = R, \lambda^{opt} = \lambda$ 
        End if
      End For
    End For
  End For
End For

 $Q^{opt}, m^{opt}, T_l^{opt}, P^{opt}, \lambda^{opt}, R^{opt} = \text{argmin } T.C_{sc}^{opt}$ 
 $T.C_{sc}^{opt} = T.C_{sc}(Q^{opt}, m^{opt}, T_l^{opt}, P^{opt}, \lambda^{opt}, R^{opt})$ 

```

Figure 3.5: Pseudocode for the integrated solution (homogeneous suppliers)

3.2.4.2 Stackelberg (sequential) solution

The second solution adopts a sequential two-stage optimization approach, where the manufacturer takes the lead in decision-making. This structure aligns with the Stackelberg game framework (see Yue and You, 2014), in which the manufacturer acts as the leader and the suppliers as followers. As the principal decision-maker, the manufacturer determines key production parameters, including the production rate (P), reorder point (R), batch size (T_l), and the number of deliveries per supplier in each production cycle (m). These decisions are made to minimize the manufacturer's cost per unit time while meeting expected demand. To ensure the timely receipt of raw materials in accordance with the manufacturer's production schedule and to minimize overall production lead time, it is assumed that suppliers produce the first batch at the

maximum feasible rate $P_s = \lambda_m \alpha P_m$, ensuring that the manufacturer receives materials as quickly as possible. However, after this initial batch, the suppliers strategically adjust their raw material production rate to minimize their own costs, primarily reducing production and holding costs, while still fulfilling their obligation to supply materials to the manufacturer.

This adjustment optimizes the supplier's cost structure without disrupting the manufacturer's production schedule, making it a true Stackelberg game. The manufacturer leads by setting operational constraints, and the supplier follows by making an optimal decision that balances their cost efficiency with the manufacturer's requirements. This two-stage optimization ensures a cost-effective and synchronized supply chain, benefiting both parties through structured decision-making and strategic cost reduction. Figure 3.6 presents the profile for the Stackelberg solution, and to implement it, new interpretations of the decision variables for the suppliers are required, which are presented in Table 3.4.

P_s	<i>Suppliers' production rate (homogeneous suppliers), which for the Stackelberg solution is the production rate suppliers switch to after producing and delivering the first batch at maximum rate</i> $P_{sm} = \lambda_m \alpha P_m, P_s = \lambda \alpha P$
λ	<i>Suppliers' production rate relative to the manufacturer's production rate after the adjustment (homogeneous suppliers)</i>
P_{si}	<i>Supplier i's production rate (heterogeneous suppliers), which for the Stackelberg solution is the production rate suppliers switch to after producing and delivering the first batch at maximum rate</i> $P_{smi} = \lambda_{mi} \alpha P_m, P_{si} = \lambda_i \alpha P$
λ_i	<i>Supplier i's production rate relative to the manufacturer's consumption rate after the adjustment (heterogeneous suppliers)</i>

Table 3.4: Additional Notation for Stackelberg/Sequential Solution.

We first derive the costs per unit time under the assumption of homogeneous suppliers. The holding cost for suppliers is recalculated using Equation (3.29) by determining the area of the quadrilateral shown in

Figure 3.6. Note that in the homogeneous scenario, all suppliers operate with the same production rate and use batches of equal size. However, their expected delivery times to the manufacturer's warehouse vary.

To derive the holding costs of a supplier, the area under the production curve in Figure 3.6 can be partitioned into a simple triangle, a trapezium and a rectangle. The area of the triangle is $\frac{T_l^2}{2\lambda_m\alpha P_m}$. The area of the trapezium is determined by multiplying the average of the length of the two vertical borders by the length (the time required for the supplier to produce $m-1$ batches at the rate adapted to the manufacturer's production rate), i.e. $\frac{1}{2} \times (T_l + mT_l) \times \frac{(m-1)T_l}{\lambda\alpha P} = \frac{(m^2-1)T_l^2}{2\lambda\alpha P}$. The area of the rectangle is the product of the length of two adjacent sides $\frac{(m-1)nT_l}{\alpha P} - \frac{(m-1)T_l}{\lambda\alpha P}$ and mT_l , i.e.

$$\frac{m(m-1)T_l^2}{\alpha P} \left[n - \frac{1}{\lambda} \right].$$

The area under the supply curve in Figure 3.6 is the sum of the area of the shapes described above, i.e.

$$\frac{T_l^2}{2\lambda_m\alpha P_m} + \frac{(m^2-1)T_l^2}{2\lambda\alpha P} + \frac{m(m-1)T_l^2}{\alpha P} \left[n - \frac{1}{\lambda} \right]$$

As for Figure 3.1, we need to determine the area under the supply curve (the staircase region) in Figure 3.6 to compute the amount of raw material inventory held in stock per supplier per unit time. For this region the mean height is $\frac{T_l + (m-1)T_l}{2} = \frac{mT_l}{2}$ and the length is $\frac{(m-1)nT_l}{\alpha P}$. Hence, the area is

$$= \frac{m(m-1)nT_l^2}{2\alpha P}$$

The cost of the inventory held in stock by a supplier during a production cycle is proportional to the area between the production and the supply curve. This is given by

$$\frac{T_l^2}{2\lambda_m\alpha P_m} + \frac{(m^2-1)T_l^2}{2\lambda\alpha P} + \frac{m(m-1)T_l^2}{\alpha P} \left[n - \frac{1}{\lambda} \right] - \frac{m(m-1)nT_l^2}{2\alpha P}.$$

This simplifies to

$$\frac{T_l^2}{2\lambda_m\alpha P_m} + \frac{m(m-1)nT_l^2}{\alpha P} - \frac{(m-1)^2T_l^2}{2\lambda\alpha P}$$

The average inventory holding cost per unit time for each supplier is thus

$$\frac{E[\Lambda]h_s\alpha}{mnT_l} \left(\frac{T_l^2}{2\lambda_m\alpha P_m} + \frac{m(m-1)nT_l^2}{\alpha P} - \frac{(m-1)^2T_l^2}{2\lambda\alpha P} \right)$$

For n suppliers, the average inventory holding cost per unit time H_{ns} is

$$H_{ns} = \frac{E[\Lambda]h_s\alpha}{mT_l} \left(\frac{T_l^2}{2\lambda_m\alpha P_m} + \frac{m(m-1)nT_l^2}{\alpha P} - \frac{(m-1)^2T_l^2}{2\lambda\alpha P} \right). \quad (3.29)$$

The production cost per unit time for the suppliers is also recomputed as follows:

Firstly, we determine the production cost of the first batch of raw materials delivered to the manufacturer

$$\left(\frac{g_1}{\lambda_m\alpha P_m} + b_1[\lambda_m\alpha P_m]^{\beta_1} \right) T_l.$$

and subsequently, the costs of the remaining $m - 1$ batches delivered are computed as

$$\left(\frac{g_1}{\lambda\alpha P} + b_1[\lambda\alpha P]^{\beta_1} \right) (m-1)T_l.$$

Hence, the production cost of raw materials for each supplier per unit time is

$$\left(\left(\frac{g_1}{\lambda_m\alpha P_m} + b_1[\lambda_m\alpha P_m]^{\beta_1} \right) T_l + \left(\frac{g_1}{\lambda\alpha P} + b_1[\lambda\alpha P]^{\beta_1} \right) (m-1)T_l \right) \frac{E[\Lambda]\alpha}{mnT_l}$$

For the n suppliers, the total production cost of raw materials per unit time is given by

$$S_{PC} = \left(\left(\frac{g_1}{\lambda_m\alpha P_m} + b_1[\lambda_m\alpha P_m]^{\beta_1} \right) T_l + \left(\frac{g_1}{\lambda\alpha P} + b_1[\lambda\alpha P]^{\beta_1} \right) (m-1)T_l \right) \frac{E[\Lambda]\alpha}{mT_l} \quad (3.30)$$

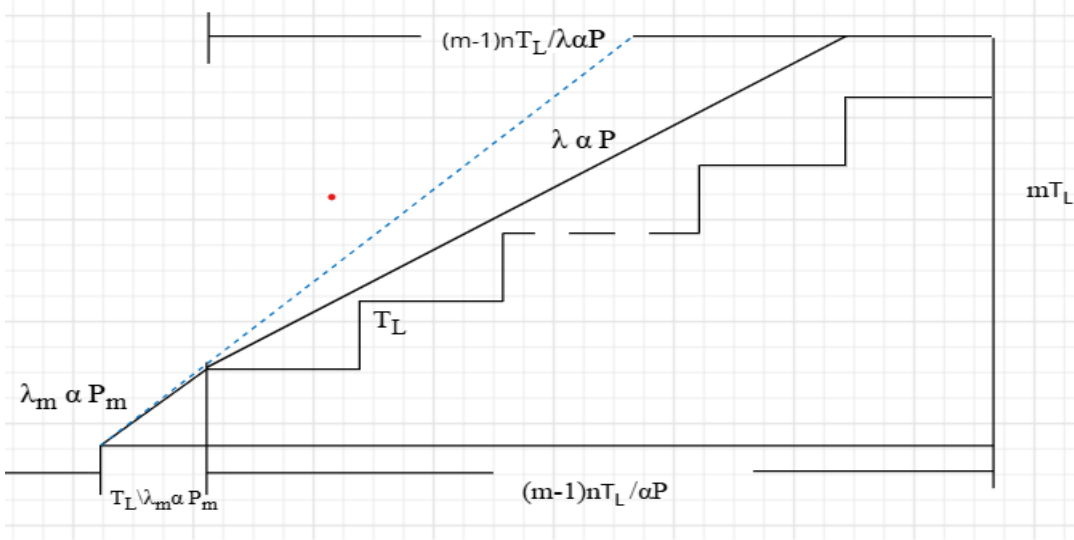


Figure 3.6: Supplier i 's inventory profile for m shipments delivered to the manufacturer's warehouse under the Stackelberg approach.

In the heterogeneous scenario, the suppliers' production rates and batch sizes vary, unlike the homogeneous case where all suppliers have identical values for both. The corresponding holding costs per unit time for the heterogeneous case are derived by computing the area corresponding to the raw material inventory at hand in a quadrilateral similar to Figure 3.6, where $\frac{nT_l}{\alpha P}$ is replaced by $\frac{\sum_{i=1}^n T_{li}}{\alpha P}$ due to the non-homogeneity of suppliers. As before, the quadrilateral consists of a triangle, a trapezium and a rectangle.

The area of the triangle is $\frac{T_{li}^2}{2\lambda_{mi}\alpha P_m}$. The area of the trapezium is $\frac{(m^2-1)T_{li}^2}{2\lambda_i\alpha P}$, and the area of the rectangle is

$\frac{m(m-1)T_{li}}{\alpha P} \left(\left[\sum_{j=1}^n T_{lj} \right] - \frac{T_{li}}{\lambda_i} \right)$. Hence, the area of the quadrilateral is

$$\frac{T_{li}^2}{2\lambda_{mi}\alpha P_m} + \frac{(m^2-1)T_{li}^2}{2\lambda_i\alpha P} + \frac{m(m-1)T_{li}}{\alpha P} \left[\left(\sum_{j=1}^n T_{lj} \right) - \frac{T_{li}}{\lambda_i} \right]$$

The area of the staircase region is $\frac{m(m-1)T_{li}\sum_{j=1}^n T_{lj}}{2\alpha P}$. The holding costs incurred by supplier i are proportional

to the difference between these two areas (the quadrilateral and the staircase), which is

$$\frac{T_{li}^2}{2\lambda_{mi}\alpha P_m} + \frac{(m^2 - 1)T_{li}}{2\lambda_i\alpha P} + \frac{m(m - 1)T_{li}}{\alpha P} \left[\left(\sum_{j=1}^n T_{lj} \right) - \frac{T_{li}}{\lambda_i} \right] - \frac{m(m - 1)T_{li} \sum_{j=1}^n T_{lj}}{2\alpha P}.$$

This simplifies to

$$\frac{T_{li}^2}{2\lambda_{mi}\alpha P_m} + \frac{m(m - 1)T_{li} \sum_{j=1}^n T_{lj}}{2\alpha P} - \frac{(m - 1)^2 T_{li}^2}{2\lambda_i\alpha P}$$

Summing over the suppliers, the total holding costs incurred by the suppliers during a production cycle are

$$\sum_{i=1}^n \left(\frac{T_{li}^2}{2\lambda_{mi}\alpha P_m} + \frac{m(m - 1)T_{li} \sum_{j=1}^n T_{lj}}{2\alpha P} - \frac{(m - 1)^2 T_{li}^2}{2\lambda_i\alpha P} \right).$$

Dividing by the average length of a production cycle, $\frac{m \sum_{i=1}^n T_{li}}{\alpha E[\Lambda]}$, and multiplying by the unit holding cost,

the total holding costs incurred by the suppliers per unit time are

$$\frac{\alpha E[\Lambda]}{m \sum_{i=1}^n T_{li}} \sum_{i=1}^n \left(\frac{T_{li}^2}{2\lambda_{mi}\alpha P_m} + \frac{m(m - 1)T_{li} \sum_{j=1}^n T_{lj}}{2\alpha P} - \frac{(m - 1)^2 T_{li}^2}{2\lambda_i\alpha P} \right) \quad (3.31)$$

The production costs for raw materials incurred in a production cycle by supplier i when suppliers are assumed to be heterogeneous is

$$\left(\left(\frac{g_1}{\lambda_{mi}\alpha P_m} + b_1[\lambda_{mi}\alpha P_m]^{\beta_1} \right) T_{li} + \left(\frac{g_1}{\lambda_i\alpha P} + b_1[\alpha\lambda_i P]^{\beta_1} \right) (m - 1)T_{li} \right)$$

Hence, the average production costs for supplier i per unit time are given by

$$\left(\left(\frac{g_1}{\lambda_{mi}\alpha P_m} + b_1[\lambda_{mi}\alpha P_m]^{\beta_1} \right) T_{li} + \left(\frac{g_1}{\lambda_i\alpha P} + b_1[\alpha\lambda_i P]^{\beta_1} \right) (m - 1)T_{li} \right) \frac{\alpha E[\Lambda]}{m \sum_{i=1}^n T_{li}}.$$

Note: For convenience here, it is assumed that the cost parameters (g_1, b_1, β_1) are the same for each of the suppliers. This can easily be generalized by substituting in the appropriate parameters for each supplier into the above formula.

The production cost for n heterogeneous suppliers per unit time is

$$S_{PC} = \frac{\alpha E[\Lambda]}{m \sum_{i=1}^n T_{li}} \sum_{i=1}^n \left(\left(\frac{g_1}{\lambda_{mi} \alpha P_m} + b_1 [\lambda_{mi} \alpha P_m]^{\beta_1} \right) T_{li} + \left(\frac{g_1}{\lambda_i \alpha P} + b_1 [\lambda_i \alpha P]^{\beta_1} \right) (m-1) T_{li} \right) \quad (3.32)$$

The pseudocode describing the Stackelberg solution is presented in Figure 3.7.

```

T.Cmopt = big W, T.Csopt = big X
For Tl = Tsmin: step size : Tsmin
  For m = mmin: step size :  $\frac{1\alpha}{nT_l}$ 
    For P = Pmin: step size : Pmax
       $\mu_L = \left( \frac{mnT_l}{\alpha P} + d_1 + \frac{T_l}{\lambda_m \alpha P_m} \right)$ 
       $\sigma_L = \sigma \sqrt{\left( \frac{mnT_l}{\alpha P} + d_1 + \frac{T_l}{\lambda_m \alpha P_m} \right)}$ 
       $\lambda = \lambda_m$ 

      Determine  $Q = \frac{mnT_l}{\alpha}$ 
       $R = F^{-1} \left( 1 - \frac{h_m mnT_l}{\alpha \pi E[\Lambda]} \right)$ 
      Compute T.Csc(Q, Tl, P, m, R)

      If T.Cm ≤ T.Cmopt && dj - d1 ≤  $\frac{(j-1)T_l}{\alpha P} \forall j$  && Tsmin ≤ Tl ≤ Tsmax && dj - dj-1 ≤  $\frac{T_l}{\alpha P}$  && λm ≤ 1 && nλm ≥ 1
        T.Cmopt = Min T.Cm(Q, Tl, P, m, R)
        Qopt = Q, mopt = m, Tlopt = Tl, Popt = P, Ropt = R.
      End If
    End For
  End For
End For

For Psl = Psmin: step size : λmαPmax
  For i = 1, 2, 3, 4, ..., n
    λi =  $\frac{P_{sl}}{\alpha P}$ 
    Compute T.Cs(λi, Psl)
  End For
  If T.Cs ≤ T.Csopt &&  $\sum_{i=1}^n P_{sl} > \alpha P$ 
    T.Csopt = Min T.Cs(λi, Psl)
    λiopt = λi, Pslopt = Psl
  End If
End For

T.Csc = Compute T.Cm(Q, Tl, P, m, R) + T.Cs(λi, Psl)
Q, Tl, P, m, R, λi, Psl = argmin T.Csc
T.Cscopt = T.Csc(Q, Tl, P, m, R, λi, Psl)

```

Figure 3.7: Pseudocode for the Stackelberg solution (Sequential approach)

Note: It should be stressed that the above code directly minimizes the total costs of the suppliers given the batches ordered by the manufacturer. Under the homogeneous model, each of the suppliers incurs the same costs and thus minimization of the total costs per unit time of the suppliers directly leads to the minimization of the costs of individual suppliers per unit time. Under a model with heterogeneous suppliers, the manufacturer essentially orders a set of deliveries from each supplier individually. Each of the suppliers can individually minimize their costs per unit time given this delivery profile. This leads to minimizing the total costs of the suppliers per unit time. Hence, the relationship between the suppliers has no effect on such a solution. We may treat the suppliers as a single firm or as n independent firms.

3.3 Chapter Summary

In this chapter, we have established a structured, yet flexible, framework for optimizing the supply chain by defining key constraints and assumptions. We considered limitations on the batch size to align with real-world transportation capacities, production rate adjustments to balance efficiency and cost, and supplier participation to ensure financial sustainability. The cost structures for suppliers were analyzed, with separate formulations for homogeneous and heterogeneous scenarios, capturing variations in production rates, batch sizes, schedules, and associated costs. Additionally, we have discussed the solution methodology, which includes two key approaches: the integrated solution and the Stackelberg solution. In the integrated approach, all of the decision variables are jointly determined within a centralized system to achieve the best feasible solution for the logistics chain as a whole. In contrast, the Stackelberg approach allows the manufacturer to optimize its process independently, while suppliers adapt their operations to meet the manufacturer's targets at minimal cost to themselves. Regarding scheduling, the model is structured so that all suppliers make an equal number of deliveries in both homogeneous and heterogeneous scenarios. However, in the heterogeneous case, batch sizes vary in proportion to suppliers' production rates.

This structured, yet adaptable, approach ensures efficiency while maintaining supplier participation and sustainability.

CHAPTER 4

NUMERICAL EXAMPLE AND SENSITIVITY ANALYSIS

4.1 Numerical Example

In this chapter, a numerical analysis and sensitivity study are conducted to examine how key supply chain parameters influence critical decision-making processes. The analysis focuses on the effects of mean demand, variability in demand, suppliers' production rates relative to the manufacturer's consumption rate, the number of units of raw material required per unit of finished product, and manufacturer's holding costs on system performance. To illustrate the model, a base-case scenario is first considered with three identical suppliers and a single manufacturer. The parameters for this problem are presented in Table 4.1, while Table 4.3 introduces additional parameters to extend the analysis to the heterogeneous suppliers' case, involving two distinct suppliers. The algorithms described in the pseudocode in Figures 3.5 and 3.7 are applied to generate the results, which are reported in Table 4.2 for the homogeneous supplier setting and in Table 4.4 for the heterogeneous supplier setting. This example provides a foundation for evaluating how variations in the supply chain parameters shape decision variables such as the production rate, batch size, number of deliveries in a production cycle, reorder levels, production lot size, as well as the associated cost structure. The analysis further demonstrates how optimization improves overall system efficiency and cost-effectiveness, thereby linking the conceptual development of the model to its practical application in managing complex supply chain environments.

<i>Model Parameter</i>	<i>Description</i>	<i>Value</i>
As_i	<i>Supplier i's set up cost</i>	675
A_m	<i>Manufacturer's set up cost for production</i>	2500
β	<i>Power term in the production cost function for the finished goods,</i>	1
b	<i>Coefficient of the power term in the production cost formula for the finished goods</i>	2.35
g	<i>Direct production cost parameter per unit of time for the finished goods</i>	2000

β_1	<i>Power term in the production cost function for the raw materials,</i>	1
b_1	<i>Coefficient of the power term in the production cost formula for the raw materials</i>	0.95
g_1	<i>Direct production cost parameter per unit of time for the raw materials</i>	650
$E[\Lambda]$	<i>Expected demand per unit time</i>	120
σ_A	<i>Standard deviation of demand per unit time.</i>	40
$h_{r,m}$	<i>Cost of holding a unit of raw material in manufacturer's warehouse per unit time</i>	3
h_s	<i>Cost of holding a unit of raw material in supplier's warehouse per unit time</i>	4
h_m	<i>Cost of holding a unit of the finished good in manufacturer's warehouse per unit time</i>	5
λ_m	<i>Maximum production rate of supplier i relative to the manufacturer's maximum consumption rate</i>	0.35
C	<i>Unit acquisition cost of raw material</i>	3.5
K	<i>Fixed cost term associated with transporting a batch: clearance, licence, and other documentations</i>	150
K_r	<i>Transport cost per unit of raw material: associated with loading and offloading a unit of raw material.</i>	4
V	<i>Transport cost per unit of raw material per unit time</i>	60
Z	<i>Constant ordering cost per supplier</i>	500
α	<i>Units of raw material per unit product</i>	2
P_m	<i>Maximum production rate of the manufacturer</i>	500
d_1	<i>Time required for a delivery from supplier 1</i>	2
d_2	<i>Time required for a delivery from supplier 2</i>	5
d_3	<i>Time required for a delivery from supplier 3</i>	10
π	<i>Penalty cost incurred per unit shortage by the manufacturer</i>	2000
T_{smin}	<i>Minimum batch size (units of raw materials)</i>	450
T_{smax}	<i>Maximum batch size (units of raw materials)</i>	4500

Table 4.1: Model data for the base case scenario and numerical experimentation.

	$E[\Lambda]$	Q	R	$T_{\downarrow}$	m	λ	P	P_s	$T.C_s$	$T.C_m$	$T.C_{sc}$
Integrated solution	120	6156	6135	1368	3	0.3392	171	116	33692	81782	115470
Stackelberg Solution	120	3744	3702	1248	2	1.1218 (0.3397)*	156	350 (106)*	56798	67992	124790
% change in costs									65.58	-16.86	8.07

*Table 4.2: Results for the base case: example with homogeneous suppliers. * Under the Stackelberg solution the suppliers produce the first batch at the maximum possible rate and then can reduce their production rate. This second rate is given in brackets. The values of λ give the suppliers' production rate relative to the chosen consumption rate of the manufacturer.*

Results from Table 4.2 show that the length of the production cycle (which is proportional to the lot size) is greater at the integrated solution than at the Stackelberg solution, while the batch sizes are comparable. The Stackelberg solution aims to minimize the costs of the manufacturer. It thus treats, e.g. the holding costs of the manufacturer as a priority, rather than the holding costs of the suppliers. By reducing the number of deliveries per production cycle, the Stackelberg solution reduces the holding costs of the manufacturer. However, these savings come at the cost of shifting costs onto the suppliers', whose storage costs are higher. This leads to an appreciable increase in total costs compared to the integrated solution. It should be noted that since the integrated solution minimizes the overall costs, adoption of a Stackelberg solution will lead to higher overall costs, unless the Stackelberg solution allows the actors greater flexibility in their actions.

A_{s1}	A_{s2}	g_{s1}	g_{s2}	λ_{m1}	λ_{m2}
675	1000	650	850	0.30	0.75

Table 4.3: Additional parameters for heterogeneous suppliers' settings

	$E[\Lambda]$	Q	R	T_{l1}	T_{l2}	m	λ_{s1}	λ_{s2}	P	P_{s1}	P_{s2}	$T.C_s$	$T.C_m$	$T.C_{sc}$
Integrated solution – Independent batch sizes	120	6084	6050	966	1062	6	0.5000	0.5500	161	161	177	43312	79947	123260
Integrated solution –	120	6762	6727	966	2415	4	0.3000	0.7500	161	97	243	52461	82870	135330

proportional batch sizes														
Stackelberg Solution	120	3738	3731	1068	2670	2	0.2949	0.7219	178	105	257	96765	74690	171460
% change in costs (third solution compared to first solution)												123.41	-6.58	39.10

Table 4.4: Results of the base case example with heterogenous suppliers

Also, for the case of heterogeneous suppliers, Table 4.4 shows that under the integrated solution, allowing flexible and optimally determined batch sizes for the two suppliers leads to an approximate 10% cost reduction compared with the scenario in which batch sizes and suppliers' production rates are constrained to be proportional to the suppliers' capacities. Although Supplier 2 has a much higher capacity than Supplier 1, Supplier 1 has lower fixed unit costs of production per unit of time. For this reason, at the optimal solution with independent batch sizes, Supplier 1 produces only at a slightly lower rate than Supplier 2. At the solution with proportional batch sizes, Supplier 1 produces at 40% of the rate of Supplier 2. Hence, when there are clear asymmetries between the costs of the suppliers, independent batch sizes give the manufacturer enough flexibility to reduce the costs of an integrated system. In addition to the assumption of proportional batch sizes, the Stackelberg solution shortens the production cycle to reduce the storage costs of the manufacturer. However, this comes at a significant increase in the costs incurred by the suppliers. As a result, the average costs incurred under the Stackelberg solution are almost 40% higher than under the optimal integrated solution.

4.2 Sensitivity Analysis

We conducted additional numerical experiments using the model with homogeneous suppliers, as the results from the base case scenarios indicated that the integrated solution provides better financial savings compared to the Stackelberg approach. The objective of this analysis is to examine the impact of various parameters on decision variables and costs per unit time, with costs per unit time serving as the performance metric. The parameters analyzed include the mean and variance of demand per unit time, the number of raw material units required per unit of the finished product, the supplier's relative production

rate, and the holding costs for the finished product. The results for both the integrated and Stackelberg solutions, along with the percentage savings for the manufacturer, the relative increase in total costs per unit time, and the costs incurred by the suppliers, are presented in Tables 4.5–4.19.

Effect of Demand for the Final Product ($E[\Lambda]$)

Tables 4.5, 4.6, and 4.7 illustrate how the model responds to changes in demand using both the integrated solution and the Stackelberg solution. Table 4.5 presents the results obtained from the integrated solution for different levels of demand. It was observed that the manufacturer's and suppliers' production rate increase almost directly in proportion with the demand rate.

Integrated Solution

$E[\Lambda]$	Q	R	T_l	m	λ	P	P_s	$T.C_s$	$T.C_m$	$T.C_{sc}$
120	6156	6135	1368	3	0.3390	171	116	33692	81782	115470
140	7046	7037	1568	3	0.3590	196	141	46049	102900	148950
160	10128	10073	1688	4	0.3340	211	141	52087	132930	185020
180	11328	11269	1888	4	0.3410	236	161	65581	159610	225190
200	12612	12611	2102	4	0.3370	261	176	78237	189060	267340

Table 4.5: Effect of demand on the integrated solution.

Stackelberg Solution

$E[\Lambda]$	Q	R	T_l	m	λ	P	P_s	$T.C_s$	$T.C_m$	$T.C_{sc}$
120	3744	3702	1248	2	1.1218 (0.3397)	156	350 (106)	56798	67992	124790
140	4464	4390	1488	2	0.9409 (0.3387)	186	350 (126)	68957	88943	157900
160	5346	5346	1782	2	0.8294 (0.3341)	211	350 (141)	81420	112150	193570
180	5904	5849	1968	2	0.7114 (0.3374)	246	350 (166)	95733	139340	235070
200	6624	6619	2208	2	0.6341 (0.3370)	276	350 (186)	110350	168770	279120

Table 4.6: Effect of demand on the Stackelberg solution

$E[\Lambda]$	% change in suppliers' costs	% change in manufacturer's costs	% change in total costs
120	68.58	-16.86	8.07
140	49.74	-13.56	6.01
160	56.31	-15.63	4.62
180	45.98	-12.70	4.39
200	41.05	-10.73	4.41

Table 4.7: Percentage changes in costs at the Stackelberg solution relative to the integrated solution.

Additionally, the batch size and the number of deliveries per production cycle increase gradually to meet demand. This adjustment results from the fact that as demand increases, the intensity of production has to increase. This results in the length of the production cycle increasing. Table 4.6 gives the results from the Stackelberg solution. They follow a similar pattern with the production rates and batch sizes increasing in proportion to the demand rate. However, the number of deliveries per production cycle is fixed. This results in lower holding costs for the manufacturer, at the cost of the suppliers. Table 4.7 compares the results from both solution approaches, showing that the integrated solution provides significant savings overall and for the suppliers, while the manufacturer benefits from savings under the Stackelberg solution. In a real-world partnership, particularly in small and medium-sized enterprises (SMEs), the principal partner typically manages, coordinates, and finances most of the operations. This partner often determines the terms that govern the business. Therefore, while the integrated solution may increase costs for the manufacturer, it ultimately offers better savings to the suppliers, contributing to a reduction in the overall cost of the supply chain system. It should be noted that when an integrated system is set up, then these benefits can be shared between the players according to an appropriate agreement.

Effect of Raw Material Consumption Coefficient (α)

Integrated Solution

α	Q	R	T_l	m	λ	P	P_s	$T.C_s$	$T.C_m$	$T.C_{sc}$
0.5	2784	2783	464	1	0.9005	191	86	6090	69047	75137
1	3984	3963	664	2	0.6988	166	116	16837	68988	85825
2	6156	6135	1368	3	0.3392	171	116	33692	81782	115470

3	6228	6227	2076	3	0.3333	171	171	68630	84507	153140
4	7728	7587	2576	4	0.3354	161	216	111630	91280	202910

Table 4.8: Effect of manufacturers' raw material consumption coefficient on the integrated solution.

Stackelberg Solution

α	Q	R	T_l	m	λ	P	P_s	$T.C_s$	$T.C_m$	$T.C_{sc}$
0.5	2724	2721	454	1	0.9215 (-)	191	88* (-)	6145	68773	74918
1	3744	3702	624	2	1.1218 (0.3590)	156	175 (56)	16122	65652	81774
2	3744	3702	1248	2	1.1218 (0.3397)	156	350 (106)	56798	67992	124790
3	3744	3702	1872	2	1.1218 (0.3333)	156	525 (156)	123090	70332	193420
4	3744	3702	2496	2	1.1218 (0.3381)	156	700 (211)	216290	72672	288960

Table 4.9: Effect of manufacturer's raw material consumption coefficient on the Stackelberg solution.

* - Since there is only one delivery per production cycle, the suppliers produce at the maximum possible rate.

α	% change in suppliers' costs	% change in manufacturer's costs	% change in total costs
0.5	0.90	-0.40	-0.29
1	-4.25	-4.84	-4.72
2	68.58	-16.86	8.07
3	79.35	-16.77	26.30
4	93.76	-20.39	42.41

Table 4.10: Percentage changes in costs at the Stackelberg solution relative to the integrated solution.

The effect of the manufacturer's raw material consumption coefficient, α , is illustrated in Tables 4.8 to 4.10. Table 4.8 presents the results of the integrated solution, while Table 4.9 reports the corresponding outcomes under the Stackelberg solution. In both coordination schemes, an increase in α leads to larger raw material batch sizes delivered by suppliers, higher manufacturer production lot sizes, and increased reorder levels. These adjustments reflect the higher raw material requirement per unit of finished product.

A key structural difference between the two solutions lies in the treatment of supplier production decisions and the number of deliveries per production cycle. Under the Stackelberg framework, the manufacturer

commits to its production and inventory parameters, while suppliers subsequently adjust their production rates to minimize their own costs without disrupting the manufacturer's plan. When only one delivery is made per cycle ($m = 1$), suppliers operate at their maximum feasible production rate. However, when multiple deliveries are required ($m \geq 2$), the first batch is produced and delivered at the maximum rate, while subsequent batches are produced at an adjusted, lower rate. This adjustment prevents excessive inventory accumulation at the supplier level and aligns production more closely with the manufacturer's consumption pattern.

The results show that, for lower values of α , $\alpha \leq 1$, the Stackelberg solution yields lower total supply chain costs compared to the integrated solution. This improvement is driven by reduced supplier production rates, smaller delivery batch sizes, and fewer inventory holding costs throughout the system. The decoupling of supplier and manufacturer production rates allows suppliers to better match their output to actual demand, resulting in more efficient inventory control.

As α increases beyond unity, the integrated system increasingly relies on multiple deliveries per production cycle to cope with higher raw material demand. Over this region, responsibility for raw material storage gradually shifts toward the suppliers, consistent with lean manufacturing principles. The suppliers' adjusted production rates become approximately proportional to α , reflecting the proportional increase in raw material demand for a fixed final product demand. At the same time, the manufacturer increases the size of its production lots (and hence the length of the production cycles, since mean demand is fixed). This leads to a tendency of the manufacturer to reduce its production rate, since there is a longer period available to produce the appropriate stocks of the finished good to avoid stock out.

In terms of cost distribution, the integrated solution seeks to balance costs across all supply chain participants. In contrast, the Stackelberg solution increasingly favors the manufacturer as α rises, leading to substantial cost savings for the manufacturer, but significantly higher production and inventory costs for suppliers. This imbalance results in higher total system costs for $\alpha > 1$, as confirmed by the large positive

percentage increases in total supply chain cost. Consequently, while the Stackelberg solution is more advantageous for $\alpha \leq 1$, the integrated solution becomes more suitable when raw material consumption is high, as it better controls inventory growth and cost escalation across the entire supply chain.

Effect of Manufacturer's Holding Cost for the final product (h_m)

Integrated Solution

h_m	Q	R	T	m	λ_{so}	P	P_s	$T.C_s$	$T.C_m$	$T.C_{sc}$
4	6156	6154	1368	3	0.3392	171	116	33692	76375	110070
5	6156	6135	1368	3	0.3392	171	116	33692	81782	115470
6	6156	6119	1368	3	0.3392	171	116	33692	87172	120860
7	4704	4691	1568	2	0.3342	196	131	36870	89264	126130
8	4704	4680	1568	2	0.3342	196	131	36870	93185	130060

Table 4.11: Effect of the manufacturer's holding costs for the final product using the integrated solution

Stackelberg Solution

h_m	Q	R	T	m	λ	P	P_s	$T.C_s$	$T.C_m$	$T.C_{sc}$
4	3744	3715	1248	2	1.1218 (0.3397)	156	350 (106)	56789	64520	121320
5	3744	3702	1248	2	1.1218 (0.3397)	156	350 (106)	56798	67992	124790
6	3744	3691	1248	2	1.1218 (0.3397)	156	350 (106)	56798	71453	128250
7	3744	3682	1248	2	1.1218 (0.3397)	156	350 (106)	56798	74904	131700
8	2352	2339	1568	1	0.8929 (-)	196	350 (-)	82499	77933	160430

Table 4.12: Effect of the manufacturer's holding costs for the final product using the Stackelberg solution.

h_m	% change in suppliers' costs	% change in manufacturer's costs	% change in total costs
4	68.55	-15.21	10.22
5	68.58	-16.86	8.07
6	68.58	-18.03	6.12
7	54.05	-16.09	4.42
8	123.75	-16.38	23.35

Table 4.13: Percentage changes in costs at the Stackelberg solution relative to the integrated solution.

We analyzed the impact of the manufacturer's unit holding cost for the final product in Tables 4.11, 4.12, and 4.13. Table 4.11 presents the results of the integrated solution. As the manufacturer's holding cost increased, the model responded by reducing the reorder level and (at appropriate points) the production lot size together with the number of deliveries per production cycle. The corresponding shortening of the production cycle results in limiting the inventory costs as the unit costs of storage increase. When the number of deliveries per production cycle falls, the batch size and production rates increase somewhat to ensure the appropriate supply of raw materials in a cycle. In Table 4.12, the Stackelberg solution exhibits a similar response.

When comparing the costs incurred under both solution strategies, the integrated solution consistently provides greater cost savings than the Stackelberg solution. This is because the Stackelberg approach prioritizes the manufacturer's process, which limits the flexibility of the suppliers, particularly when multiple deliveries are made per production cycle. Decoupling the suppliers' production rate from the manufacturer's production rate when there is only one delivery per cycle seems to provide no advantage in cost limitation. This is due to the fact that as the manufacturer's holding costs become large, the supplier's holding costs are not such an important factor in determining the optimal strategy. In this case, it is likely that the supplier would benefit from producing raw materials at a lower rate than the maximum rate when only one delivery per production cycle is made.

Effect of the Suppliers' Maximum Production Rate Relative to the Manufacturer's Maximum Consumption Rate (λ_m)

The impact of the supplier production rate on the manufacturer's consumption rate (λ_m) is analyzed in Tables 4.14, 4.15, and 4.16. From Table 4.14, the results indicate that the integrated solution is independent of the relative production-to-consumption rate. Increasing λ_m will enlarge the feasible set of solutions. However, as long as the suppliers can deliver raw materials at a sufficiently high rate, when the demand is

fixed it does not pay to change the rate of producing the final good and the suppliers should adapt to this production rate.

Integrated Solution

λ_m	Q	R	T	m	λ	P	P_s	$T.C_s$	$T.C_m$	$T.C_{sc}$
0.35	6156	6135	1368	3	0.3392	171	116	33692	81782	115470
0.40	6156	6135	1368	3	0.3392	171	116	33692	81782	115470
0.45	6156	6135	1368	3	0.3392	171	116	33692	81782	115470
0.50	6156	6135	1368	3	0.3392	171	116	33692	81782	115470
0.55	6156	6135	1368	3	0.3392	171	116	33692	81782	115470

Table 4.14: Effect of suppliers' relative production rate on the integrated solution

Stackelberg Solution

λ_m	Q	R	T	m	λ	P	P_s	$T.C_s$	$T.C_m$	$T.C_{sc}$
0.35	3744	3741	1248	2	1.1218 (0.3397)	156	350 (106)	56798	68246	125040
0.40	3744	3686	1248	2	1.2821 (0.3397)	156	400 (106)	62363	68237	130600
0.45	3720	3719	1240	2	1.4901 (0.3344)	151	450 (101)	67542	67132	134580
0.50	3624	3601	1208	2	1.6556 (0.3344)	151	500 (101)	72979	66579	139560
0.55	3624	3574	1208	2	1.8212 (0.3344)	151	550 (101)	78612	66574	145190

Table 4.15: Effect of supplier's production rate on the Stackelberg solution.

λ_m	% change in suppliers' costs	% change in manufacturer's costs	% change in total costs
0.35	68.58	-16.55	8.29
0.40	85.09	-16.56	13.10
0.45	100.47	-17.91	16.55
0.50	116.61	-18.59	20.86
0.55	133.32	-18.60	25.74

Table 4.16: Percentage changes in costs at the Stackelberg solution relative to the integrated solution

In contrast, the Stackelberg solution shows a little sensitivity to the relative production-to-consumption rate. This is due to the assumption that the first delivery batch is produced at the fastest rate to expedite the

manufacturer's production. As the relative production-to-consumption rate increases and maximum production rate fixed, the Stackelberg approach adjusts by slowly reducing the reorder level and manufacturer's lot size and at an appropriate threshold slightly decreasing the production rates. This may well be due to faster production of the first batch resulting in a longer time horizon over which the final goods can be produced.

As shown in Table 4.16, the integrated solution outperforms the Stackelberg approach in terms of overall cost savings, particularly when considering the costs incurred by the suppliers. The integrated solution achieves greater efficiency in managing the supplier's production rate and the manufacturer's consumption rate, resulting in reduced costs for both parties.

Effect of Variation in Demand for the Finished Good (σ)

Lastly, the effect of variation in demand is analyzed in Tables 4.17, 4.18, and 4.19. Table 4.17 presents the results from the integrated solution. From these results, it is observed that the model responds to an increase in variability in demand by adjusting important factors such as the manufacturer's lot size, reorder level,

Integrated Solution

σ	Q	R	T	m	λ	P	P_s	$T.C_s$	$T.C_m$	$T.C_{sc}$
5	5976	5974	1328	3	0.3494	166	116	33858	79105	112960
10	5976	5957	1328	3	0.3645	166	121	35170	79392	114560
15	6156	6095	1368	3	0.3392	171	116	33692	81494	115190
20	6156	6135	1368	3	0.3392	171	116	33692	81782	115470
25	6156	6116	1368	3	0.3538	171	121	35010	82063	117070

Table 4.17: Effect of variation in demand using the integrated solution.

Stackelberg Solution

σ	Q	R	T	m	λ	P	P_s	$T.C_s$	$T.C_m$	$T.C_{sc}$
5	3624	3573	1208	2	1.1589 (0.3344)	151	350 (101)	56194	65579	121770
10	3624	3612	1208	2	1.1589 (0.3344)	151	350 (101)	56194	65835	122030
15	3744	3663	1248	2	1.1218 (0.3397)	156	350 (106)	56798	67738	124540

20	3744	3702	1248	2	1.1218 (0.3397)	156	350 (106)	56798	67992	124790
25	3744	3741	1248	2	1.1218 (0.3397)	156	350 (106)	56798	68246	125040

Table 4.18: Effect of variation in demand using the Stackelberg solution.

σ	% change in suppliers' costs	% change in manufacturer's costs	% change in total costs
5	66.00	-17.10	7.80
10	59.78	-17.08	6.52
15	68.58	-16.88	8.12
20	65.58	-16.86	8.07
25	62.23	-16.84	6.81

Table 4.19: Percentage changes in costs at the Stackelberg solution relative to the integrated solution delivery batch size, and the production rate of both the manufacturer and the suppliers. One expects that as the variation in demand increases, the reorder point should also increase, in order to minimize the likelihood of shortages. The re-order point does generally rise. However, there are other ways of minimizing the likelihood of shortages. One could increase the size of a production lot while fixing the length of time for which the production line is action, i.e. increasing Q while keeping Q/P fixed. Such a change is observed when σ increases from 10 to 15. One could also decrease the length of time between initiation and completion of production. Increasing the suppliers' production rate while leaving everything else fixed is such a change (observed when σ changes from 5 to 10 and from 20 to 25).

For the Stackelberg solution, presented in Table 4.18, the model reacts in a similar way to the integrated solution. However, Table 4.19 shows that the integrated solution results in better financial savings, particularly due to the accumulated savings at the supplier's end. A key distinction between the two solutions is that, for the Stackelberg approach, the impact of variability is felt primarily at the manufacturer's end of the supply chain. This is because the suppliers adjust only based on the manufacturer's order and production parameters provided to them. In contrast, the integrated solution takes a more holistic view of the supply chain, considering both the manufacturer's and suppliers' end. As a

result, it makes a centralized decision that jointly and moderately affects all stakeholders, leading to a more balanced and cost-effective outcome across the entire supply chain.

4.3 Chapter Summary

This chapter has provided an in-depth analysis through numerical examples and sensitivity analysis to assess how different parameters influence the performance of both the integrated and Stackelberg solution approaches. The analysis of the mean demand, variability in demand, the relative production rate of suppliers, the holding costs of the final product, and the raw material consumption coefficient (i.e., the quantity of raw material required per unit of the final product) highlighted the different ways in which these factors influence key decisions such as production rate, lot size, reorder level, batch size, and the associated costs. A key observation is that when there are very few deliveries of the raw material per production cycle savings are possible by adopting the Stackelberg solution compared to the integrated solution. Such situations occur when the consumption coefficient (α) is small, i.e. demand for the raw material is small. This is possible because under the Stackelberg solution the production rate of the suppliers is not coupled to the production rate of the producer.

Through these numerical examples, it has become evident that the integrated solution typically provides a more balanced and cost-effective approach for both the manufacturer and the suppliers, while the Stackelberg solution tends to offer greater savings for the manufacturer, often at the expense of the suppliers. The sensitivity analysis also revealed important insights into how changes in demand, supplier production rates, and holding costs affect the overall supply chain. In particular, the integrated approach's centralized decision-making process allowed for more efficient cost savings across the entire system, while the Stackelberg approach's focus on optimizing the manufacturer's process leads to higher costs for suppliers.

Building on the results of the mathematical analysis, Chapter 5 extends the study by applying a Discrete Event Simulation (DES) model. The simulation framework is used to relax a key assumption of the

analytical formulation, namely the synchronization of dispatch times for raw material deliveries from multiple suppliers to the manufacturer's warehouse. Relaxing this assumption allows the DES model to represent more realistic operational conditions and to provide additional insights into supply chain optimization.

CHAPTER 5

SIMULATION STUDY OF SUPPLYCHAIN SYSTEMS

5.1. Introduction

In this chapter, we extend the previous model by relaxing the assumptions regarding simultaneous or synchronized dispatch times. We achieve this through simulation optimization using Arena simulation software. The main objectives of this chapter are as follows:

1. To develop a discrete-event simulation (DES) model representing the multiple suppliers, single manufacturer supply chain system illustrated in Figure 1.1.
2. To validate the simulation model by comparing its results with the analytical model presented in Chapter 3.
3. To extend the model from Chapter 3 by removing the assumption of homogeneous suppliers, allowing suppliers, for example, to deliver identical batch sizes of raw materials at varying production rates.

The chapter is structured as follows: Section 5.2 outlines the key steps involved in conducting the simulation study. Section 5.3 focuses on the development and validation of the DES simulation model for multiple homogeneous suppliers, single manufacturer system. Section 5.4 systematically relaxes the assumption of simultaneous dispatch times in the mathematical model.

5.2 Procedure for Simulation Study

Simulation is a powerful technique used to replicate the behavior of real-world systems as they evolve over time. Discrete Event Simulation (DES) models represent system operations as a sequence of logically ordered events, capturing interactions between key components within the system.

For the two-echelon supply chain problem under study, Arena simulation software, along with OptQuest optimization, is utilized to develop and analyze the DES model. The process of building an effective simulation model involves several key steps, outlined as follows:

1. **Formulation of the Problem:** The first step involves clearly defining the problem to be studied. This requires close collaboration between the decision-maker and the analyst to establish a shared understanding of the system's key challenges, constraints, and objectives.
2. **Setting Objectives and Planning the Study:** At this stage, the scope of the simulation is determined. This includes defining the level of detail required, configuring the system (such as time frames, available resources, and operational constraints), and selecting the appropriate software tools. Additionally, performance metrics—such as the total cost incurred by suppliers, optimal production lot size, or average shortages per unit time—are identified to assess the effectiveness of the model.
3. **Data Collection and Refinement of the Problem:** Gathering accurate and relevant data is critical to constructing a reliable model. This step involves defining the conceptual framework of the simulation, identifying key parameters, and setting baseline conditions to ensure that the model realistically reflects a real-world supply chain system.
4. **Validation of the Conceptual Model:** Before proceeding with implementation, the conceptual model must be reviewed and validated. The project manager plays a crucial role in ensuring that all critical details are accounted for during the modeling phase, minimizing the risk of major revisions later in the process.
5. **Verification of the Model:** Using specialized simulation software (such as Arena), a computer-based model of the supply chain is built. The verification process ensures that the model functions as intended, accurately reflecting the system's logical structure and operational flow.
6. **Validation of the Model:** The accuracy of the simulation model is assessed by comparing its outputs against known performance metrics or real-world data. This benchmarking process ensures that the model produces reliable and meaningful results.
7. **Experiment Design, Simulation Optimization and Output Analysis:** In the final stage, various operational parameters and scenarios are specified for analysis. This includes running multiple

simulations to test different conditions, optimizing performance using tools such as OptQuest, and thoroughly documenting the findings. The results provide valuable insights into supply chain dynamics, enabling informed decision-making and strategic improvements. Figure 5.1 illustrates this process of modeling simulations in a structured manner.

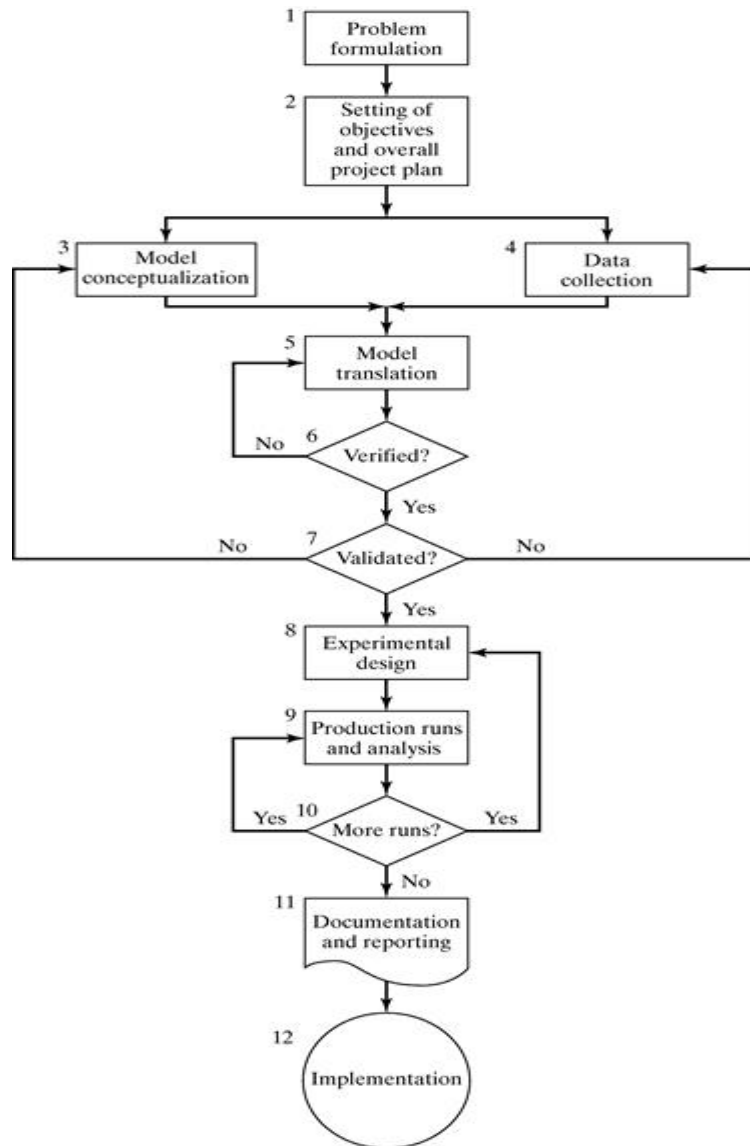


Figure 5.1: A flow chart depicting the basic steps of simulation, (Banks et.al 2007)

Since the problem addressed in this chapter is theoretical, we systematically adopt and modify steps 3, 5, 6, and 7 from Adegbola (2023). The detailed modifications are outlined below:

1. **Development of a Conceptual Model:** The conceptual model is designed to accurately represent the supply chain structure illustrated in Figure 1.1. This model serves as the foundation for the simulation by defining the interactions between multiple suppliers and a single manufacturer under different production rate settings.

2.Computer Modeling and Verification: Arena Simulation Software (version 16.0) is used to develop the simulation model, replicating the multiple suppliers, single manufacturer (MSSM) supply chain problem under both homogeneous and heterogeneous production rates among the suppliers. Appendix B provides a brief summary of the functions of the simulation modules used in model development. Detailed explanations of the simulation model's construction are presented in Section 5.3, while Appendix C outlines the logic used in the simulations for both homogeneous and heterogeneous production scenarios. To verify the accuracy and correctness of the simulation model, the following verification techniques were applied

- a.
 - i. Animation of entities in the simulation model to visualize system dynamics
 - ii. Systematic inspection of sub-models to ensure consistency and correctness.
- b. The simulation model's accuracy is further assessed by comparing the estimated costs with results from the analytical model. Specifically, the optimal values of the decision variables obtained from the analytical model are used as inputs in the simulation model, and the resulting costs are compared under the assumption of deterministic demand.

3. Experiment Design, Validation (Simulation Optimization), and Output Analysis:

I. Output Analysis-The integrated MSSM supply chain model, assuming a homogeneous production rate, is simulated for an infinite planning horizon (which is implicitly implied by the goal of minimizing costs per unit time). Since simulations must terminate in a reasonable time, determining the steady-state behavior of a system is essential. To achieve this:

- A warm-up period is established using Arena's Output Analyzer to remove initial transient effects.

- The running time and number of replications are set sufficiently high to ensure reliable and statistically significant results.

II. System Optimization- OptQuest, an advanced optimization tool within Arena, is employed to optimize the MSSM supply chain model. It facilitates the search for an optimal solution to minimize the long-run average cost by evaluating different values of the decision variables. Using OptQuest, users can define:

- Decision variables (key controllable parameters in the model).
- Constraints (limitations on system performance).
- Objective function (performance metric to be optimized, such as cost minimization).

The optimized simulation results are then compared with those from the analytical model to assess possible improvements in performance.

III. Experiment Design: Experimental simulations are conducted by varying key model parameters to analyze the robustness of the system. This variation helps evaluate performance under different conditions and provides valuable managerial insights based on observed trends and behavior patterns. These steps ensure a structured approach to developing, validating, optimizing, and analyzing the simulation model for the MSSM supply chain problem.

5.3 The Homogeneous Multiple Suppliers, Single Manufacturer Simulation Model

To develop and validate a simulation model for a heterogeneous Multiple Suppliers, Single Manufacturer (MSSM) supply chain, where suppliers dispatch raw materials at different times, it is first necessary to establish a validated simulation model for the homogeneous MSSM system. This homogeneous model serves as the baseline scenario, as it has already been mathematically formulated and optimized.

The validation process begins by comparing the results of the homogeneous MSSM simulation model with those obtained from the analytical model. If the simulation model accurately replicates the analytical results, it confirms the correctness of the simulation approach. Once validated, the homogeneous model is systematically extended to determine the optimal batch sizes to be supplied by the suppliers, while

simultaneously relaxing the assumption of synchronized dispatch times imposed in the mathematical formulation. This step-by-step refinement ensures a logical transition from the base case to a more complex and realistic heterogeneous MSSM supply chain model. Figure 1.1 illustrates the homogeneous version of the problem, where suppliers are assumed to be homogeneous in terms of their production rate and batch size. To develop and validate the homogeneous MSSM simulation model, Arena Simulation Software 16.0 and the OptQuest optimization package were used. The conceptual logical framework for developing the simulation model for the multiple suppliers, single manufacturer (MSSM) supply chain problem is presented in Figure 5.2.

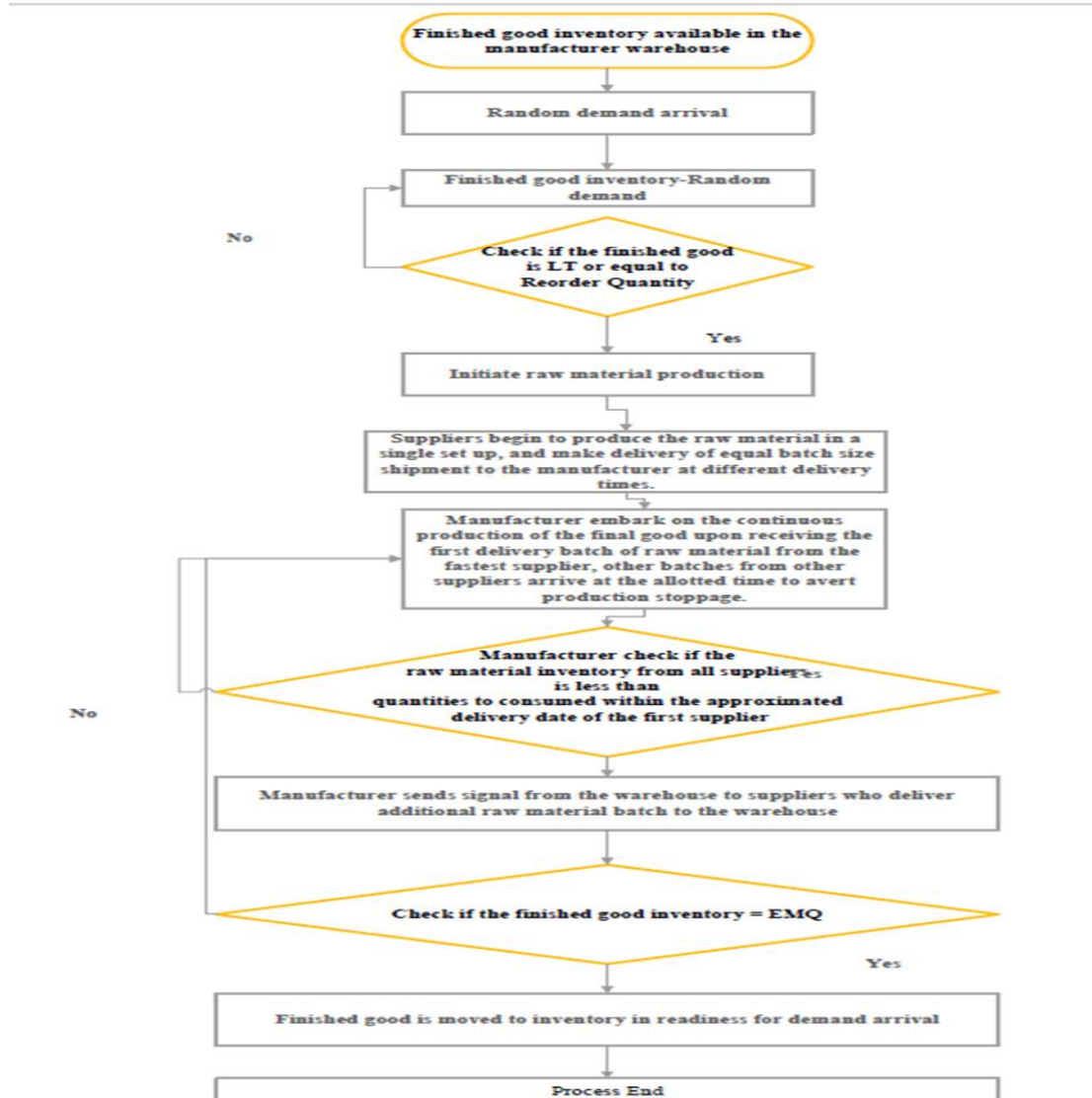


Figure 5.2: The conceptual model for simulating the homogenous multiple suppliers, single manufacturer supply chain problem

To provide insight into model development, the system was divided into three sub-models: (1) the finished goods warehouse of the manufacturer, (2) the raw material warehouse and production system of the manufacturer, and (3) the raw material storage and production systems of the suppliers. The development of each sub-model is enumerated below:

1. The finished goods warehouse of the manufacturer- The volume of finished goods in storage is defined by a variable called the inventory level. The create module in Arena generates customers, and the time between each customer's arrival is fixed using the expression in the advanced process module as shown in Figure 5.3 and Figure 5.4. The inventory of finished goods is randomly depleted by demand which is assumed to be normally distributed with a known mean and standard deviation i.e. $N(\mu, \sigma)$.

Figure 5.3: Create module used in generating customers at a specified time between arrivals.

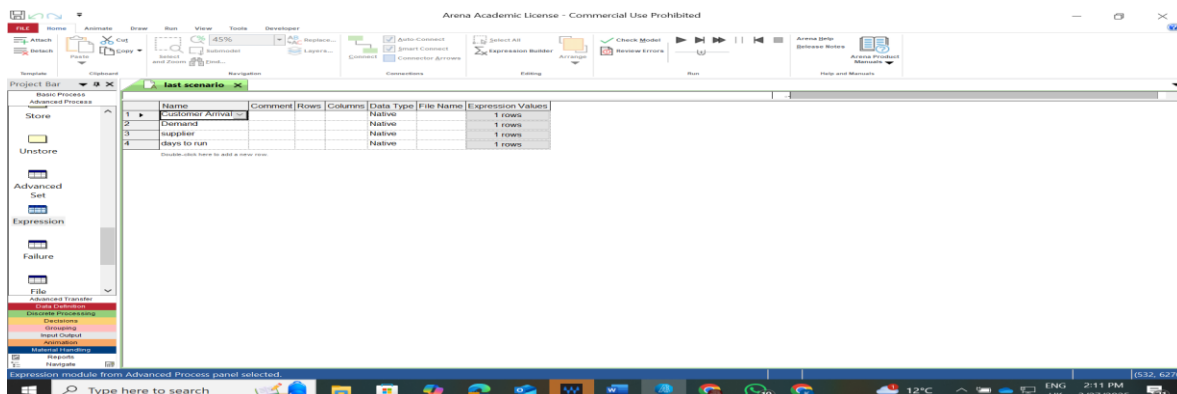


Figure 5.4: Expression module used in defining customers' arrival times

Next, through the assign module, the inventory level is updated by taking away the random demand from the current inventory level. The decision module continuously checks whether the inventory level is less than or equal to the reorder level. If so, then a joint order is made through a signal that is sent to suppliers to initiate raw material production. The logic of the simulation for the manufacturer's end of the supply chain is presented in Figure 5.5

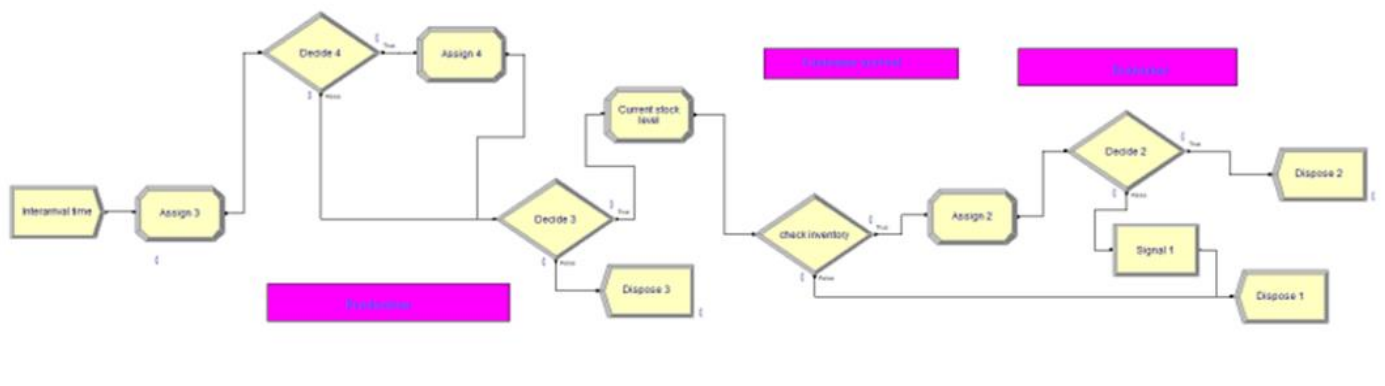


Figure 5.5: Logic of the Arena simulation used for the consumption of the manufactured products and the ordering of raw materials from the suppliers

Similar to the manufacturer sub-model, modeling of a supplier's activities in the simulation commence with the *Create* module. This module is configured to generate a single arrival containing 999,999,999 entities, as illustrated in Figure 5.6, to reflect long-term planning in the supply chain model. These entities are then placed in a queue by the *Hold* module, as depicted in Figure 5.7. The system releases only one entity per cycle upon receiving a signal from the manufacturer, indicating that the supplier should initiate raw material production. Each supplier operates with a single production setup, with the first batch being delivered immediately after production, as represented by the first delivery line in Figure 5.8. Within a production cycle, subsequent batches of raw materials are transferred into the supplier's inventory, as shown in Figure 5.9, where they remain until a replenishment order is issued (at no cost) by the manufacturer's warehouse. This order is triggered when the consumption of raw materials reduces the amount of raw materials

remaining from a delivery cycle (both delivered and to be delivered later in the current cycle) to a level equal to the quantity required to cover the lead time of the fastest supplier, thereby preventing any interruption in production. This continues until the m -th batch has been ordered. Since it is assumed that the supply of raw materials is deterministic and ensures a sufficient supply to the manufacturer a) these orders do not require a supplier to reinitiate production and b) in practice, such a policy is equivalent to one where the delivery times of all of the batches sent by the suppliers in a production cycle are determined when the manufacturer initiates a production cycle.

It is important to note that the initial replenishment order for raw materials is triggered when the inventory of finished goods available for sale falls below the predetermined reorder level as a result of sales activities managed at the supervisory level. Subsequent replenishment requests are generated endogenously by the production system when the available raw material inventory decreases to the quantity required to sustain production over the lead time of the fastest supplier. Upon receipt of a replenishment signal from the production system, the immediate dispatch of a batch of raw materials is initiated by each supplier over the remaining $(m - 1)$ independent delivery epochs, given that the first delivery is initiated from the management level. These dispatches are fulfilled from the suppliers' accumulated inventories and are transported to the manufacturer's warehouse according to supplier-specific delivery lead times, which are primarily determined by the geographical proximity of each supplier to the manufacturer. Furthermore, owing to the non-terminating nature of the simulation model, a renewal condition is imposed to ensure a well-defined and repeatable cycle structure. Specifically, the inventory level of finished goods available for sale within each cycle must fall to or below the reorder quantity before suppliers are re-engaged to produce raw materials within a single production run. This condition prevents overlapping production decisions across successive cycles and maintains consistency in supplier engagement. Figure 5.10 illustrates the logical framework governing supplier operations as implemented in the ARENA simulation environment.

Create

Name: Entity Type:

Time Between Arrivals

Type: Expression: Units:

Entities per Arrival: Max Arrivals: First Creation:

Figure 5.6: Create module used in generating entities required to initiate the supplier's system of raw material production

Hold

Name: Type:

Wait for Value: Limit:

Queue Type:

Queue Name:

Figure 5.7: Hold module waiting for signal 83 to illustrate how the production and delivery of raw materials is initiated through signal received from the manufacture. Note 83 denotes an arbitrary signal used to control and coordinate the Hold module.

Lastly, the model of the manufacturer's warehouse and production system sub-model, illustrated in Figure 5.11, begins with the *Create* module, which generates a sufficient number of entities. These entities are then placed on hold within the *Hold* module, awaiting a signal from the supplier sub-model. This signal is triggered when the raw material in transit is delivered to the *Station* module via the *Route* module (part of the *Advanced Transfer* system) after a predetermined transportation time, as shown in Figure 5.10.



Figure 5.8: Arena simulation logic used for batching and dispatching the first delivery from Supplier i to the manufacturer's warehouse

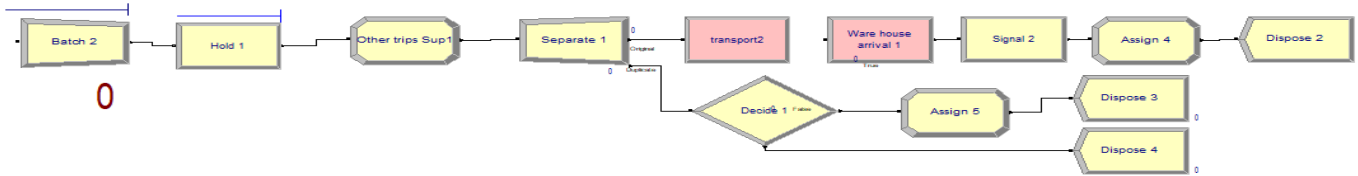


Figure 5.9: Arena simulation logic for batching, holding, and dispatching $(m - 1)$ deliveries from Supplier i to the manufacturer's warehouse

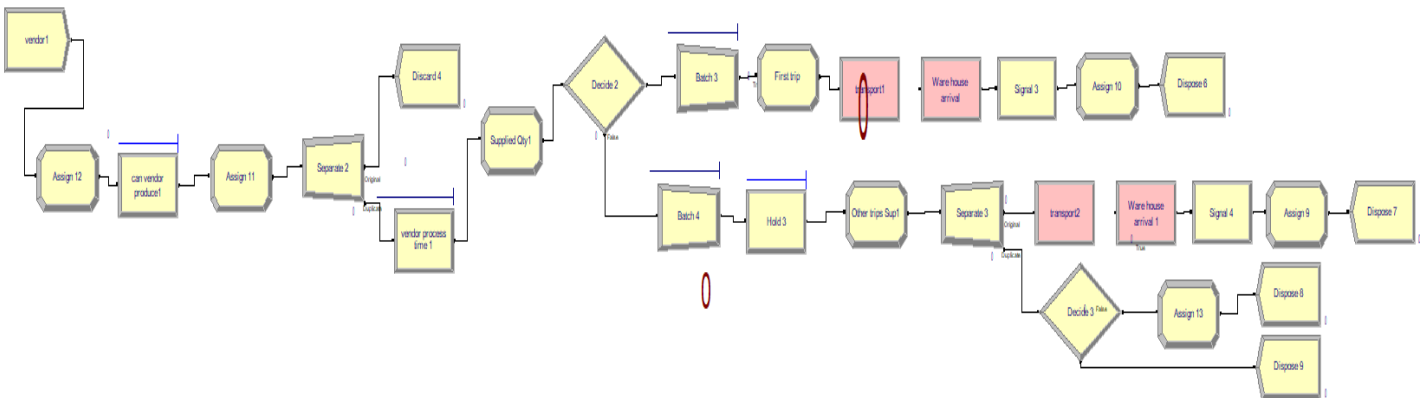


Figure 5.10: Logic of simulating the batching and dispatching of raw materials from Supplier i to the manufacturer

Upon receiving the signal, a single entity is released and expanded into a batch of the required size through the *Separate* module. Each entity within the batch then moves into another *Separate* module, where it is split into two streams. The first entity proceeds directly to production via the *Process* module, which seizes, delays, and releases it according to a unit processing time. This sequence of operations determines the manufacturer's lot size, which is then added to the inventory of the final product at the end of a production cycle.

The second split entity is sent to an identical *Process* module, operating at the same unit rate as the first. However, this module specifically monitors the amount of unconsumed raw materials from a delivery cycle. When this amount falls to a predetermined threshold, which is typically the quantity required to sustain production during the transit period of the fastest supplier, the system triggers a replenishment order. It is important to note that each supplier completes ' m ' deliveries per production cycle. The first delivery is initiated when the inventory of the final good drops below the reorder point. Subsequent deliveries are scheduled on the basis of the amount of unconsumed raw materials from a delivery cycle (as described above). The logical framework of this system in *Arena* is presented in Figure 5.11.

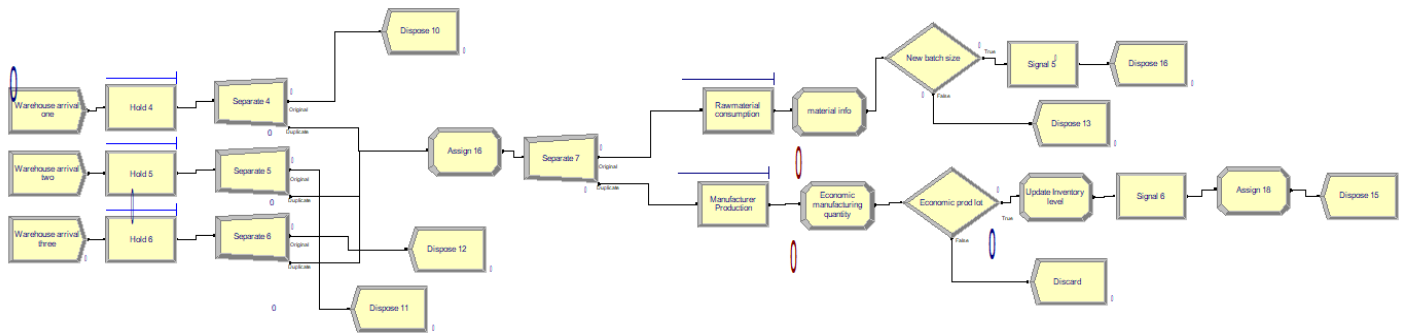


Figure 5.11: Logic of simulating the replenishment of raw materials and the manufacturer's production system

5.3.1 Verification and validation of the homogeneous MSSM supply chain simulation model

Verification of the Model

The model of the MSSM supply chain was verified to ensure it was correctly implemented and aligned with its conceptual framework. This was conducted through:

1. Entity Animation in the Simulation Model – Entities were animated to visualize system dynamics and confirm that model behavior matched real-world expectations.

2. Systematic Inspection of Sub-Models – Each sub-model was carefully examined to check for consistency and correctness, ensuring that all system components interacted as intended.
3. Using the values of the optimal decision variables from the analytical model as inputs into the simulation model and ensuring deterministic demand by setting the standard deviation of demand to a very low value to eliminate stochastic effects.

Validation of the Model

To further strengthen the validation process, output analysis and system optimization were conducted as follows:

I. Output Analysis

Since the MSSM supply chain model, under the assumption of a homogeneous production rate, is non-terminating, its steady-state behavior was determined through:

- Determination of the Warm-Up Period – A six-month warm-up period was established using the Output Analyzer to eliminate initial transient effects and allow the model to reach a stable operational state.
- Simulation Runtime and Replications – The model was simulated for 50 years with 20 replications, ensuring sufficiently large sample sizes for obtaining reliable and statistically significant results.

II. System Optimization

After comparing results for the purpose of verification, the model was further validated using OptQuest in Arena as an optimization tool. Under this condition:

- The simulation model was no longer dependent on solutions derived from the analytical model.

- Instead, independent optimization was performed based on predefined parameters, decision variables, constraints, and objective functions.

For the homogeneous production rate MSSM supply chain model, the decision variables (controls) included:

1. Batch Size – The quantity of raw materials in each delivery.
2. Jointly Determined Production Rate – The rate at which production occurs, considering the demand and supply constraints.
3. Manufacturer's Lot Size – The quantity of finished goods produced per cycle.
4. Reorder Level – The level of inventory of the final good that triggers a new production cycle.
5. Number of Deliveries – The number of batches delivered by each supplier in a production cycle.

The constraints applied to the model were:

1. Uniform Batch Sizes – All suppliers must supply raw materials in batches of identical size.
2. Synchronized Dispatch Times – The i -th batch from each supplier in a delivery cycle is dispatched at the same time ($i = 1, 2, \dots, m$).
3. Restriction on the Manufacturer's Lot Size – The reorder level cannot exceed the lot size.
4. Minimum Requirement for Raw Material Stocks – When production is active, the manufacturer must always have at least one unit of raw material available to guarantee continuous production.
5. Relation between Lot Size and Consumption of Raw Materials – The manufacturer's lot size must be equal to the amount of raw materials delivered in a production cycle divided by the consumption conversion parameter, α .

The objective of the optimization process was to minimize the long-run average cost, thus ensuring the most cost-effective operation of the MSSM supply chain model.

Using data presented in Table 4.1, Table 5.1 presents the statistical data collected from the simulation model. Tables 5.2 to 5.3 provide a detailed comparison of the cost performance between the simulation model and the analytical model as part of the verification process. This comparison was conducted using both the integrated solution and Stackelberg solution approaches discussed in Chapter 3, with the standard deviation of demand per unit time being equal to 0.0001.

Furthermore, Tables 5.4 to 5.13 present the results obtained during the validation phase using the OptQuest Optimizer in Arena. These results demonstrate the optimized performance of the MSSM supply chain model under both the integrated and Stackelberg solution approaches, further validating the model's accuracy and effectiveness.

<i>Supplier set up cost</i>	<i>Output</i>
<i>Supplier holding cost</i>	<i>Time persistent</i>
<i>Supplier acquisition cost</i>	<i>Output</i>
<i>Manufacturer holding cost for in-sale finished good</i>	<i>Time persistent</i>
<i>Shortage cost</i>	<i>Time persistent</i>
<i>Holding cost of manufacturer for finished goods in production</i>	<i>Time persistent</i>
<i>Raw material holding cost in manufacturers warehouse</i>	<i>Time persistent</i>
<i>Production cost of raw material</i>	<i>Output</i>
<i>Production cost of finished good</i>	<i>Output</i>
<i>Manufacturer Set up cost and transportation cost</i>	<i>Output</i>
<i>Raw material ordering cost</i>	<i>Output</i>
<i>Raw material acquisition cost by the manufacturer</i>	<i>Output</i>
<i>Long run average total cost</i>	<i>Output</i>

Table 5.1: Statistics gathered from the simulation model.

$E[\Lambda]$	Q	R	T_L	m	P	P_s	Mathematical Modelling			Simulation Modelling			% Difference		
							$T.C_s$	$T.C_m$	$T.C_{Sc}$	$T.C_s$	$T.C_m$	$T.C_{Sc}$	$T.C_s$	$T.C_m$	$T.C_{Sc}$
120	5976	5934	1328	3	166	116	33858	78815	112670	33841	78752.49	112593.49	0.05	0.08	0.07
140	7056	6996	1568	3	196	131	43013	101750	144760	43547.84	101339.19	144887.03	-1.23	0.41	-0.08
160	7956	7953	1768	3	221	151	55189	125210	180400	55237.51	125109.53	180347.03	-0.09	0.08	0.03
180	11088	11066	1848	4	231	161	65940	155730	221671	66802.80	155335.84	222138.64	-1.29	0.25	-0.21
200	12288	12263	2048	4	256	181	80946	184520	265470	80924.39	184770.16	265694.55	0.03	-0.14	-0.08

Table 5.2: Verification of the simulation model using the integrated solution approach

$E[\Lambda]$	Q	R	T_L	m	P	P_s	Mathematical Modelling			Simulation Modelling			% Difference		
							$T.C_s$	$T.C_m$	$T.C_{Sc}$	$T.C_s$	$T.C_m$	$T.C_{Sc}$	$T.C_s$	$T.C_m$	$T.C_{Sc}$
120	3762	3762	1254	2	146	350 (101)	56538	65153	121690	56747	65643	122390	-0.37	-0.75	-0.57
140	4224	4203	1408	2	176	350 (121)	68310	84127	152440	69378	83700	153078	-1.53	0.51	-0.41
160	4944	4913	1648	2	206	350 (141)	81179	107420	188600	82344	106922	189226	-1.42	0.47	-0.33
180	5664	5651	1888	2	236	350 (161)	94867	133540	228400	95508	133233	228741	-0.67	0.23	-0.20
200	6504	6439	2168	2	271	350 (181)	109330	165100	274430	110484	164523	275008	-1.05	0.35	-0.21

Table 5.3: Verification of the simulation model using the Stackelberg solution approach

From Tables 5.2 and 5.3, the results of the verification of the simulation model are presented by comparing its cost outputs with those of the analytical model under two solution methodologies: the integrated solution and the Stackelberg solution. The simulation model closely replicates the cost performance of the analytical model, with the percentage differences across supplier cost ($T.C_s$), manufacturer cost ($T.C_m$), and total supply chain cost ($T.C_{Sc}$) remaining within a narrow margin (less than $\pm 1.6\%$ for both the suppliers and the manufacturers and less than $\pm 0.6\%$ for overall costs). This level of accuracy confirms that the simulation model is a valid representation of the analytical model, especially under deterministic conditions where the standard deviation of demand is reduced to near zero.

In terms of performance comparison, the integrated solution, which involves centralized decision-making across the supply chain, tends to produce slightly lower overall costs. Conversely, the Stackelberg Solution, which assumes a leader–follower dynamic where the manufacturer optimizes first and suppliers adapt accordingly, results in only marginally higher costs but offers a more decentralized and practically implementable approach especially in systems where centralized control is not feasible.

The minimal cost differences and consistent trends across varying demand levels demonstrate that simulation modeling can serve not only as a robust verification tool, but also as an independent platform for analyzing and managing complex, real-world supply chain scenarios that involve dynamic lead times and decision interdependencies.

Next, we present the results from OptQuest under both the integrated solution and Stackelberg solution techniques for validating the simulation model. The process is assumed to be continuous over an infinite horizon, with a steady state achieved after a warm-up period of six months. To ensure statistical reliability, the simulation was run for 50 years with 20 replications. Additionally, OptQuest was truncated after approximately 70,000 replications due to the extensive computational requirements of performing a local search over the solution space of the decision variables. Figures 5.12 showed optimization performed using OptQuest software in Arena.

Model validation using OptQuest optimizer under the integrated solution approach

	$E[\Lambda]$	Q	R	T_L	m	λ_s	P	P_s	$T.C_s$	$T.C_m$	$T.C_{sc}$
Mathematical Modelling	120	5976	5934	1328	3	0.349	166	116	33858	78815	112670
Simulation Modelling	120	5976	5956	1328	3	0.339	166	113	32962.41	79142.35	112104.76
Percentage (%) difference									2.71	-0.41	0.50

Table 5.4: Comparison of optimal results from OptQuest (simulation) and integrated solution (mathematical modeling).

	$E[\Lambda]$	Q	R	T_L	m	λ_s	P	P_s	$T.C_s$	$T.C_m$	$T.C_{Sc}$
Mathematical Modelling	140	7056	6996	1568	3	0.334	196	131	43013	101750	144760
Simulation Modelling	140	7056	7030	1568	3	0.332	195	130	43179.17	101288.72	144407.89
Percentage (%) difference									-0.39	0.46	0.24

Table 5.5: Comparison of optimal results from OptQuest (simulation) and integrated solution (mathematical modeling).

	$E[\Lambda]$	Q	R	T_L	m	λ_s	P	P_s	$T.C_s$	$T.C_m$	$T.C_{Sc}$
Mathematical Modelling	160	7956	7953	1768	3	0.342	221	151	55189	125210	180400
Simulation Modelling	160	7956	7952	1768	3	0.342	221	151	54975.10	125253.54	180228.64
Percentage (%) difference									0.39	-0.04	0.10

Table 5.6: Comparison of optimal results from OptQuest (simulation) and integrated solution (mathematical modeling).

	$E[\Lambda]$	Q	R	T_L	m	λ_s	P	P_s	$T.C_s$	$T.C_m$	$T.C_{Sc}$
Mathematical Modelling	180	11088	11066	1848	4	0.349	231	161	65940	155730	221671
Simulation Modelling	180	11088	11082	1848	4	0.345	231	160	66078.98	155249.82	221328.80
Percentage (%) difference									-0.21	0.31	0.16

Table 5.7: Comparison of optimal results from OptQuest (simulation) and integrated solution (mathematical modeling).

	$E[\Lambda]$	Q	R	T_L	m	λ_s	P	P_{s1}	$T.C_s$	$T.C_m$	$T.C_{Sc}$
Mathematical Modelling	200	12288	12263	2048	4	0.354	256	181	80946	184520	265470
Simulation Modelling	200	12288	12273	2048	4	0.350	256	179	80918.90	184034.02	264952.93
Percentage (%) difference									0.03	0.26	0.20

Table 5.8: Comparison of optimal results from OptQuest (simulation) and integrated solution

(mathematical modeling).

Model validation using OptQuest optimizer under Stackelberg solution approach

	$E[\Lambda]$	Q	R	T_L	m	λ_s	P	P_s	$T.C_s$	$T.C_m$	$T.C_{sc}$
Mathematical Modelling	120	3762	3762	1254	2	1.1986 (0.3459)	146	350 (101)	56538	65153	121690
Simulation Modelling	120	3762	3761	1254	2	1.1986 (0.3459)	146	350 (101)	56736.58	65120.44	121857.02
Percentage (%) difference									-0.35	0.05	-0.14

Table 5.9: Comparison of optimal results from OptQuest (simulation) and Stackelberg solution (mathematical modeling).

	$E[\Lambda]$	Q	R	T_L	m	λ_s	P	P_{s1}	$T.C_s$	$T.C_m$	$T.C_{sc}$
Mathematical Modelling	140	4224	4203	1408	2	0.9943 (0.3438)	176	350 (121)	68310	84127	152440
Simulation Modelling	140	4224	4203	1408	2	0.9943 (0.3438)(	176	350 (121)	69377.89	83700.39	153078.29
Percentage (%) difference									-1.53	0.51	-0.41

Table 5.10: Comparison of optimal results from OptQuest (simulation) and Stackelberg solution (mathematical modeling).

	$E[\Lambda]$	Q	R	T_L	m	λ_s	P	P_{s1}	$T.C_s$	$T.C_m$	$T.C_{sc}$
Mathematical Modelling	160	4944	4913	1648	2	0.8495 (0.3422)	206	350 (141)	81179	107420	188600
Simulation Modelling	169	4944	4911	1648	2	0.8495 (0.3398)	206	350 (140)	81300.95	107066.70	188367.60
Percentage (%) difference									-0.15	0.33	0.12

Table 5.11: Comparison of optimal results from OptQuest (simulation) and Stackelberg solution (mathematical modeling).

	$E[\Lambda]$	Q	R	T_L	m	λ_s	P	P_s	$T.C_s$	$T.C_m$	$T.C_{sc}$
Mathematical Modelling	180	5664	5651	1888	2	0.7415 (0.3411)	236	350 (161)	94867	133540	228400
Simulation Modelling	180	5664	5653	1888	2	0.7415 (0.3432)	236	350 (162)	94677.64	133780.8	228458.50
Percentage (%) difference									0.20	-0.18	-0.00026

Table 5.12: Comparison of optimal results from OptQuest (simulation) and Stackelberg solution (mathematical modeling).

	$E[\Lambda]$	Q	R	T_L	m	λ_s	P	P_s	$T.C_s$	$T.C_m$	$T.C_{sc}$
Mathematical Modelling	200	6504	6439	2168	2	0.6458 (0.3339)	271	350 (181)	109330	165100	274430
Simulation Modelling	200	6504	6439	2168	2	0.6458 (0.3339)	271	350 (181)	110484.36	164523.22	275007.58
Percentage (%) difference									-1.05	0.35	-0.21

Table 5.13: Comparison of optimal results from OptQuest (simulation) and Stackelberg solution (mathematical modeling).

From Tables 5.4 to 5.13, a detailed comparison is presented between the optimized outcomes from the simulation model using OptQuest, and those obtained through the analytical model under both the integrated and Stackelberg solution approaches. The purpose of this comparison is to validate the effectiveness and accuracy of the developed simulation model in replicating or closely approximating the results achieved through mathematical modeling. Across the tables, it is evident that the percentage differences between the total costs (supplier cost ($T.C_s$), manufacturer cost ($T.C_m$), and total supply chain cost ($T.C_{sc}$)) obtained from the simulation and those from the analytical model remain consistently low, generally within $\pm 2\%$ and within $\pm 0.5\%$ for total costs. This minimal variation demonstrates that the simulation model, when optimized using OptQuest, is capable of closely mirroring the cost structures derived analytically, even under varying levels of demand ($E[\Lambda]$).

More importantly, this validation reinforces the credibility of simulation as a robust alternative in cases where mathematical formulation becomes highly complex or intractable, particularly in dynamic and stochastic environments. For instance, when assumptions such as simultaneous dispatch times are relaxed, or when the variability in supplier proximity and production rates introduces additional layers of complexity, analytical solutions may become cumbersome or NP-hard. In contrast, the simulation model retains its applicability and accuracy in such scenarios. Thus, the results affirm that simulation modeling,

supported by optimization tools like OptQuest, can effectively address complex supply chain problems, providing reliable solutions even under conditions where mathematical models may fail or be limited in scope. This establishes simulation as not only a valid surrogate for analytical methods, but also a potentially more flexible and scalable approach for supply chain optimization under realistic and uncertain operational settings.

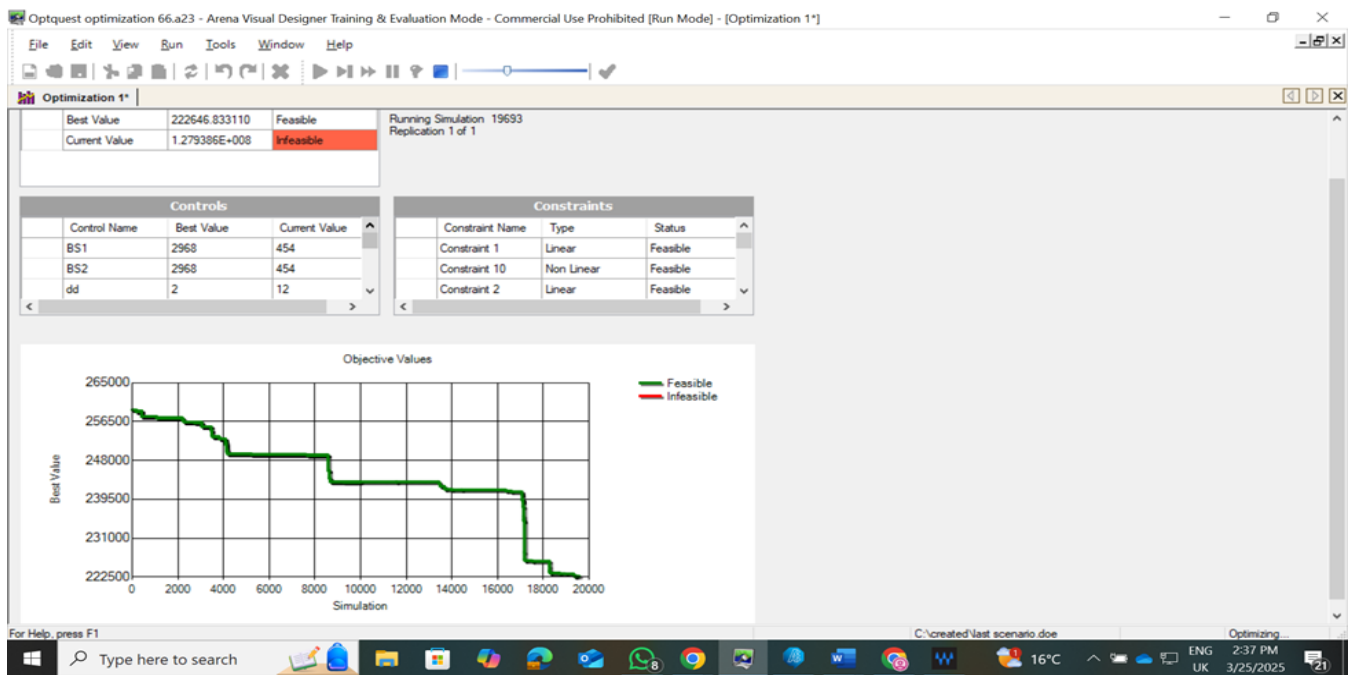


Figure 5.12: Optimization of decision variables using OptQuest.

5.4 Simulation of the Heterogenous Multiple Supplier, Single Manufacturer Model with Varying Dispatch Times

The heterogeneous multiple suppliers–single manufacturer (MSSM) simulation model, characterized by varying dispatch times, is a systematic extension of the homogeneous MSSM model previously described in Section 5.3. While both models address the coordination between multiple suppliers and a single manufacturer, the key distinction lies in the production capacities of the suppliers. In the homogeneous case, all of the suppliers were assumed to have equal (both maximal and actual) production rates, although their proximity to the manufacturer varied. This assumption simplified logistical timetabling, e.g. making

it relatively easy to determine which supplier would be the first to deliver a batch to the manufacturer based on distance. In contrast, the heterogeneous setting introduces a more complex and realistic scenario in which suppliers possess different capacities, meaning that they naturally produce raw materials at different rates. Despite these differences, under various approaches suppliers may all be either required to deliver batches of the same size or make the same number of shipments to the manufacturer in a production cycle. However, the combination of different production rates and delivery distances creates a dynamic environment where determining the sequence of the deliveries from the suppliers becomes non-trivial. For instance, a batch from a faster-producing supplier located far away from the manufacturer may arrive later than a slower-producing supplier located nearby. This dynamic interplay between production and delivery times means that the lead time associated with any given supplier may become crucial, as it is now a function of production rate, batch size and the time required for delivery (i.e., proximity to the manufacturer's warehouse).

Unlike the previously discussed homogeneous case, where simultaneous dispatch times allowed for easier analytical treatment, this level of heterogeneity adds significant complexity to the modeling process. The traditional mathematical modeling approach becomes rigid and increasingly impractical, especially when accounting for real-world constraints and variation. As a result, we adopt simulation modeling and optimization techniques to address this problem more effectively. Simulation offers the flexibility to model such complex, dynamic systems while still providing insight into system performance and decision-making. To support this approach, Figure 5.13 presents a sample inventory profile that captures the flow of raw materials under a heterogeneous MSSM setting, where production rates, dispatch times, and delivery distances differ, but there is a common batch size. Additionally, Figure 5.14 illustrates the conceptual simulation model developed to represent this extended framework. These visualizations provide further evidence of the applicability of simulation in addressing challenges to supply chain coordination that are otherwise difficult to capture through purely analytical methods.

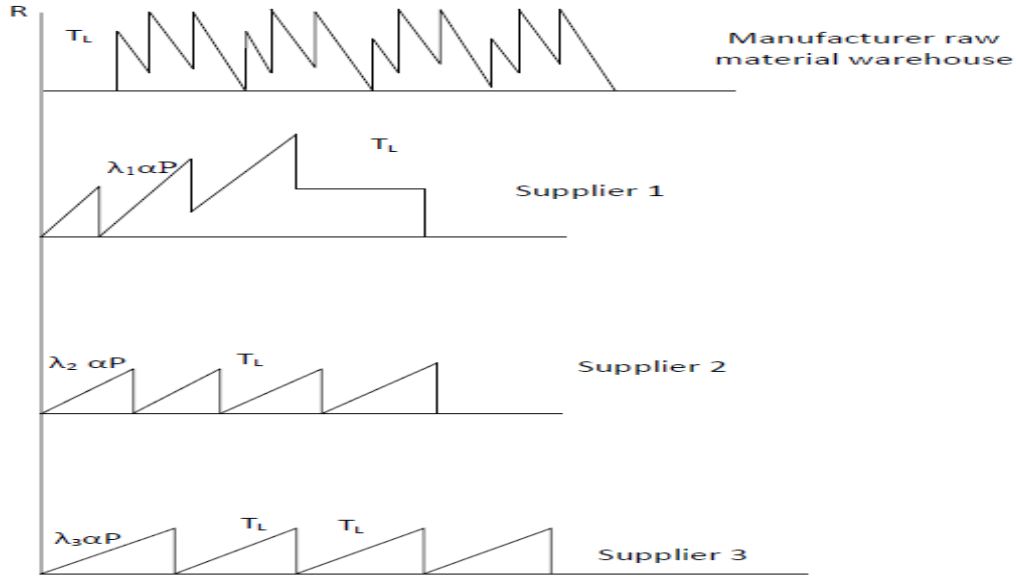


Figure 5.13: Heterogeneous suppliers dispatching batches of equal size to the manufacturer's warehouse at differing production rates.

As shown in Figure 5.13, heterogeneous suppliers are each required to dispatch the same number of batches of identical size to the manufacturer while operating at differing production rates. Despite the uniformity in batch size and the number of shipments per production cycle, the variation in production capacities among the suppliers introduces significant complexity in the delivery process.

This heterogeneity leads to differences in the suppliers' ability to meet replenishment requests at the time they are required by the manufacturer. Specifically, only those suppliers with faster production rates and sufficient accumulated inventory (equal to or greater than the required batch size) can respond promptly to the manufacturer's demand and dispatch a shipment without delay. These high-capacity suppliers often continue production at a rate that allows them to build and maintain controlled inventory levels. Consequently, they may fulfill future replenishment orders from existing stocks and even suspend production midway through the manufacturer's production cycle once the inventory suffices.

In contrast, suppliers with lower production rates typically lack the inventory necessary to meet immediate demand for replenishment. Such suppliers must continue production beyond the point of request until the

required batch size is completed, after which shipment occurs. Unlike their high-capacity counterparts, such suppliers operate under near zero-inventory conditions and deliver raw materials just-in-time. Overproduction or partial deliveries are not permissible; all of the suppliers are required to deliver the exact batch size as requested, with no flexibility for an excess or shortfall.

A major distinction between the two modeling frameworks lies in how replenishment timing at the manufacturer is treated. In the analytical model, replenishment follows a predetermined cyclic structure. The manufacturer initiates a new order when the remaining raw material in the current cycle reaches the level required to sustain production until the first delivery of the subsequent cycle. Under this formulation, reorder timing is implicitly fixed once the production cycle is defined, resulting in a structured and synchronized decision rule.

In contrast, the simulation model endogenizes replenishment timing by treating it as a decision variable. Rather than adhering to a fixed cycle, the timing of orders is dynamically optimized within the discrete-event simulation environment. This allows replenishment decisions to adjust in real time to variations in supplier production rates, delivery lead times, and operational constraints. By embedding reorder decisions within event-based system states, the simulation captures the operational interdependencies inherent in a heterogeneous multi-supplier configuration. This approach is consistent with prior research that models dynamic reorder timing through discrete-event logic rather than static analytical expressions (Grewal et al., 2015), although the present study extends the concept to a coordinated multi-supplier setting with explicit optimization of timing decisions. The complexity and timing sensitivity of the heterogeneous supplier–manufacturer system underscores the limitations of rigid analytical synchronization assumptions. These characteristics justify the adoption of simulation coupled with optimization, which provides a flexible and robust framework for analyzing such interdependent systems. By relaxing the strict synchronization

embedded in the analytical formulation, the simulation model offers a more realistic representation of operational decision-making in multi-supplier supply chains.

The simulation–optimization experiments were conducted using the data in Tables 4.1 and 4.3 together with the assumptions specified in Section 5.3.1, except for supplier homogeneity. Sensitivity analyses on key system parameters were performed to evaluate robustness. The results for the equal-batch policy solved through discrete-event simulation with optimization are presented in Tables 5.14(a)–5.17(a). A parallel sensitivity analysis was carried out for the heterogeneous MSSM configuration with unequal batch deliveries, solved using the analytical optimization model developed in Chapter 3, and the corresponding results are reported in Tables 5.14(b)–5.17(b). Percentage variations in supplier costs, manufacturer costs, and total supply chain costs are summarized in Tables 5.14(c)–5.17(c), enabling a structured managerial comparison of the two reordering policies.

In both frameworks, the initial replenishment is sales-driven: suppliers commence production and dispatch when the manufacturer’s finished goods inventory Q falls below the reorder level r . Subsequent replenishment decisions, however, are production-driven and depend on raw material consumption at the manufacturer. Under the simulation-based policy, suppliers dispatch independently and deliveries arrive unsynchronized. Equal batch sizes are imposed across suppliers, with batch size treated as a decision variable. Replenishment timing is governed by a dynamic reorder timing buffer d , such that an order is triggered when available raw materials decline to the quantity expected to be consumed over the next d time units. The parameter d therefore regulates when an order is placed rather than how much is ordered, introducing timing flexibility into the system.

By contrast, the analytical model assumes synchronized dispatch from suppliers, although arrivals remain unsynchronized due to proximity differences. Batch quantities are unequal and proportional to supplier

production capacities. After the initial sales-based trigger, replenishment occurs when raw material inventory falls to the quantity required during the delivery lead time of the fastest supplier, yielding a more structured, lead-time-based control policy.

Together, these two modeling approaches provide a rigorous basis for evaluating the operational and cost implications of dynamic timing flexibility versus synchronized analytical coordination in a heterogeneous multi-supplier supply chain.

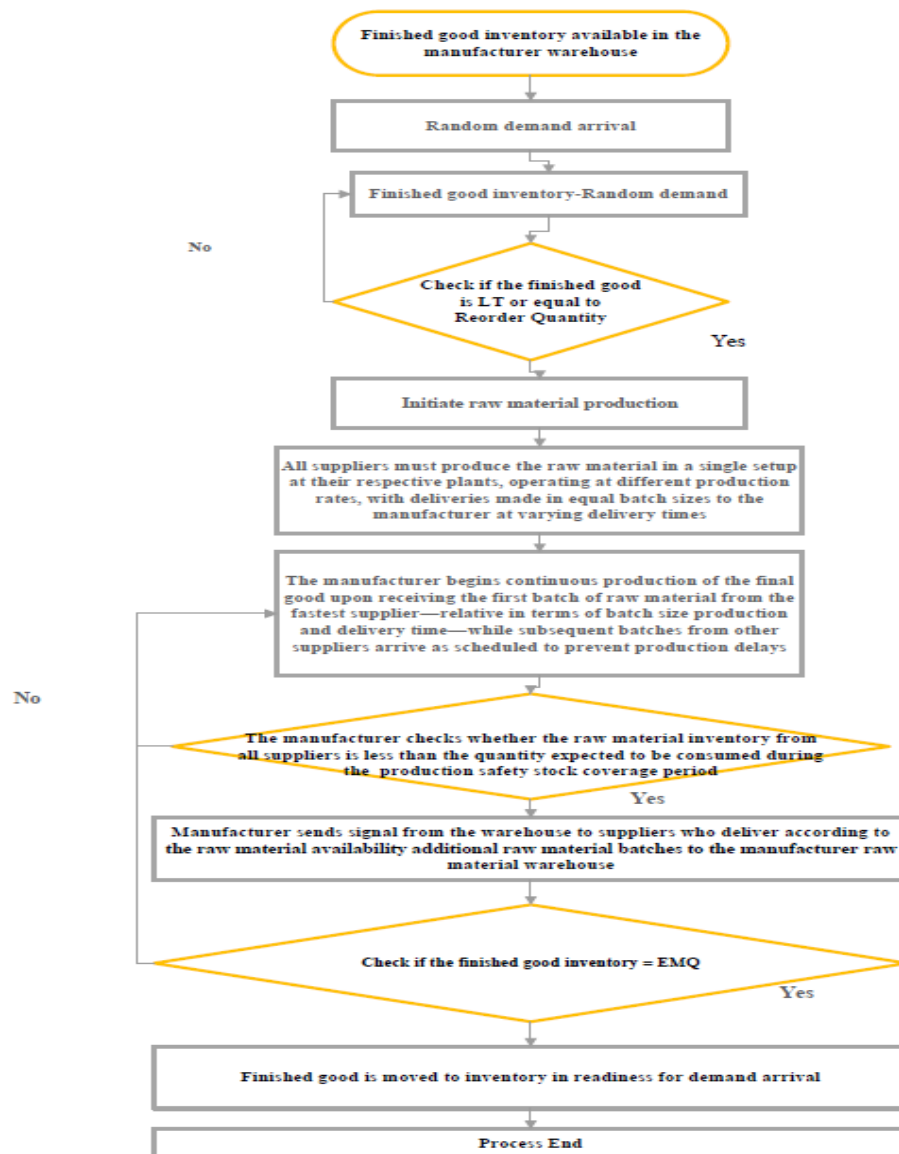


Figure 5.14: The conceptual model for simulating the heterogeneous multiple suppliers, single manufacturer supply chain problem with varying dispatch times

Effect of Manufacturer's Demand ($E[\Lambda]$)

$E[\Lambda]$	Q	R	λ_{s1}	λ_{s2}	T_{l1}	T_{l2}	m	d	P	P_{s1}	P_{s2}	$T.C_s$	$T.C_m$	$T.C_{sc}$
120	5138	5138	0.50	0.51	734	734	7	4	146	146	149	37560.87	70065.70	107626.57
140	5138	5120	0.49	0.52	734	734	7	3	173	170	180	50403.40	85939.70	136343.10
160	6608	6606	0.49	0.51	944	944	7	3	196	192	200	64317.01	110323.71	174640.72
180	8022	8018	0.50	0.53	1146	1146	7	5	216	216	229	81596.69	135513.98	217110.67

Table 5.14(a): Effect of demand on the heterogeneous MSSM supply chain using simulation optimization under an equal batch size policy

$E[\Lambda]$	Q	R	λ_{s1}	λ_{s2}	T_{l1}	T_{l2}	m	P	P_{s1}	P_{s2}	$T.C_s$	$T.C_m$	$T.C_{sc}$
120	4152	4118	0.51	0.53	1014	1062	4	166	169	177	43798	71048	114850
140	5550	5535	0.50	0.55	1062	1158	5	176	177	193	54527	88873	143400
160	7380	7315	0.50	0.52	1206	1254	6	201	201	209	67773	116430	184210
180	9480	9468	0.49	0.54	1500	1660	6	231	225	249	87856	149190	237046

Table 5.14(b): Effect of demand on the heterogeneous MSSM supply chain using mathematical optimization allowing batches of differing sizes

$E[\Lambda]$	% change in suppliers' costs	% change in manufacturer's costs	% change in total costs
120	-14.24	-1.38	-6.29
140	-7.56	-3.30	-4.92
160	-5.10	-5.25	-5.20
180	-7.12	-9.17	-8.41

Table 5.14(c): Cost Comparison of 5.14(a) and 5.14(b)

Table 5.14(a) presents the results of the simulation-based model under the first setting, where two heterogeneous suppliers, differing in production rates and distances to the manufacturer, are obligated to deliver batches of the same size. As demand per unit time increases from 120 to 180 units, the system responds through a series of coordinated adjustments across several key decision variables, including the

production lot size Q , reorder level R , production rate P , and the decision variable d , which determines the minimum level of the availability of raw materials before requesting batch deliveries from the suppliers. The values of Q and R steadily increase as demand grows, reflecting the manufacturer's need to maintain higher inventory levels to avoid shortages. This increase is accompanied by proportional increases in the manufacturer's production rate P , and the suppliers' production rates of P_{s1} and P_{s2} , which are dynamically adjusted based on their respective relative production capacities and cost effectiveness. These adjustments ensure that the equal batch size condition is met for all of the deliveries. Despite having different production capacities, both suppliers are required to make the same number of deliveries during the manufacturer's production cycle and must deliver equal batch sizes. This constraint enforces uniformity in the number of dispatches, but allows flexibility in the timing of deliveries, as the completion times of batches depend on each supplier's production rate. The arrival times of the batches at the manufacturer's warehouse also differ due to the unequal distances of the suppliers from the manufacturer, an important factor considered in the model to reflect real-world logistical challenges.

A central feature of the simulation model is the use of d as a decision variable to regulate the timing of deliveries. This mechanism allows for proactive yet flexible inventory management that is sensitive to real-time consumption and supplier responsiveness. In practice, this means that suppliers dispatch their batches not on a fixed calendar schedule, but in response to a dynamic signal from the manufacturer tied to raw material inventory depletion forecasts. Suppliers that have already produced the required batch immediately ship their lot, while others continue production and dispatch as soon as their batch becomes available. The cost performance of the system reflects this careful balance. As demand rises, the total supplier cost $T.C_s$, manufacturer cost $T.C_m$, and overall supply chain cost $T.C_{sc}$ increase in a controlled and predictable manner. This growth is expected as higher demand translates into larger batch sizes, more deliveries within a production cycle, and longer production cycles. Nevertheless, the increase in costs is moderated by the system's built-in optimization logic, which dynamically coordinates production and replenishment.

Table 5.14(b) presents the results of the analytical optimization model under the second policy setting, in which two suppliers are permitted to deliver batches of different sizes to the same manufacturer. This contrasts with the first setting where a single batch size was enforced. While suppliers continue to make the same number of deliveries per production cycle, the flexibility in batch sizing enables a more tailored supply approach, particularly important given the differences in their production capacities and distances to the manufacturer's warehouse. The model adjusts to these variations while seeking to minimize the total cost across the supply chain. As demand per unit time $E[\Lambda]$ increases from 120 to 180 units, the system responds by increasing the production lot size Q of the manufacturer, which aligns with rising production needs. For instance, Q rises from 4152 units at a demand of 120 to 9480 units at a demand of 180, with a near-linear response to growth in demand. Correspondingly, the reorder level R is also adjusted upward to ensure that new production is triggered timely enough to avoid shortages, taking into account the arrival times of raw material batches from both suppliers. Unlike in the simulation-based model, reordering of batches from the production system is tied directly to the lead time arrival of the first batch, often from the faster supplier. This makes the system relatively rigid, as inventory replenishment depends not just on internal consumption patterns, but also on supplier responsiveness and delivery lead times. This rigidity may reduce flexibility in managing inventories, especially under fluctuating demand.

The manufacturer's production rate P increases with demand, as expected, and so do the suppliers' production rates P_{s1} and P_{s2} adjusted according to the suppliers' relative production capacities and efficiency. These production rates ensure that even though batch sizes are unequal, deliveries remain coordinated across the cycle.

The number of deliveries per cycle m also increases with demand, moving from four (4) deliveries at 120 units of demand to six (6) deliveries at 180 units. This increment helps prevent large accumulations of raw material inventory and distributes the logistical burden more evenly over the production horizon.

From a cost perspective, both the total supplier cost $T.C_s$ and the manufacturer cost $T.C_m$ increase with demand. This is consistent with the behavior observed in the first setting. For instance, $T.C_s$ increases from 43,798 at 120 units of demand to 87,856 at 180 units, while $T.C_m$ increases from 71,048 to 149,190 over the same interval. The overall supply chain cost $T.C_{sc}$ rises steadily, reflecting the aggregate impact of scaling up both production and logistics operations. This analytical model offers operational flexibility in batch sizing, allowing each supplier to operate within its capacity limits. However, this flexibility comes at the cost of a more rigid reordering mechanism, which is less adaptive to real-time consumption trends than the simulation-based approach. Furthermore, the asymmetry in travel distances and batch sizes introduces complexity in synchronizing deliveries and may lead to increased holding or shortage costs if not carefully managed.

A comparative evaluation of the two models under increasing demand, as presented in Table 5.14(c), highlights the superior performance of the simulation-based optimization setting in terms of overall cost savings. Across all tested levels of demand, the simulation model consistently achieves notable reductions in both suppliers' and manufacturer's costs. For instance, suppliers' costs decrease by as much as 14.24% at the lowest demand level and remain favorably lower even as demand increases. Meanwhile, the manufacturer's cost savings become more pronounced with rising demand, reaching a peak reduction of 9.17% at the highest demand level. These combined effects yield total savings in the supply chain ranging from 4.92% to 8.41%. The enhanced performance of the simulation-based approach is largely due to its adaptive reordering mechanism, which initiates deliveries once the quantity of raw materials available is only sufficient for production over d days, where d is a decision variable determined by the interaction of other model parameters. This responsiveness mitigates the risk of early ordering, excess inventory, or production delays, especially under fluctuating or increasing demand. It enables a more appropriate alignment of inventory flow with consumption patterns, improving efficiency and reducing unnecessary costs across the supply chain. One other advantage of the dynamic replenishment approach is that the form

of the optimal policy is less sensitive to changes in the demand than the unequal batch size policy. This suggests that the dynamic replenishment approach is more robust to changes in the market.

Effect of Manufacturer's Holding Cost for the Final Good (h_m)

h_m	Q	R	λ_{s1}	λ_{s2}	T_{l1}	T_{l2}	m	d	P	P_{s1}	P_{s2}	$T.C_s$	$T.C_m$	$T.C_{Sc}$
5	5138	5138	0.50	0.51	734	734	7	4	146	146	149	37557.47	70080.82	107638.29
7	5138	5124	0.50	0.51	734	734	7	3	146	146	149	37557.47	79432.94	116990.41
9	5138	5108	0.50	0.51	734	734	7	3	146	146	149	37557.59	88773.68	126326.27
11	4648	4628	0.50	0.56	664	664	7	4	146	146	164	38761.11	93459.34	132220.46

Table 5.15(a): Impact of manufacturer's holding cost for the final good on the heterogeneous MSSM supply chain using simulation optimization under an equal batch size policy

h_m	Q	R	λ_{s1}	λ_{s2}	T_{l1}	T_{l2}	m	P	P_{s1}	P_{s2}	$T.C_s$	$T.C_m$	$T.C_{Sc}$
5	4152	4118	0.51	0.53	1014	1062	4	166	169	177	43798	71048	114850
7	4152	4096	0.51	0.53	1014	1062	4	166	169	177	43798	78492	122290
9	4152	4078	0.51	0.53	1014	1062	4	166	169	177	43798	85895	129690
11	3402	3385	0.50	0.57	1062	1206	3	176	177	201	47583	88151	135730

Table 5.15(b): Impact of manufacturer's holding cost on the heterogeneous MSSM supply chain using mathematical optimization under a policy allowing unequal batch sizes

h_m	% change in suppliers' costs	% change in manufacturer's costs	% change in total costs
5	-14.25	-1.36	-6.28
7	-14.25	1.20	-4.33
9	-14.25	3.35	-2.59
11	-18.54	6.02	-2.59

Table 5.15(c): Cost comparison of 5.15(a) and 5.15(b)

Table 5.15(a), like 5.14(a), presents results from a simulation-based model where two heterogeneous suppliers are required to deliver equal batch sizes to a single manufacturer. Despite having different production capacities, both suppliers must make the same number of deliveries per production cycle. Importantly, while the batch sizes are equal, the potential production rates of the suppliers and travel times are different, reflecting the reality that e.g. suppliers are located at different distances from the

manufacturer. As such, both arrival times and dispatch times are unsynchronized. At a low manufacturer holding cost $h_m = 5$, the system operates with a production lot size $Q = 5138$, reorder level $R = 5138$, and delivery structure comprising seven shipment cycles ($m = 7$). The production rates for Supplier 1 and Supplier 2 are $P_{s1} = 146$ and $P_{s2} = 149$, respectively. Although these production rates differ slightly, the dispatch timing is managed such that both suppliers fulfill their deliveries without causing delays in production. As the manufacturer's holding cost increases, the simulation model responds by reducing the reorder level and eventually decreasing the lot size with a corresponding decrease in the batch size and a slight increase in the production rate of Supplier 2. This reflects a strong response to the rising pressure of inventory costs. Increasing inventory costs makes shorter production cycles, obtained by increasing the production rate and/or decreasing the lot size, more favorable. This reinforces Supplier 2's role as the faster producer. Such changes in the strategy reduce the average inventory level to minimize exposure to holding costs, since long production cycles and early replenishment lead to higher storage costs for finished goods. Notably, the batch sizes, production rates, and number of shipments per production cycle remain unchanged over a relatively wide range of holding costs, demonstrating that the model retains structural stability while fine-tuning ordering policies. This helps ensure that raw material supply begins promptly and that manufacturer production is not delayed. This is particularly important given that production must not be interrupted due to assumed high setup costs. Throughout these scenarios, the asymmetry in supplier travel times plays an important role. Even though suppliers dispatch batches of the same size, the arrival time at the manufacturer's warehouse varies based on distance. Although Supplier 2 has a greater production capacity, Supplier 1 may deliver earlier, due to its proximity to the manufacturer. This reinforces the need for coordination through the simulation model, which allows dispatch timing to be flexible and dependent on both production readiness and logistical constraints. The simulation model effectively balances the trade-offs introduced by asymmetric travel times, fixed delivery obligations, and rising inventory costs. By adjusting reorder levels and lot sizes while keeping delivery structures stable, the model minimizes

escalations in costs. Supplier costs remain consistent due to equal batch sizes and delivery mandates, while the manufacturer absorbs the bulk of the increase in costs due to its higher holding costs.

Table 5.15(b), on the other hand, presents the results from a mathematical optimization model that captures the behavior of a supply chain system involving two suppliers delivering unequal batch sizes to a single manufacturer. Unlike the simulation-based scenario in Table 5.15(a), where batch sizes are equal, this formulation allows suppliers to produce and dispatch batches of a size that better reflects their individual production capacities, while still maintaining coordination with the manufacturer's production needs.

In this configuration, suppliers are situated at unequal distances from the manufacturer, resulting in asymmetric travel times, Supplier 1 is assumed to have a faster delivery time relative to Supplier 2, who has a higher production capacity. Despite the differing batch sizes, both suppliers are obligated to make a batch delivery in every shipment cycle during the manufacturer production runs, so that the total number of batches at the manufacturer is the number of suppliers multiplied by the number of deliveries per production cycle. However, due to unequal production rates, the dispatch batch sizes vary, and deliveries are synchronized (equal time dispatch) to prevent disruption in the manufacturer's production schedule. Compared to the model in which the batch sizes must be equal, the lot sizes are higher and number of deliveries per production cycle are smaller, while the production rates are greater. This results in shorter production cycles aimed at reducing the costs of holding the final good.

At a manufacturer holding cost of $h_m = 5$, the model yields a production lot size of $Q = 4152$, reorder level $R = 4118$, and a production cycle consisting of four delivery shipments ($m = 4$). Supplier 1 and Supplier 2 produce at rates of $P_{s1} = 169$ and $P_{s2} = 177$, respectively. The relative production strength of Supplier 2 is reflected in its ability to handle a larger batch size, thereby distributing supply responsibilities efficiently across the network. Despite the difference in batch sizes, both suppliers deliver once per delivery cycle, adhering to the requirement of coordinated delivery frequency. As the manufacturer's holding cost increases, the lot size and reorder point fall and the production rate increases. These changes control the

costs of holding the final good.

It is important to highlight that although deliveries differ in size, they are still dispatched in a synchronized manner, both in timing and frequency, with each supplier delivering once per shipment cycle. The variation in batch sizes is a direct consequence of the suppliers' differing production rates and cost structures. Supplier 2, having the higher production capacity, consistently handles batches of larger size, reducing the burden on Supplier 1. Nonetheless, due to the differing proximity of the suppliers to the manufacturer, the arrival times of these unequal batches at the manufacturer warehouse differ, which may influence the timing of manufacturing startup or inventory buildup. The model carefully balances these differences through the optimization of reorder level and lot size, ensuring that neither supplier delays the manufacturer's production cycle. The mathematical optimization effectively coordinates suppliers with heterogeneous capacities and travel times by adjusting key decision variables such as reorder levels, lot sizes, and shipment frequency. The model maintains a consistent delivery rhythm while allocating supply responsibilities proportionally. The response to rising holding costs is well-modulated, reflecting robustness in planning and operational adaptability.

Indeed, a critical distinction between the two models lies in how the timing of replenishment is managed, particularly through the incorporation of the decision variable d in the simulation model. In the simulation-based equal batch size delivery setting, d represents the flexible time threshold used to trigger supplier dispatch based on the consumption rate of raw materials at the manufacturer. This flexibility allows the system to dynamically regulate when the manufacturer initiates a replenishment order, thereby influencing the holding cost of raw materials and finished goods, and also reducing the likelihood of shortages. As the amount of available raw materials depletes to the level expected to be consumed within d units of time, suppliers that have completed the production of a batch are prompted to dispatch, while others continue production and deliver a batch immediately upon its completion. This staggered, but coordinated, dispatch ensures that the manufacturer's production is sustained without interruption, while also controlling for

unnecessary inventory accumulation and the associated holding costs.

By contrast, the analytical model follows a more rigid approach, where replenishment timing is synchronized with the arrival of the fastest supplier. Since dispatch from suppliers is simultaneous, the manufacturer's reordering decision is not made flexibly based on consumption, but instead tied to the supplier with the shortest lead time. This fixed ordering structure, although orderly, limits the model's ability to adapt to real-time inventory dynamics or variations in supplier readiness. As a result, while it simplifies planning, it may lead to inefficiencies in managing inventory and cost, especially when suppliers differ significantly in production capacities and distances from the manufacturer.

The integration of d in the simulation model, therefore, emerges as a powerful mechanism for cost optimization. It enables the system to intelligently balance the timing of supply with actual material needs, which is especially important in environments where production costs, travel times, and supplier capabilities are heterogeneous. This decision-driven flexibility is central to the simulation model's superior cost performance, as it allows both holding and shortage costs to be proactively managed in response to system dynamics, something the analytical model does not accommodate. Consequently, the use of d not only enhances responsiveness, but also reinforces the robustness of the supply chain against fluctuations in demand and supply timing.

Effect of the Raw Material Consumption Coefficient (α)

α	Q	R	λ_{s1}	λ_{s2}	T_{l1}	T_{l2}	m	d	P	P_{s1}	P_{s2}	$T.C_s$	$T.C_m$	$T.C_{sc}$
0.5	3600	3592	0.89	0.90	450	450	2	8	166	74	75	5765.79	65606.12	71371.90
1	3528	3516	0.55	0.56	588	588	3	6	169	93	95	13346.85	66357.66	79704.51
2	5138	5138	0.50	0.53	734	734	7	4	146	146	149	37560.87	70065.70	107626.57
3	5472	5469	0.50	0.51	1026	1026	8	3	143	214	218	80275.79	74034.56	154310.35

Table 5.16(a): Effect of the raw material consumption coefficient on the heterogeneous MSSM supply chain using simulation optimization under an equal batch size policy

α	Q	R	λ_{s1}	λ_{s2}	T_{l1}	T_{l2}	m	P	P_{s1}	P_{s2}	$T.C_s$	$T.C_m$	$T.C_{sc}$
0.5	3904	3874	0.61	0.70	456	520	2	186	57	65	5540	72283	77824
1	3192	3178	0.78	0.83	516	548	3	166	129	137	18032	65057	83089
2	4152	4118	0.51	0.53	1014	1062	4	166	169	177	43798	71048	114850
3	6412	6390	0.51	0.53	1350	1398	7	146	225	233	85056	78867	163920

Table 5.16(b): Effect of the raw material consumption coefficient on the heterogeneous MSSM supply chain using mathematical optimization under a policy allowing unequal batch sizes

α	% change in suppliers' costs	% change in manufacturer's costs	% change in total costs
0.5	3.91	-10.18	-9.04
1	-35.10	1.96	-4.25
2	-16.61	-1.40	-6.71
3	-5.96	-6.53	-6.22

Table 5.16(c): Cost comparison of 5.16(a) and 5.16(b)

Table 5.16(a) presents the effect of varying the raw material consumption coefficient α on the performance of the heterogeneous multi-supplier single manufacturer (MSSM) system under the equal batch delivery setting solved through simulation-based optimization. The parameter α represents the number of raw material units required to produce one finished product. As expected, changes in α have a direct impact on the production lot size of the manufacturer Q , the reorder level R , supplier and manufacturer production rates (P_{s1}, P_{s2}), as well as the cost structure of the supply chain. As the amount of raw materials required increases, unsurprisingly the batch size and rate of producing raw materials increases. The number of deliveries per production cycle increases markedly, while d falls. Intuitively, as the amount of raw materials needed increases, it becomes more important to control the costs of their storage. There is a tendency for the lot size to increase (although these changes are not uniform) and the rate of producing the final good to fall. The results in Table 5.16(a) clearly show that as the raw material consumption coefficient increases, supplier costs rise disproportionately, leading to higher overall supply chain costs. While reducing the coefficient α is financially attractive in products with flexible, modular formulations, since it minimizes raw material intensity, reduces lot sizes, and lowers both supplier and system-wide costs, this strategy may

come at the expense of product quality. A lower consumption coefficient may imply the use of fewer or cheaper raw materials per finished unit, which, although cost-efficient, risks compromising durability, functionality, or market acceptance of the final product. For unit-sensitive products, altering the coefficient α is impossible, as the formulation requires fixed proportions of inputs that cannot be substituted or reduced without compromising product integrity.

Table 5.16(b) presents the results for the policy which allows batches of different size. Heterogeneous suppliers dispatch materials at the same time, but in different batch sizes determined by their respective production rates. Unlike the equal batch size setting in Table 5.16(a), where replenishment was governed by the dynamic reorder buffer d , replenishment here is regulated by the lead time of the fastest supplier. This structural difference significantly affects how costs are distributed across the supply chain. As the raw material consumption coefficient α increases, the pattern of changes observed in the optimal solution are very similar to those observed under the model of dynamic replenishment. The system operates at relatively small lot sizes, with supplier production rates proportionally aligned to that of the manufacturer. The batch size increases at a generally faster rate. This may well result from the lower flexibility in acquiring raw materials. Again, the policy with unequal batch sizes shifts a high share of the cost burden to the suppliers, while insulating the manufacturer from the volatility introduced by increasing raw material consumption. The efficiency of the dynamic replenishment approach again is fairly uniform over the range of the parameters considered (from about 4% to 9%).

From a managerial perspective, while the policy which allows unequal batch sizes supports alignment with individual supplier capacities, the equal batch strategy with dynamic replenishment proves to be more cost-efficient for the overall supply chain. Its advantages lie in lower production costs, smoother manufacturer operations, and adaptive replenishment timing. Firms must therefore balance the trade-off between

supplier-level flexibility and system-wide cost efficiency when determining optimal batch sizes and replenishment policies.

Effect of Variability in Demand

σ	Q	R	λ_{s1}	λ_{s2}	T_{l1}	T_{l2}	m	d	P	P_{s1}	P_{s2}	$T.C_S$	$T.C_m$	$T.C_{Sc}$
15	5138	5116	0.50	0.51	734	734	7	4	146	146	149	37540.87	69946.40	107487.27
20	5138	5138	0.50	0.51	734	734	7	4	146	146	149	37560.87	70065.70	107626.57
25	5180	5176	0.50	0.51	740	740	7	5	146	146	149	37516.45	70426.03	107781.20
30	5180	5180	0.50	0.51	740	740	7	4	146	146	149	37532.01	70448.34	107980.36

Table 5.17(a): Effect of variation in demand on the heterogeneous MSSM supply chain using simulation optimization under an equal batch size policy

σ	Q	R	λ_{s1}	λ_{s2}	T_{l1}	T_{l2}	m	P	P_{s1}	P_{s2}	$T.C_S$	$T.C_m$	$T.C_{Sc}$
15	4152	4079	0.51	0.53	1014	1062	4	166	169	177	43798	70786	114580
20	4152	4118	0.51	0.53	1014	1062	4	166	169	177	43798	71048	114850
25	4950	4942	0.51	0.54	966	1014	5	157	161	169	42241	72862	115100
30	5070	4975	0.50	0.55	966	1062	5	161	161	177	43175	74479	117650

Table 5.17(b): Effect of variation in demand on the heterogeneous MSSM supply chain using mathematical optimization under a policy allowing unequal batch sizes

σ	% change in suppliers' costs	% change in manufacturer's costs	% change in total costs
15	-16.67	-1.20	-6.60
20	-16.61	-1.40	-6.71
25	-12.59	-3.46	-6.79
30	-15.04	-5.72	-8.96

Table 5.17(c): Cost comparison of 5.17(a) and 5.17(b)

Table 5.17(a) evaluates the effect of variability in demand, represented by the standard deviation σ , on the optimal strategy for an integrated supply chain under an equal batch policy. As σ increases there is a moderate increase in the lot size, reorder level R and the batch size. These adjustments reflects the system's response to higher uncertainty in demand by maintaining slightly higher inventory levels to buffer against

stockouts. The increase in average costs is relatively small compared to the magnitude of demand variability introduced,. In addition, the production rates are stable indicating that the equal batch policy, coupled with a dynamic reorder timing buffer ' d' ', provides resilience and mitigates the potential cost escalation associated with uncertain demand.

These results highlight that the equal batch policy is not only cost-efficient, but can also be operationally stable in environments with stochastic demand. The gradual adjustment in reorder levels and batch sizes ensures that the manufacturer maintains service reliability without incurring disproportionate costs.

Table 5.17(b) examines the effect of increasing variability in demand (σ) on the unequal batch policy with heterogeneous suppliers. This strategy is more sensitive to an increase in the variability in demand compared to the equal batch policy with dynamic replenishment. The reorder level (R) and lot size increase by a larger proportion than under the dynamic replenishment approach. The batch sizes and production rates are stable (at a higher level than under the dynamic replenishment policy). This increase in lot size is associated with an increase in the number of deliveries per production cycle. Again, these adjustments reflect the system's attempt to buffer against the heightened risk of stockouts under increasingly uncertain demand.

The comparative assessment of the two shipment policies under varying levels of demand uncertainty (σ) is presented in Table 5.17(c). The results indicate that the equal batch shipment policy consistently achieves lower costs than the unequal batch policy across all examined levels of variability (between 6% and 9% overall). In addition, the form of the optimal strategy under the dynamic replenishment policy is more robust against changes in the variability in demand than the unequal batch policy.

From a managerial perspective, the evidence highlights important trade-offs in shipment policy. Equal batch shipment offers a stable and cost-effective approach by harmonizing supplier outputs into uniform delivery

lots and leveraging the dynamic reorder timing buffer to synchronize replenishment across the chain. This ensures smoother manufacturer operations, avoids premature or excess ordering, and reduces both supplier and manufacturer costs under fluctuating demand. In contrast, the unequal batch policy aligns replenishment with the capacities of the individual suppliers, which may seem beneficial in heterogeneous supply settings but creates rigidity in ordering decisions and increases system-wide costs. For managers, these results suggest that dynamic replenishment is more valuable in reducing costs, and also enhances the robustness of the supply chain, compared to flexibility in the size of batches. Unequal batching should only be adopted in cases where supplier-specific constraints outweigh the benefits of system-level efficiency.

5.5 Managerial Insights

Both the analytical and simulation models provide valuable decision support for managing multi-supplier, single-manufacturer supply chains under stochastic demand and flexible supplier production rates. In both models, supplier production capacities vary, reflecting real-world operational flexibility. The analytical model assumes unequal batch sizes, synchronized dispatch from independent suppliers, and unsynchronized arrival at the manufacturer, capturing the impact of supplier heterogeneity under a fixed coordination schedule. In contrast, the simulation model assumes equal batch sizes, unsynchronized dispatch, and unsynchronized arrival, providing greater operational flexibility to adapt to variable demand and supply conditions.

From a managerial perspective, these differences are critical. The analytical model highlights cost trade-offs and coordination strategies under structured rules but is limited by its rigid dispatch assumption, which can lead to higher inventory levels or early deliveries. The simulation model demonstrates that flexible dispatch policies, aligned with demand patterns and production rates, can reduce total supply chain costs, smooth inventory levels, and improve responsiveness. This insight emphasizes the value of demand-

sensitive, adaptive planning strategies, showing that even with equal batch sizes, allowing dispatch timing to vary dynamically can outperform rigid, batch-size-driven coordination.

These findings have practical applications across industries where multiple suppliers serve a single production facility and demand is volatile. Examples include automotive manufacturing, pharmaceutical production, electronics assembly, food and beverage processing, and consumer packaged goods, where efficient coordination of multiple suppliers directly impacts production continuity, inventory costs, and service levels.

Ultimately, combining insights from both models equips managers with a comprehensive toolkit: the analytical model provides clarity on fundamental cost and coordination trade-offs, while simulation allows testing of flexible, operationally realistic strategies that enhance efficiency and responsiveness in multi-supplier, single-manufacturer systems.

CHAPTER 6

SUMMARY AND FUTURE WORK

6.1 Chapter Overview

This final chapter presents the overarching conclusions of the research and highlights potential directions for future study. It consolidates the core contributions, findings, and insights derived from both the analytical and simulation models developed throughout the dissertation. The research has systematically extended existing models of supply chain coordination, particularly in systems involving multiple suppliers and one or more manufacturers operating under stochastic demand. By relaxing several simplifying assumptions and integrating additional real-world complexities, the study has proposed generalized frameworks and solution approaches that enhance decision-making in multi-echelon supply chain systems.

The chapter is organized into two sections. Section 6.2 provides a comprehensive summary of the research and its principal conclusions, tracing the progression from analytical formulations to simulation-based evaluations. Section 6.3 outlines promising avenues for future research, including possible extensions of the models, methodological refinements, and broader applications that could enhance both academic inquiry and practical supply chain management.

6.2 Summary and Conclusion

This research has developed a generalized model for a multiple-supplier, single manufacturer supply chain system under the assumption of stochastic demand. The primary objective from the manufacturer's perspective is to ensure the timely delivery of raw materials, which is achieved through suppliers who produce and deliver on demand. Conversely, suppliers benefit from vertical collaboration with manufacturers, who effectively sustain their businesses by indirectly absorbing all associated costs.

The study extends the work of Aldurgam et al. (2017), which focused on a single-vendor, single-manufacturer system under stochastic demand, assuming a fixed vendor production rate and a known batch size. In contrast, this research relaxes those assumptions by introducing multiple independent suppliers (firstly assuming homogeneous suppliers and then considering heterogeneous suppliers) with varying production-to-consumption rate ratios. The model assumes that suppliers have distinct, known delivery times and that no single supplier can independently meet the manufacturer's total requirements for raw materials. Instead, suppliers collectively fulfill demand for the raw material, which is appropriately transformed to produce the final product.

A key contribution of this research is its treatment of delivery batch size as a decision variable rather than a fixed parameter, incorporating factors such as loading, mileage, and other haulage costs. Supplier heterogeneity is primarily characterized by differences in production rates. Two solution approaches are proposed: the integrated approach, which determines all decision variables jointly to minimize the overall cost of the logistic chain; and the Stackelberg approach, where suppliers adjust their systems to meet the parameters set by the manufacturer. Here, the suppliers and the manufacturer aim to minimize their costs independently. Results from the analytical model indicate that the integrated approach yields superior financial savings compared to the Stackelberg approach. Game theory predicts that this will be the case unless decoupling the production processes of the suppliers and manufacturer can give significant advantages. Sensitivity analysis on the raw material consumption coefficient shows that the integrated solution performs better when the consumption coefficient (α) is greater than one ($\alpha > 1$), whereas the Stackelberg approach can be more advantageous when $\alpha \leq 1$. When the demand for raw materials is relatively small, then this demand can often be met effectively by only having one delivery from each supplier per production cycle. In such cases, there is an advantage from freeing the production rate of the supplier from the manufacturer's production rate. However, in the case where multiple deliveries are

required per production cycle (this is the case when demand for the raw material is high), then it becomes favorable to tie the supplier's production rate to the manufacturer's production rate, in order to control warehouse costs. By definition, the Stackelberg approach leads to lower costs for the manufacturer. However, these savings lead to the suppliers incurring higher costs.

It is important to note that, while the heterogeneous supplier scenario serves as a general case, the analytical study explored only one setting of the heterogeneous scenario, where suppliers delivered unequal batch sizes to the manufacturer, but dispatched at the same time. The assumption of equal dispatch times, despite varying batch sizes, may not always reflect practical situations. For instance, manufacturers may demand equal batch sizes regardless of suppliers' production rates, which could result in scenarios where faster suppliers dispatch first, but deliver later due to the transport time to the manufacturer.

To better capture such complexities, the study introduced simulation optimization as a more flexible and realistic approach. Using Arena simulation software, a model of the homogeneous supplier system was developed and validated with OptQuest optimization software under both integrated and Stackelberg approaches. This model was then systematically extended to relax the assumptions of unequal batch sizes and uniform dispatch times, previously adopted in the analytical modeling of heterogeneous suppliers.

Under this extended framework, a comparative analysis was conducted to gain deeper insights into the operational performance of heterogeneous supplier systems. Specifically, two alternative heterogeneous replenishment policies were examined. The first policy considers multiple independent suppliers delivering unequal batch sizes, reflecting differences in suppliers' production capacities, while maintaining synchronized dispatch times. Although orders are released simultaneously, actual arrivals at the manufacturer's warehouse are unsynchronized due to differences in suppliers' geographic proximity and

transportation lead times. This policy corresponds to the heterogeneous setting previously examined using analytical modeling.

The second policy, in contrast, requires a fixed batch size from heterogeneous suppliers but relaxes the assumption of synchronized dispatch, permitting suppliers to dispatch at different times according to their respective production capacities. Owing to its greater structural flexibility and ability to capture time-dependent interactions among demand, production, and logistics processes, this policy was analyzed using simulation optimization. Comparative results indicate that the second policy consistently outperforms the first in terms of total system cost.

A key reason for this difference in performance lies in the inherent rigidity of the first policy, which is tightly anchored to the delivery lead time of the fastest supplier. Reorder decisions are triggered solely based on this shortest lead time, implicitly forcing all suppliers, regardless of their production rates or transportation distances, to conform to a common replenishment schedule. This rigidity limits the system's ability to respond to variability in demand and supply conditions and often results in early deliveries, elevated average inventory levels, and higher holding costs.

By contrast, the second policy introduces temporal flexibility by decoupling the reorder decision from the fastest supplier's lead time. Through the incorporation of a dynamic reorder timing buffer, represented by a parameter d and endogenously determined via simulation, replenishment timing is adjusted based on the joint interaction of demand for the finished product, raw material consumption rates, supplier production capacities, and delivery lead times. This flexibility enables better alignment between material inflows and actual production requirements, smooths inventory profiles, and reduces total system cost, even when equal batch sizes are imposed across suppliers.

The results of the simulations reinforce the managerial value of adaptive and demand-sensitive planning strategies by showing that tailoring replenishment timing and batch size decisions to underlying demand and supply conditions leads to measurable reductions in total supply chain cost in multi-supplier, single-manufacturer environments.

6.3 Future Research Directions

While this study has made significant contributions by extending traditional single-supplier, single-manufacturer supply chain models to more realistic multiple-supplier, single and multiple-manufacturer configurations under stochastic demand, several promising avenues for future research remain open. These extensions can further broaden the applicability and robustness of the developed models and offer deeper insights into the dynamics of complex supply chains. The following directions are proposed:

1. Incorporation of Supplier Heterogeneity in Cost Structures and Lead Times - Although this research considered heterogeneity primarily in terms of suppliers' production rates, future work could integrate more nuanced forms of heterogeneity. Specifically, differences in suppliers' cost structures, production lead times, transportation distances, and reliability levels could be modeled explicitly to reflect more realistic supplier profiles. Such an extension would enable more tailored and effective supplier selection and order allocation strategies.
2. This study models a non-preemptive supply chain using a (Q, R) policy, where replenishment is triggered when the inventory of the finished good reaches the reorder level R . Each production cycle starts with the inventory level at R and finishes after dispatching the corresponding production lot with the inventory level at $Q + S$, with S denoting the safety stock. Future research could extend the model to a preemptive reordering framework, applicable under the condition of a relatively long lead-time, where a production cycle begins when the inventory level is at R , and the lot ordered in the previous

cycle arrives within the current production cycle. Such an approach would incorporate lead-time variability and overlapping orders into the optimization structure.

3. **Integration of Multi-Echelon and Reverse Logistics** - An extension to multi-echelon supply chain systems, where raw materials, intermediates, and finished products flow across multiple layers (suppliers, manufacturers, distributors, and retailers), could provide a comprehensive end-to-end view of supply chain performance. Furthermore, integrating the principles of reverse logistics, such as returns management, recycling, or remanufacturing, would make the model more applicable to modern sustainable supply chains.
4. **Modeling Supplier Disruption and Risk Management** - Given the growing importance of supply chain resilience, future research could explicitly model the risk of supplier disruptions due to factors such as capacity failures, financial insolvency, or geopolitical issues. Strategies for mitigating such risks — through backup suppliers, inventory buffers, or dynamic sourcing policies — could be evaluated within the proposed framework.
5. **Incorporating Negotiation and Contractual Mechanisms** - This research focused primarily on operational coordination through delivery batch sizing and production planning. Future studies could explore the role of negotiation, incentive mechanisms, and contractual agreements (e.g., quantity discounts, buyback contracts, revenue-sharing schemes) between suppliers and manufacturers. Such mechanisms could further align incentives and improve the stability and profitability of collaborative supply chains.
6. **Application of Advanced Solution Techniques and Metaheuristics** - While this study employed integrated and Stackelberg solution approaches, future research could explore hybrid algorithms, advanced metaheuristics (such as genetic algorithms, particle swarm optimization, or artificial bee colony algorithms), and machine learning-based optimization to handle larger, more complex instances or highly nonlinear systems more efficiently.

7. An important extension of this research is the relaxation of the single-manufacturer assumption to a multiple-manufacturer setting, which can be effectively examined using simulation and simulation–optimization techniques. Future studies may use simulation to evaluate alternative supplier allocation and delivery coordination policies, including first-come-first-served allocation, random delivery sequencing, equal batch size deliveries, unequal batch size deliveries, demand-proportional allocation, priority-based dispatching, and service-level-driven replenishment rules. Such an extension would enable a systematic assessment of the trade-offs among cost efficiency, responsiveness, and fairness when multiple manufacturers compete for shared supplier resources under stochastic demand.
8. Real-World Case Study Validation - Finally, empirical validation through real-world case studies or industrial data would strengthen the practical relevance of the models. Collaborations with manufacturing firms, logistics companies, or supply chain consortia could provide valuable datasets to calibrate and validate the models, test their applicability, and demonstrate their impact in actual business settings.

APPENDIX A

Mathematical Derivation of the Costs for a Heterogeneous Multiple-Supplier, Single-Manufacturer Model under the Assumption of Simultaneous Dispatch

This appendix presents detailed mathematical derivation supporting the proposed heterogeneous Multiple-Supplier, Single-Manufacturer supply chain model under the assumption of simultaneous dispatches from the suppliers. The corresponding proof for the homogeneous case is provided in Chapter 3 of this dissertation. The derivation in this appendix establishes the theoretical foundation for the optimization framework applied to this setting, as discussed in the main text.

1. Supplier's set up costs per unit time

$$\begin{aligned} V_{sn} &= \frac{E[\Lambda]A_{s1}}{Q} + \frac{E[\Lambda]A_{s2}}{Q} + \frac{E[\Lambda]A_{s3}}{Q} + \dots\dots + \frac{E[\Lambda]A_{sn}}{Q} \\ &= \frac{E[\Lambda] \sum_{i=1}^n A_{si}}{Q}. \quad (A.1) \end{aligned}$$

Since the size of a production lot is given by the total amount of raw materials divided by the conversion

coefficient, $Q = \frac{m \sum_{i=1}^n T_{ij}}{\alpha}$

2. Supplier's holding cost per unit time

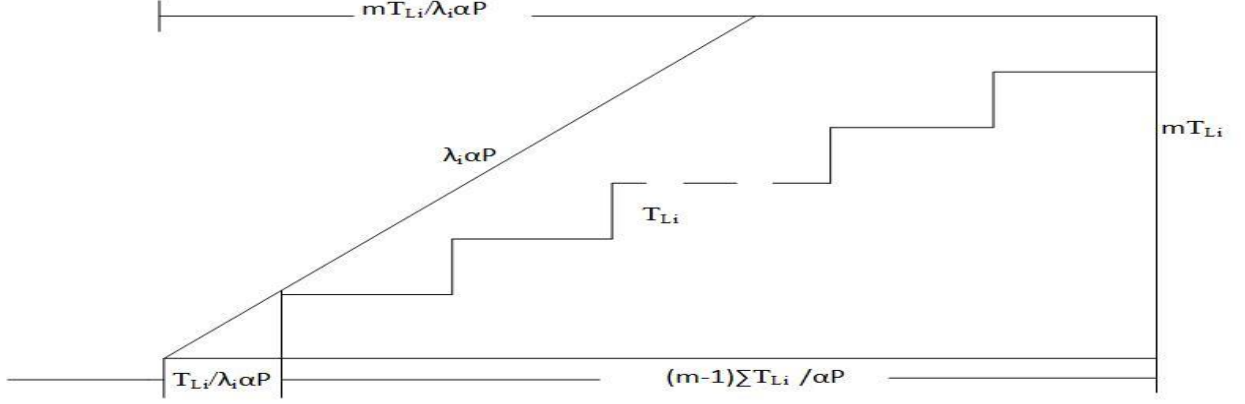


Figure A.1: The inventory profile of Supplier i when m shipments are made to the manufacturer's warehouse per production cycle under the assumption of heterogeneous suppliers delivering batches of size proportional to their capacity.

To calculate the amount of inventory held in storage by supplier i , we compute the area of a trapezium formed by the inventory-time profile, where the longer base is given by $\frac{T_{li}}{\lambda_i \alpha P} + (m-1) \sum_{j=1}^n \frac{T_{lj}}{\alpha P}$, the shorter base is $\frac{T_{li}}{\lambda_i \alpha P} (1-m) + \frac{m-1}{\alpha P} \sum_{j=1}^n T_{lj}$, and the height is $m T_{li}$; the area of this trapezium represents the cumulative inventory produced by Supplier i during the production cycle and is computed as $\frac{m T_{li}^2 (2-m)}{2 \lambda_i \alpha P} + \frac{m(m-1) T_{li} \sum_{j=1}^n T_{lj}}{\alpha P}$. Next, the staircase region representing the cumulative inventory dispatched with mean height $\frac{T_{li} + (m-1) T_{li}}{2}$ and length $\frac{(m-1) \sum_{j=1}^n T_{lj}}{\alpha P}$ is considered. The area is $\frac{m(m-1) T_{li} \sum_{j=1}^n T_{lj}}{2 \alpha P}$. The area between these two shapes represents the profile of the inventory held at supplier i 's warehouse during the production cycle. This is given as $\frac{m(m-1) T_{li} \sum_{j=1}^n T_{lj}}{2 \alpha P} + \frac{m T_{li}^2 (2-m)}{2 \lambda_i \alpha P}$. The average holding cost per unit time for supplier i is given by the holding costs for a production cycle divided (the previous expression multiplied by the unit holding cost) by the expected length of this cycle.

$$\frac{h_s E[\Lambda]}{Q} \left[\frac{m(m-1) T_{li} \sum_{j=1}^n T_{lj}}{2 \alpha P} + \frac{m T_{li}^2 (2-m)}{2 \lambda_i \alpha P} \right] = \frac{h_s T_{li} E[\Lambda]}{2P} \left[m-1 + \frac{T_{li} (2-m)}{\lambda_i \sum_{j=1}^n T_{lj}} \right].$$

Summing over all the suppliers, the total holding costs per unit time incurred by the suppliers are given by

$$\frac{h_s E[\Lambda]}{2P} \sum_{i=1}^n T_{li} \left[m-1 + \frac{T_{li} (2-m)}{\lambda_i \sum_{j=1}^n T_{lj}} \right] \quad (A.2)$$

3. Mean production cost per unit time for raw materials: The production cost per unit of raw material for supplier i is $\left[\frac{g_{1i}}{\lambda_i \alpha P} + b_1 [\lambda_i \alpha P]^{\beta_1} \right]$. The cost of producing the m batches in a delivery cycle is $\left[\frac{g_{1i}}{\lambda_i \alpha P} + b_1 [\lambda_i \alpha P]^{\beta_1} \right] m T_{li}$. Diving by the mean length of a production cycle, the mean production cost per unit time for supplier i is $\left[\frac{g_{1i}}{\lambda_i \alpha P} + b_1 [\lambda_i \alpha P]^{\beta_1} \right] \frac{\alpha E[\Lambda] T_{li}}{\sum_{j=1}^n T_{lj}}$. Summing over all n suppliers, the mean production cost for raw materials per unit time is

$$\sum_{i=1}^n \left[\left[\frac{g_{1i}}{\lambda_i \alpha P} + b_1 [\lambda_i \alpha P]^{\beta_1} \right] \frac{\alpha E[\Lambda] T_{li}}{\sum_{j=1}^n T_{lj}} \right] \quad (A.3)$$

4. Average transportation cost- The transportation cost consists of three components: a fixed cost for each batch that remains unchanged regardless of the size of a batch, a loading cost that depends on the batch size, and a mileage cost assumed to be proportional to both the batch size and the journey time. For the fixed cost component, the n suppliers collectively incur a total cost of nmK , which remains constant regardless of the production rate. In addition, they incur a loading cost of $K_r m \sum_{i=1}^n T_{li}$. The mileage cost depends on the charge rate V . Summing all three components, the average transportation cost is given by:

$$T_c = \frac{mE[\Lambda]}{Q} \left((nK + K_r \sum_{i=1}^n T_{li}) + V \sum_{i=1}^n T_{li} d_i \right) \quad (A.4)$$

5. Costs for holding the raw material in the manufacturer's warehouse per unit time- The inventory profile follows the same structure as the one presented in Figure 3.2 and Figure 3.3. The only difference is that, whereas the manufacturer previously received equal batch sizes of ingredients from all suppliers, in this case, the suppliers deliver different batch sizes. These varying batch sizes are determined by each supplier's individual production rate. However, all of the suppliers are assumed to dispatch their

shipments at the same time during each dispatch cycle (synchronized dispatch). Adopting the same analysis as in figure 3.3, it is assumed that the first delivery in a cycle is made at time 0. This profile is repeated m times per production cycle and the holding costs are proportional to the area between the supply and consumption profile. The consumption curve in a delivery cycle forms a triangle of height equal to the supply of raw materials, i.e. $\sum_{i=1}^n T_{li}$. The length of this cycle is equal to the time required for the manufacturer to consume these raw materials, i.e. $\frac{\sum_{i=1}^n T_{li}}{\alpha P}$. Hence, the area under the consumption curve is $\frac{(\sum_{i=1}^n T_{li})^2}{2\alpha P}$. The area under the supply curve can be calculated as the area of n rectangles orientated horizontally. The height of the i -th rectangle is T_{li} . Its length is the time required for the manufacturer to consume the raw materials received in a delivery batch minus the difference between the time of arrival of the batch from the i -th supplier and the time of arrival of the batch from the first supplier, $\frac{\sum_{i=1}^n T_{li}}{\alpha P} + d_i - d_1$. It follows that the area under the supply curve is given by

$$\sum_{i=1}^n T_{li} \left[\frac{\sum_{i=1}^n T_{li}}{\alpha P} - d_i + d_1 \right].$$

It follows that the area between the supply curve and the consumption curve in a delivery cycle is given by

$$\frac{(\sum_{i=1}^n T_{li})^2}{2\alpha P} - \sum_{i=1}^n T_{li} d_i + d_1 \sum_{i=1}^n T_{li}$$

The costs incurred by the manufacturer per unit for holding the raw materials in a production cycle are proportional to both the unit holding costs and the number of deliveries made in a cycle and inversely proportional to the length of a production cycle, i.e.

$$\frac{E[\Lambda] m h_{rm}}{Q} \left[\frac{(\sum_{i=1}^n T_{li})^2}{2\alpha P} - \sum_{i=1}^n T_{li} d_i + d_1 \sum_{i=1}^n T_{li} \right] = \frac{E[\Lambda] \alpha h_m}{\sum_{i=1}^n T_{li}} \left[\frac{(\sum_{i=1}^n T_{li})^2}{2\alpha P} - \sum_{i=1}^n T_{li} d_i + d_1 \sum_{i=1}^n T_{li} \right]$$

(A.5)

It is important to note that in modeling both the manufacturer's holding cost per unit time for the finished goods inventory $H_{fp} = h_m \left[\frac{1}{2} \left[Q - \frac{m \sum_{i=1}^n T_{li} E[\Lambda]}{\alpha P} \right] + R - E[\Lambda] \left[d_1 + \frac{T_{l1}}{\lambda_1 \alpha P} \right] \right]$, and the manufacturer's raw material acquisition cost $\frac{cmE[\Lambda] \sum_{i=1}^n T_{li}}{Q}$, the same approach is used for both the homogeneous supplier setting and the heterogeneous supplier setting. The only difference in modeling is that the term $\frac{mnT_l}{\alpha}$ used in the homogeneous case to represent the size of a production batch is replaced by $\frac{m \sum_{i=1}^n T_{l1}}{\alpha}$ in the heterogeneous case and the time from placing an order to the first delivery changes from $d_1 + \frac{T_l}{\lambda \alpha P}$ to $d_1 + \frac{T_{l1}}{\lambda_1 \alpha P}$.

APPENDIX B

Arena Simulation Software

In this appendix, some modules of the Arena simulation software are described. These modules were used to develop the SVSM and SVMM simulation models. Arena is an application software designed to mimic real-world systems in order to analyze the impact of potential changes that might otherwise require significant capital investment or complex redesign. Arena is commonly applied in areas such as logistics, supply chain management, manufacturing, and warehousing, among others. It provides a platform for analyzing a system in its current (as-is) configuration and under a wide range of potential (to-be) alternatives, enabling informed and rational decision-making on how best to operate and improve the system.

In Arena, the three main panels that contain the modules used to define a wide range of processes are the Basic Process panel, the Advanced Process panel, and the Advanced Transfer panel. Before developing the flowchart for the supply chain system, it is essential to describe the key functions of the modules that will be used.

1. Basic Module

- **Create Module:** This module is used to generate entities based on a defined schedule or inter-arrival time. Entities created here represent items or units entering the system and begin their journey through the simulation from this point. Examples of entities include customers in a queue, cars in a service station, finished goods in a warehouse, or raw materials in a production system—depending on the type of system being modeled. Once created, these entities move to the next module to start their process flow.
- **Dispose Module:** This module is used to terminate the process flow of entities. It marks the end point

of an entity's journey through the simulation. Entities that were introduced into the system by the Create module eventually exit the simulation through the Dispose module, indicating that they have completed all required processes.

- **Process Module:** This module is used to represent activities or tasks performed in the system. It allows entities to seize resources, delay for a specified time, and then release the resources once the task is completed. The delay can be based on a fixed, random, or user-defined distribution. Like the other modules mentioned above, the Process module is part of the Basic Process panel and plays a key role in modeling operations such as service, assembly, or inspection.
- **Decision Module:** This module provides decision-making capability within the process flow. It allows entities to take different paths based on conditions (e.g., attribute values, system variables) or probabilities (chance). The Decide module is essential for modeling branching logic, such as routing an entity to different stations depending on its type, availability of resources, or random outcomes.
- **Assign Module:** This module is used to update or change information related to entities as they move through the system. It can modify entity attributes, change entity pictures, and assign new values to system variables, parameters, or other characteristics. The Assign module is useful for tracking, labeling, or dynamically adjusting how entities behave based on evolving conditions in the simulation.
- **Separate Module:** This module is used to duplicate entities for parallel or concurrent processing, allowing a single entity to be split into multiple copies. It is also used to separate previously batched entities into individual units so they can continue through the system independently. The Separate module is useful in scenarios where parts of a process need to occur simultaneously or when grouped entities need to be handled separately after processing.
- **Batch Module:** This module is used to combine multiple entities into a single grouped entity for further processing. The batching can be based on a specified number of entities, matching attributes, or arrival time. Once batched, the combined entity moves through the system as one unit until it is separated, if

needed. The Batch module is useful in modeling systems where items are processed or transported in groups, such as packaging, shipment, or assembly operations.

2. Advanced Process Module

- **Hold Module:** This module is used to retain entities in a queue until a specified condition is met or a signal is received. It operates in two main modes:

Scan Mode – holds entities until a defined logical condition becomes true.

Signal Mode – holds entities until a signal is received from another module (typically via the Signal module).

The Hold module is useful for controlling entity flow, ensuring that entities do not proceed until the system is ready or certain criteria are satisfied.

- **Signal Module:** This module is used to send a signal to one or more Hold modules, instructing them to release entities that are waiting. The Signal module can specify the number of entities to be released or simply trigger the release of all waiting entities, depending on how it is configured. It plays a crucial role in coordinating processes, especially when the release of entities depends on the completion of an event or activity elsewhere in the system.

3. Advanced Transfer Module

- **Route Module:** This module is used to transfer entities to a specified station within the model. The transfer can include a predetermined delay time, which may represent transportation, travel, or any type of transfer duration. The Route module is useful for modeling movement between different areas or departments in a system, such as moving products from one workstation to another or customers from one service point to the next.
- **Station Module:** This module defines a specific physical or logical location within the simulation where an activity or process occurs. Stations are often used in combination with Route and Transfer modules to model the movement of entities between different points. A Station represents areas such as

workstations, service counters, departments, or any location where entities perform or wait for an activity.

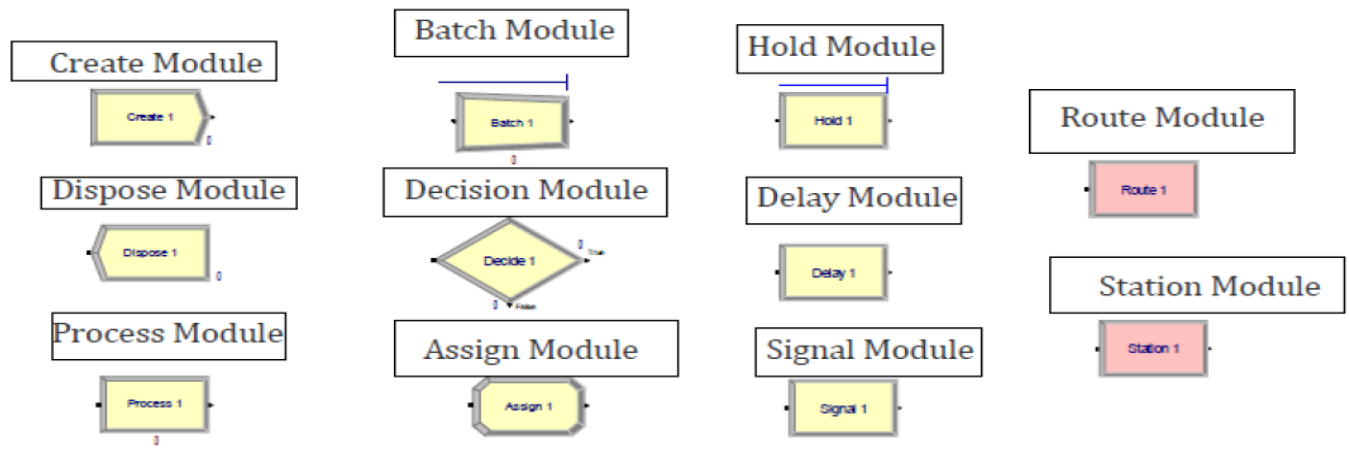


Figure B.1: Symbolic Representation of Arena Modules Used in Simulation Model Development (Including Basic, Advanced Process, and Advanced Transfer Modules)

APPENDIX C

Arena Model for the Homogeneous MSSM Supply Chain Simulation Model

This appendix presents the Arena simulation model developed for a Multiple Supplier, Single Manufacturer (MSSM) supply chain system, used as the base case scenario in this study. The modeling logic and structure were previously introduced in Chapter 5. As shown in Figures C.1 to C.3, the simulation model is divided into three interconnected submodels that collectively represent the flow of materials and information across the supply chain.

The first part of the model simulates customer demand and finished goods inventory depletion. Customer arrivals occur at random intervals following a normal distribution, reflecting real-world variability. Each customer purchase reduces the available inventory of finished goods on the retail shelf. When the inventory level falls to or below a predefined reorder level, a replenishment order is placed. Both the reorder level and the lot size of finished goods to be replenished are treated as decision variables, allowing for performance evaluation under different inventory policies. The second part of the model represents the supply side, consisting of multiple suppliers assumed to be homogeneous in terms of their production capacity, technology, and operations. However, they differ in proximity to the manufacturer, which affects their delivery times. These spatial differences are captured in Arena using the Route and Station modules from the Advanced Transfer panel. Upon receiving a requisition from the manufacturer, suppliers begin production immediately based on a specified production rate, which is also a decision variable. The first batch of raw materials or ingredients is shipped as soon as it is produced, while the remaining units are batched, held, and released sequentially. The Hold module controls the release of these batches, triggered by signals sent from the manufacturer's warehouse. Each signal releases one unit, maintaining a synchronized supply flow. The batch size and the number of delivery shipments per production cycle are

additional decision variables incorporated to study different supply and transportation strategies, with the assumption that all suppliers maintain equal delivery quantities.

The third and final part of the model captures the operations at the manufacturer's warehouse, where the raw materials received are immediately used to produce a predefined economic lot size of finished goods. This lot size, as well as the manufacturer's production rate, are modeled as decision variables, enabling cost and efficiency analysis across different production setups. After each production cycle, a signal is sent to the supplier side to release the next batch of raw materials. These released entities are then duplicated to restore the original batch size before entering the next production round. To avoid overstocking, a new replenishment order is triggered only when the quantity of raw materials from the current delivery cycle that have not yet been consumed falls to the amount required to support production during the delivery lead time of the fastest supplier.

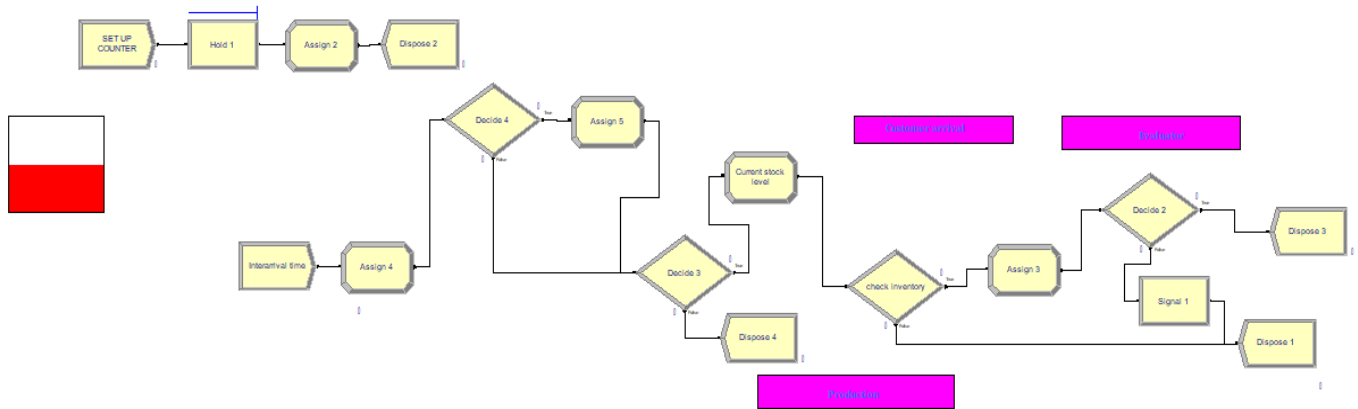


Figure C.1: Submodel 1 describing the depletion of the finished goods inventory and the manufacturer's reordering process.

Meanwhile, it is important to know that the description above refers to the integrated solution approach analyzed in the verification and validation phase of the development of the simulation model. For the Stackelberg solution, Submodel 2 defines production, storage and delivery by the suppliers according

to a slightly modified algorithm.

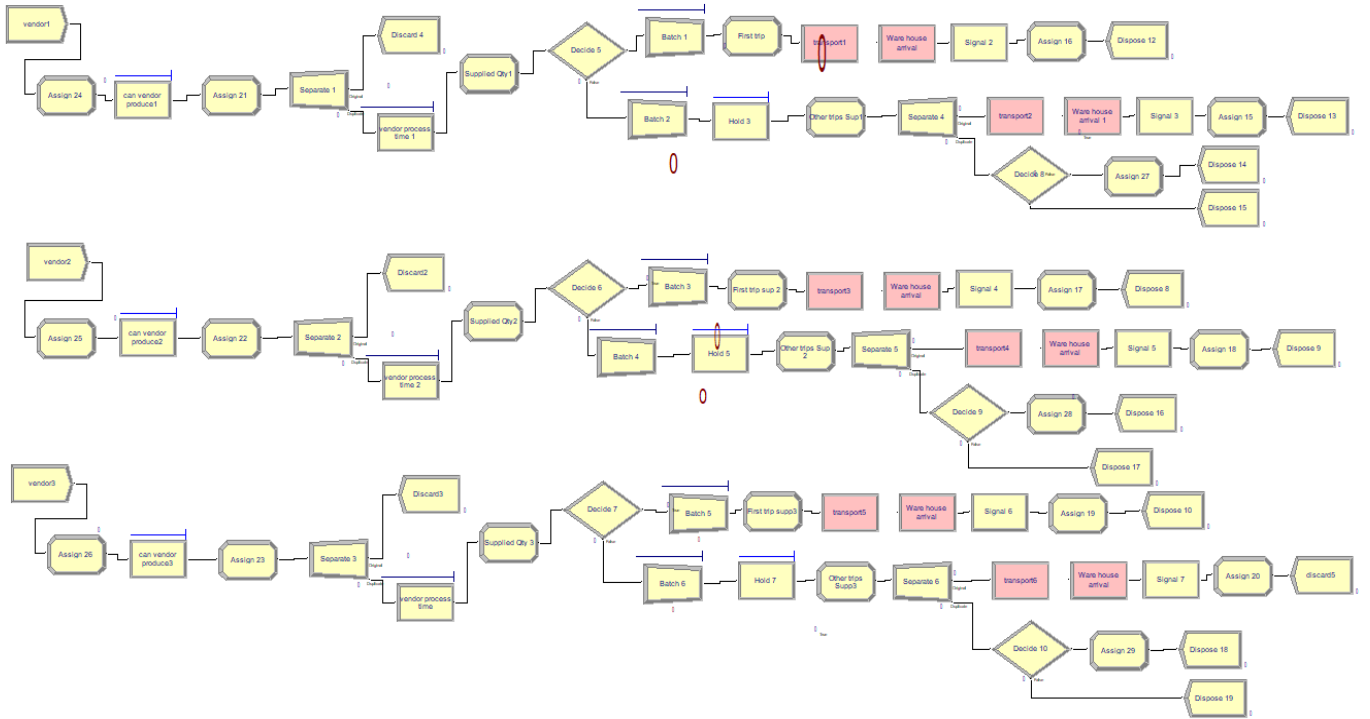


Figure C.2: Submodel 2 illustrating the production, storage, and delivery system for the three suppliers analyzed in the verification and validation section under the integrated solution approach.

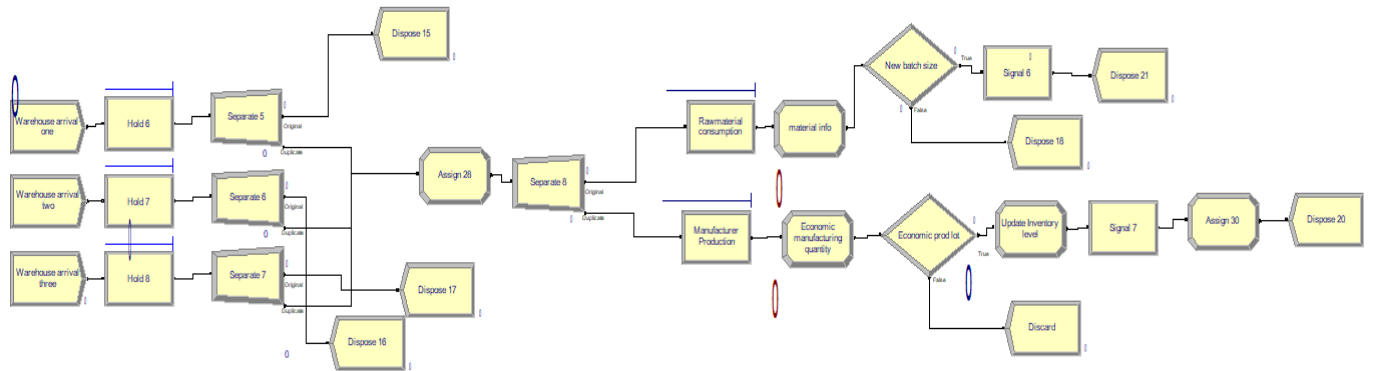


Figure C.3: Submodel 3 modeling the coordination of the manufacturer's production and raw material inventory, where batches are requested from suppliers once the amount of unconsumed raw material from the current delivery cycle falls to the amount consumed during the fastest supplier's lead time.

Stackelberg (Decentralized)

The homogeneous supply chain, involving multiple suppliers and a single manufacturer, was solved in the mathematical model using both the integrated and Stackelberg approaches. The simulation model was then adapted to replicate the Stackelberg solution. Because the Stackelberg method is a two-stage optimization process, only minor parameter adjustments were made, along with a modification of Submodel 2 in Figure C.2 to become Submodel 4 in Figure C.4. In Submodel 4, the configuration changes when a signal from the manufacturer's finished goods or sales store prompts suppliers to produce and deliver raw materials. This is achieved by assigning each supplier two production processes using the process module. Once the manufacturer's reorder level is reached, the hold module releases an entity. The separate module then multiplies this entity into the raw material quantities needed from each supplier for a production cycle, calculated as the number of delivery trips per supplier multiplied by the optimal batch size. Both variables are determined during optimization. Each separated entity enters the batch module, which groups the required number for a batch and sends a signal once filled. The number of entities processed in the batch module corresponds to the number of deliveries required per production cycle. A counter, initially set to zero, tracks these batches. The first entity is sent to the first production line, separated to match the batch size, and processed at the supplier's maximum rate using the process module's expression option. The remaining entities are routed to the second production line via the decision module, where they are separated, processed, stored, and batched before delivery to the manufacturer's warehouse.

Submodel 1, which represents the depletion of the finished goods inventory and reordering, and Submodel 3, which models coordination of production and the supply of raw materials, are the same in both the integrated and Stackelberg methods. As described in Chapter 5, the results from the mathematical model are used in the simulation to compare costs and verify the model. For validation, OptQuest is applied via two-stage optimization. The manufacturer's process is first optimized with all suppliers producing at their

maximum rate. This output is then used for the supplier side, where the first batch is processed at the maximum rate, followed by an adjustment of production rates to minimize supplier costs.

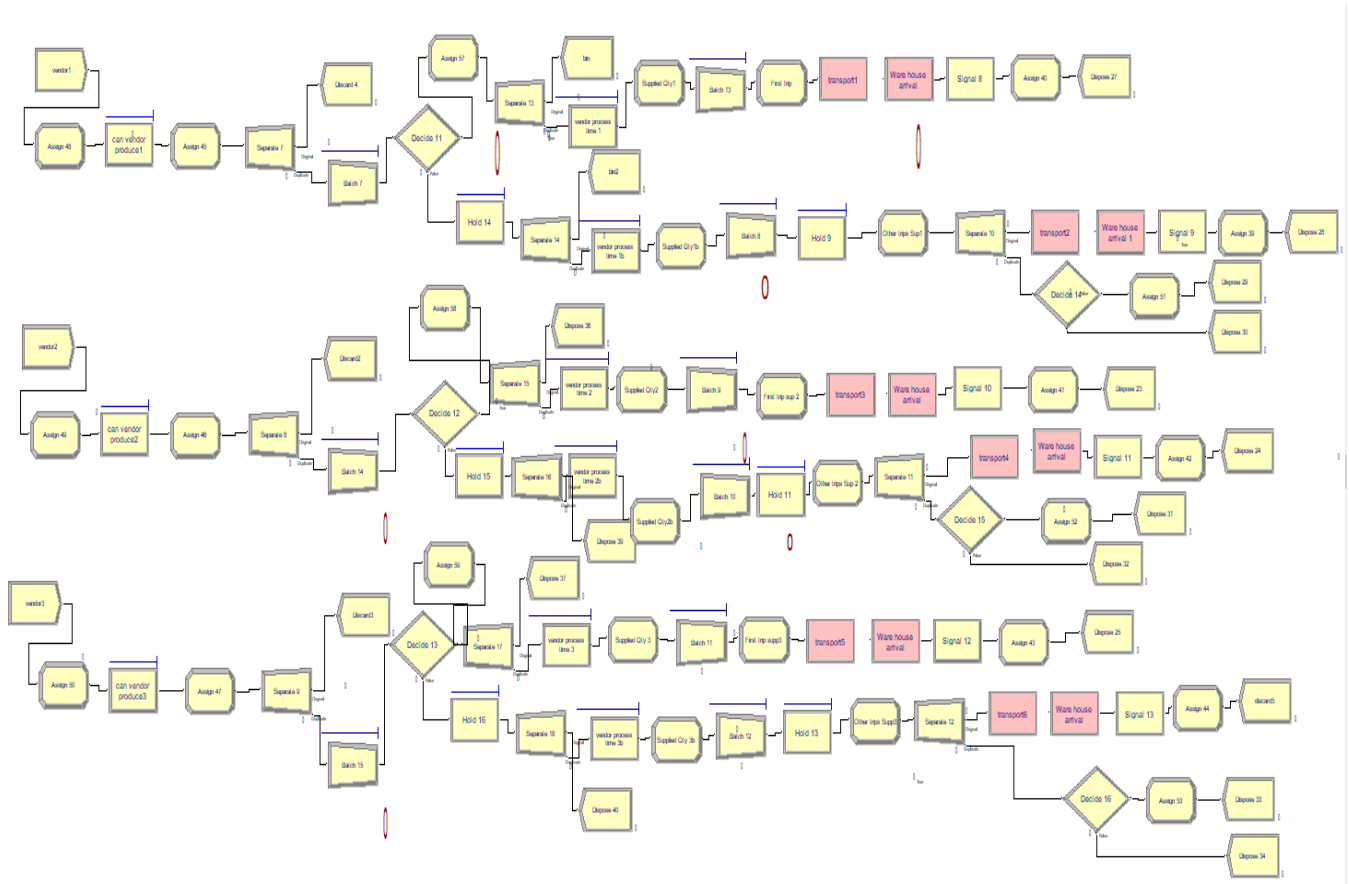


Figure C.4: Submodel 4 illustrating the production, storage, and delivery system of the three suppliers analyzed in the verification and validation section under the Stackelberg solution approach.

Arena Model for the Heterogenous MSSM Supply Chain Simulation Model

The Arena simulation model for the Heterogeneous Multiple SME Suppliers and Single Manufacturer (MSSM) supply chain closely follows the structure of the Homogeneous MSSM model, with important modifications to the replenishment logic to reflect the variability among suppliers. In the homogeneous case, raw material replenishment is consistently triggered once the amount of unconsumed raw materials from the current delivery cycle drops to a level equal to the quantity expected to be consumed

during the lead time of the fastest supplier. This approach ensures uninterrupted production and avoids excessive inventory. In contrast, the heterogeneous model employs a more dynamic and responsive replenishment mechanism to account for differences in supplier performance and delivery timing. After the initial delivery, which is coordinated directly by the manufacturer's central system, all subsequent replenishment orders are triggered from within the production process as raw materials are consumed. Instead of waiting for inventory to reach a fixed reorder point, the model introduces a decision variable, ' d ', which represents the number of production days that the remaining inventory can support. When inventory falls to the amount required for ' d ' days of production, a replenishment request is sent to the suppliers. This decision variable is determined dynamically within the simulation. It reflects the interaction of several system parameters, particularly the manufacturer's production rate, as well as inventory levels and shortage costs. As a result, the model effectively balances the cost trade-off between ordering too early, which increases inventory holding costs, and ordering too late, which can lead to shortages or production delays. Another significant feature of the model with heterogeneous suppliers is the lack of synchronization in supplier production and delivery schedules. While all suppliers are required to deliver the same batch size, they do so independently based on their respective production rates. A supplier can only dispatch a delivery once it has completed a full batch. Therefore, delivery times are staggered. During the first delivery phase, each supplier ships their batch immediately upon completing its production. However, for subsequent deliveries, only suppliers with a completed batch at the time of the replenishment request can respond immediately. Those suppliers that do not have a completed batch must wait until they accumulate the full batch size. This modeling approach reflects the real-world dynamics of supply chains involving multiple independent suppliers. By allowing replenishment decisions to be based on actual consumption levels and the variable readiness of suppliers, the model provides a more realistic and flexible simulation environment. It supports decentralized, condition-based inventory control rather than relying solely on fixed thresholds. The

Arena implementation of this system is illustrated in Figures C.5 through C.7. As shown in these Figures, the heterogeneous supplier case, like the homogeneous setting, is divided into three submodels.

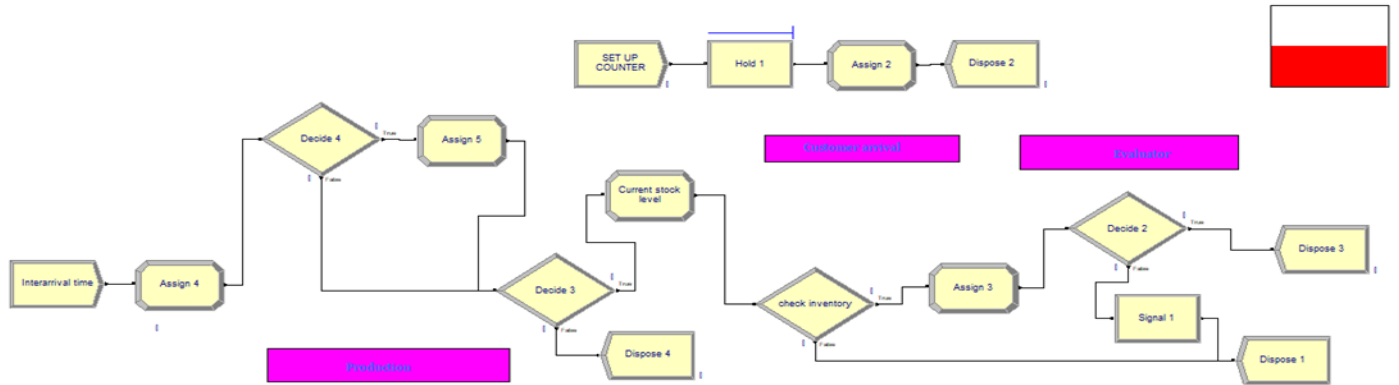


Figure C.5: Submodel 1 describing the depletion of the finished goods inventory and the manufacturer's reordering process.

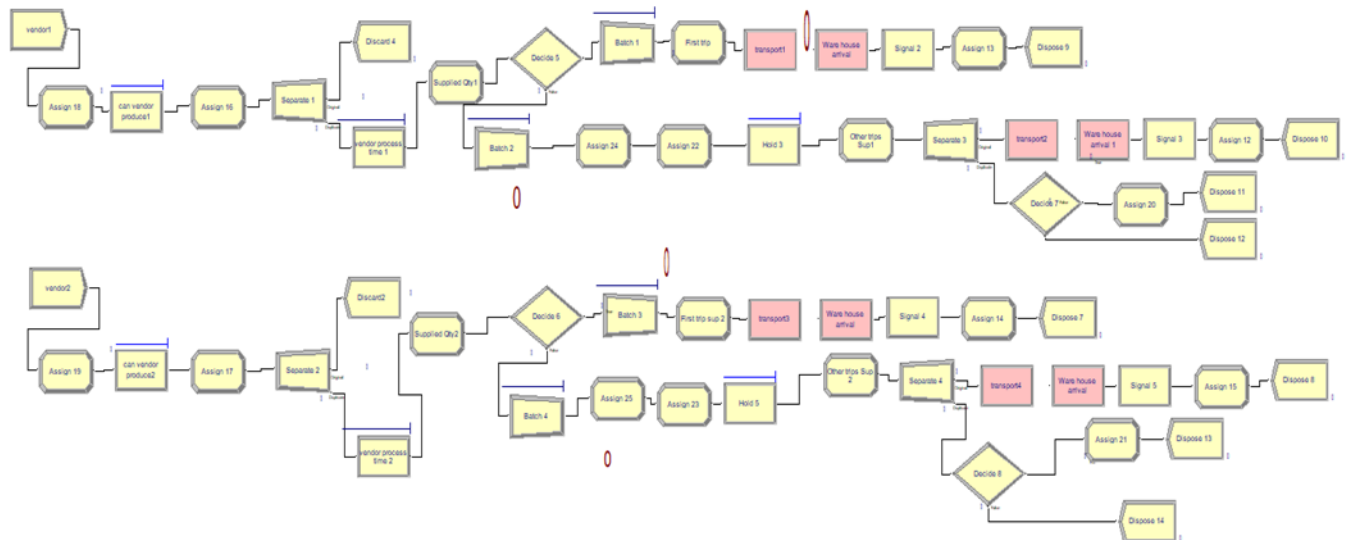


Figure C.6: Submodel 2 describing the production, storage, and delivery system of two heterogeneous suppliers.

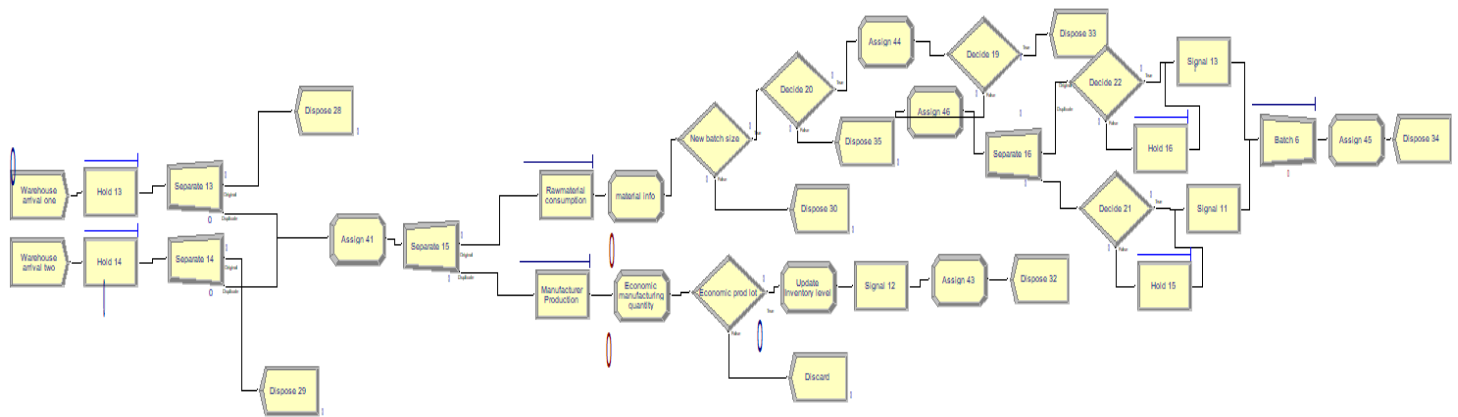


Figure C.7: Submodel 3 illustrating the coordination of the manufacturer's production and raw material inventory, batch deliveries from suppliers are triggered when work in process (WIP) raw material inventory falls to the dynamic reorder timing buffer (d).

Submodel 1 (Figure C.5) represents the depletion of the finished goods inventory and the manufacturer's reordering process, and is identical to Figure C.1, as there are no changes to the manufacturer's arrangement. In Submodel 2 (Figure C.6), the production, storage, and delivery systems of two heterogeneous suppliers are modeled using the Arena simulation software. The logic builds on the homogeneous supplier setup, but with a key addition: a variable in the assign module that links the supplier's production schedule with the manufacturer's warehouse of raw materials. This variable ensures that the manufacturer knows which supplier should make the next delivery of raw materials. The trigger occurs when the raw material inventory being processed by the manufacturer into finished goods falls to the amount required for ' d ' days of production, where ' d ' is a decision variable generated by the model. A delivery can only take place if a supplier has at least one full batch ready. If the available quantity is less than a batch, production continues until the full batch is completed before dispatching to the manufacturer. A second assign module, kept separate for easier debugging, records a parameter that acts as a unique label for each supplier. This label allows the manufacturer to receive and match orders in a just-in-time manner. Finally, a hold module is included to enable independent ordering from each supplier. Although the joint ordering point is set at ' d ' days, the simulation checks each supplier's

batch availability individually, and a dispatch is only initiated when that supplier has a complete batch ready. This batch is released as an entity after a signal is received from the manufacturer's system, and this entity is later separated according to the batch size.

Figure C.7 presents Submodel 3, illustrating the coordination between the manufacturer's production and raw material inventory. Batch deliveries are triggered when the manufacturer's work-in-process inventory of raw material reaches a dynamic reorder buffer, d . While similar to the homogeneous supplier model, this submodel modifies how replenishment orders are communicated to suppliers. Batch size signals from suppliers are separated into the appropriate batch units, with each unit duplicated to feed the production processes represented by two process modules. One process manages the replenishment of finished goods after lot completion, transferring inventory to stock for sales or customer demand. The other process monitors the level of raw material inventories using supplier-specific labels. When the manufacturer's inventory falls to the d threshold, suppliers with sufficiently large inventories dispatch immediately. If both meet this requirement, they send the appropriate batches simultaneously. If only one does, it dispatches, while the other continues production until ready.

APPENDIX D

Appendix D contains the Arena simulation output exported to Microsoft Excel for the developed models. The Excel files present the complete set of statistics for all entities tracked in the MSSM and MSMM supply chain systems. These statistics include the performance measures recorded during the simulation runs and correspond to the summary values reported in Table 5.1.

	A	B	C	D	E	F	G	H
	ProjectName	Project RunDateTime	Replication	Name	Type	Source	RecordedValue	IsPeriodic
1	Unnamed Project	2025-08-14 02:03:20	1	Avg ordering cost	Output	User Specified	93.15068493	0
2	Unnamed Project	2025-08-14 02:03:20	1	EMQ cost	Output	User Specified	128397.4071	0
3	Unnamed Project	2025-08-14 02:03:20	1	LRAC	Output	User Specified	275007.5841	0
4	Unnamed Project	2025-08-14 02:03:20	1	LRAM	Output	User Specified	164523.2223	0
5	Unnamed Project	2025-08-14 02:03:20	1	LRAV	Output	User Specified	110484.3618	0
6	Unnamed Project	2025-08-14 02:03:20	1	Material purchase cost	Output	User Specified	1395.125571	0
7	Unnamed Project	2025-08-14 02:03:20	1	PRODUCTION SET UP.NumberIn	Number In	Entity	500	0
8	Unnamed Project	2025-08-14 02:03:20	1	PRODUCTION SET UP.NumberOut	Number Out	Entity	33	0
9	Unnamed Project	2025-08-14 02:03:20	1	System.NumberOut	Number Out	System	899652	0
10	Unnamed Project	2025-08-14 02:03:20	1	Tool cost	Output	User Specified	102974.1237	0
11	Unnamed Project	2025-08-14 02:03:20	1	Transportation cost	Output	User Specified	3622.880258	0
12	Unnamed Project	2025-08-14 02:03:20	1	WAREHOUSE.NumberIn	Number In	Entity	0	0
13	Unnamed Project	2025-08-14 02:03:20	1	WAREHOUSE.NumberOut	Number Out	Entity	0	0
14	Unnamed Project	2025-08-14 02:03:20	1	customer.NumberIn	Number In	Entity	26281	0
15	Unnamed Project	2025-08-14 02:03:20	1	customer.NumberOut	Number Out	Entity	26281	0
16	Unnamed Project	2025-08-14 02:03:20	1	despatcher one.NumberIn	Number In	Entity	295848	0
17	Unnamed Project	2025-08-14 02:03:20	1	despatcher one.NumberOut	Number Out	Entity	291994	0
18	Unnamed Project	2025-08-14 02:03:20	1	despatcher three.NumberIn	Number In	Entity	291512	0
19	Unnamed Project	2025-08-14 02:03:20	1	despatcher three.NumberOut	Number Out	Entity	290579	0

Figure D.1: Example of Arena Simulation Output in Excel for MSSM Supply Chain Models.

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Curriculum Vitae



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 - **Language:** English (C2-level)
 - **Marital Status:** Married
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Educational Background:

- **Ph.D. Candidate in Operations Research and Business Intelligence**
Politechnika Wroclawska, Wroclaw, Poland
(March 2021 - October 2025):5.0/5.5
- **Master of Science (M.Sc.) in Industrial and Systems Engineering**
King Fahd University of Petroleum and Minerals, Dammam, Saudi Arabia
(2013-2015)
Cumulative GPA: 3.464/4.00
- **Bachelor of Science (B.Sc.) in Industrial and Production Engineering**
University of Ibadan, Ibadan, Oyo State, Nigeria
(2005-2009)
Major GPA: 5.60/7.00
Second Class Honors (Upper Division)

- **National Diploma (N.D.) in Industrial Maintenance Engineering**
Yaba College of Technology, Yaba, Lagos, Nigeria
(2002-2004)
Major GPA: 3.24/4.0 (Upper Credit)
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Teaching Philosophy:

My teaching philosophy is centered around bridging the gap between theoretical concepts and practical applications. I believe in fostering a student-centered learning environment where students actively engage in problem-solving, critical thinking, and analytical reasoning. My teaching approach integrates real-world case studies, digital tools, and industry collaboration to ensure students develop the competencies needed to excel in the evolving landscape of operations management and supply chain logistics. I aim to cultivate curiosity, innovation, and independent thinking in my students while promoting and preserving a deep understanding of the core principles of management science and business intelligence.

Research Interests:

My research focuses on advancing methodologies in production management, inventory control, supply chain optimization, modeling, experimental design, and operations research. I am particularly interested in developing robust and scalable models to improve supply chain efficiency, enhance vendor-buyer coordination, and mitigate risks in dynamic market environments.

A significant aspect of my research involves flexibility in manufacturing, lean manufacturing, Kanban systems, and sustainable production strategies. I explore how agile production frameworks can enhance operational resilience, reduce waste, and improve resource utilization. Additionally, I am deeply invested in sustainability, carbon footprint reduction, and Life Cycle Assessment (LCA) to assess and minimize the environmental impact of manufacturing and logistics operations.

By leveraging advanced analytical techniques, such as simulation modeling, optimization methodologies, and decision analytics, my work aims to enhance decision-making in logistics and industrial systems. My objective is to contribute meaningfully to both academia and industry by integrating theoretical frameworks with real-world applications, addressing contemporary challenges, and promoting sustainable and efficient manufacturing and supply chain practices.

Professional Affiliations:

- Member, International Association of Engineers (2015)
 - Graduate Member, Nigeria Society of Engineers (2010)
 - Graduate Student Member, Institute of Industrial Engineers (KFUPM Chapter) (2014)
-

Publications:

- AlDurgam, M., Adegbola, K., & Glock, C. H. (2017). A single-vendor single-manufacturer integrated inventory model with stochastic demand and variable production rate. *International Journal of Production Economics*, 191, 335-350.

- Adegbola, K. (2023). A simulation study of single-vendor, single and multiple-manufacturers supply chain system, with stochastic demand and two distribution policies. *Journal of decision analytics and intelligent computing*, 3(1), 62-79.
- Adegbola, K., Sanuade, O. A., Oladunjoye, M. A., & Adefehinti, A. (2020). Investigating the necessity of in-situ and laboratory data in determining thermal properties of tar sands, an experimental design approach. *Journal of King Saud University-Science*, 32(3), 2148-2156.
- Adegbola, K., & Abdulrahman, A. (2023). A proportionately increased small and equal batch delivery under consignment stock agreement for a single vendor, multiple buyers supply chain system. *International journal of research in industrial engineering*, 12(4), 414-430.
- Dinardo, M., Murino, T., & Adegbola, K. (2023). A methodology for selecting optimal knowledge acquisition through analytic hierarchy process and environment parameters impact. *Journal of applied research and technology*, 21(5), 825-849.
- Adetokunbo, P., & Sanuade, O. A., Edigbue, P., Adegbola, K., Daramola, T. (2017). Statistical analysis of data processing in some seismic refraction methods: A synthetic data example. *Geofizika*, 34(1), 93-105.
- Adekunle, A. K. (2015). *Integrated Supply Chain Models with Random Demand and Flexible Production Process* (master's thesis, King Fahd University of Petroleum and Minerals (Saudi Arabia)).
- Kehinde, A., Di Nardo, M., Murino, T., & Pandey, S. (2025). LCA energy-oriented discrete event simulation: An innovative framework for achieving sustainability in a brewery plant. *Computers & Industrial Engineering*, 111104.

Articles Under Review

- A Model of Multiple Suppliers, Single Manufacturer Inventory Control with Stochastic Demand, Variable Production Rate, and Differing Supplier Delivery Times -[Journal of Industrial and Production Engineering](#):

Work in Progress

- Optimizing Ingredient Supply Strategies in a Single-Line, Single-Product Manufacturing System with Perishable Inputs and Variable Setup Times
- A Decision Support Framework for Supplier and Location Selection in Nearshore Transitions.
- Reassessing Human-Centric Productivity Metrics in E-Commerce Warehousing Under Industry 5.0

Certifications:

Nature masterclasses certificate

- Focus on Peer Review
- Writing a Research Paper
- Publishing a Research Paper
- Writing and Publishing a Review Paper
- Persuasive Grant writing

Technical Skills:

- Proficient in Microsoft Office (Word, Visio, Excel, PowerPoint), LaTeX
- Advanced proficiency in MATLAB, Minitab, Lingo, AMPL (Optimization)

Arena, Net logo (DES & ABM Simulation)

- Strong analytical skills in optimization, risk analysis, and decision modeling
-

Awards and Honors:

- Erasmus-sponsored Apprenticeship and Trainee, University of Napoli, Federico II, Italy (*April-September, 2024*).
 - PWR Minigrant for Doctoral Student (2024)
 - Politechnika Wroclawska, PhD Study Doctoral Scholarship (2021-2025)
 - King Fahd University of Petroleum and Minerals Research Grant (2015)
 - King Fahd University Full Global Scholarship for Graduate Students (2013-2016)
 - Best Graduating Student, Department of Industrial Maintenance Engineering (2003)
 - National Merit Award in Mathematics, National Mathematical Center, Abuja (1994)
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