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Review of the Doctoral Dissertation

submitted by Willy Prastiyo, MSc, Eng.
entitled
“Balancing of Slider-Crank and Scotch-Yoke Mechanisms by the Use of Mechanical Resonance”

1. Formal Basis for the Review

The formal basis for preparing this review is Notification No. 24/04/D07/2026 of 28 April 2026 appointing me as a reviewer in the proceedings for the award of a doctoral degree. The notification was signed by Professor Renata Krzyżyńska, Vice-Rector for Research and Innovation of Wrocław University of Science and Technology, and issued pursuant to Resolution No. 410/20/RDND07/2024-2028 of the Mechanical Engineering Discipline Council of Wrocław University of Science and Technology.

2. Description of the Dissertation

The doctoral dissertation submitted for review comprises 142 pages and consists of six chapters, including a chapter containing a bibliography of 129 references. The main chapters are preceded by abstracts in English and Polish and by a list of symbols used in the dissertation. The dissertation contains numerous figures, plots, and tables illustrating the issues discussed and the results of the analyses performed. The dissertation was written in English.

The first chapter introduces the reader to the subject matter addressed in the dissertation. The Author presents two mechanical systems that are examined in the subsequent analyses: the slider-crank mechanism and the Scotch-yoke mechanism. An important element of this part of the dissertation is an extensive review of the relevant literature, presented as a tabular summary of 73 publications from the years 1902-2025. For each publication, both the mechanism under consideration and the principal contribution of the study to the development of knowledge in the field are identified.

In the subsection entitled “Scope and Contribution”, the Author presents the principal scientific achievement of the dissertation, namely a new concept for balancing mechanisms based on the deliberate use of mechanical resonance in slider-crank and Scotch-yoke mechanisms. In contrast to the traditional engineering approach, in which resonance is usually regarded as an undesirable phenomenon that should be eliminated, the Author demonstrates that, under appropriately designed conditions, resonance may be used as a beneficial dynamic effect leading to improvements in selected operating parameters of the system. The chapter concludes with a concise outline of the dissertation.

Chapter 2, entitled “Synthesis”, begins by presenting the theoretical foundations required for the subsequent analyses. The Author considers three cases of vibration of a linear single degree of freedom system modelled as a mass-damper-spring system. The analysis covers excitation by a harmonic force of constant amplitude, kinematic excitation with a constant amplitude of base motion, and inertial excitation whose amplitude is proportional to the square of the excitation frequency. For all three cases, the Author presents amplitude-frequency characteristics, highlighting the resonance phenomenon that forms the basis of the solutions proposed in the dissertation.

In the following part of the chapter, the Author proceeds to analyse the kinematics of the slider-crank and Scotch-yoke mechanisms, which constitute the principal subject of the research. Geometric constraint relations are formulated, Lagrange functions are constructed for both systems, dynamic equations of motion are derived, and the physical parameters of the systems are specified. Figures 2.11 and 2.12 present the results of preliminary simulation studies illustrating the influence of an additional elastic element on the operation of the systems under consideration. These results indicate the possibility of resonance leading to a reduction in the variability of the driving torque and in the power demand of the system. The Author also draws attention to the importance of the appropriate equilibrium position of the reciprocating mass for obtaining the desired dynamic effect and to the influence of gravitational acceleration on the behaviour of the systems. The chapter concludes with optimisation studies showing that the greatest reduction in driving torque is achieved under operating conditions close to the resonance frequency.

Chapter 3, entitled “Analysis”, develops the issues introduced in the preceding chapter. The Author derives a relationship defining the resonance frequency of the system and then uses it to formulate equations for determining the driving torque under resonance conditions for both mechanisms. In addition, relations based on Lagrange multipliers are presented for determining forces in the kinematic pairs, and equations describing the instantaneous power associated with the driving torque are formulated both as functions of time and of the angular position of the crank. For the adopted physical parameters, the results of analyses of the forces occurring in the individual kinematic pairs are presented in the form of plots and tables containing peak-to-peak and root-mean-square values. Part of the chapter is also devoted to strength analyses of the mechanisms performed in ADAMS under the assumption of small deformations. The most important part of the chapter comprises parametric studies of the effects of the reciprocating mass, the stiffness of the additional spring, and external forces, including friction forces, on the reduction in the amplitude of the variable component of the driving torque and on the work performed by the driving torque over an operating cycle. The chapter concludes with considerations concerning the inclusion of the drive motor characteristics and the influence of various multiple-slider configurations.

Chapter 4 presents the results of the experimental investigations. Two test rigs designed to verify the results obtained in the theoretical part are described: one for the slider-crank mechanism and one for the Scotch-yoke mechanism. In both cases, force sensors were used to measure the variable forces transmitted by the connecting rod. The test rigs were also equipped with systems for measuring the power drawn by the drive motor and with a system enabling regulation of the external load. The experiments focused on analysing the influence of the additional mass, spring stiffness, and load on resonance operation, on the amplitude of the forces transmitted by the connecting rod, and on the power drawn by the drive system. In both mechanisms, the occurrence of resonance and the possibility of reducing the amplitude of the variable forces were confirmed, although the effects obtained experimentally were clearly less pronounced than those predicted by the theoretical models. The chapter concludes with a design algorithm for systems using an additional elastic element to reduce the variable force components in the mechanisms under consideration.

Chapter 5 presents two examples of practical applications of the proposed concept. The first concerns a reciprocating compressor, for which the possibility of reducing power demand and driving torque by introducing an additional spring between the frame and the crosshead was analysed. The studies used coupled simulation models developed in ADAMS and Simcenter Amesim, enabling the interaction between the mechanical system and the gas flow to be represented. In addition to the simulation analyses, experimental results obtained using a prototype are also presented. The second application concerns a punch press machine in which an additional elastic element was used to reduce the driving torque during the blanking process. Both the simulation and experimental results confirmed the possibility of substantially reducing the energy demand of the systems through the use of mechanical resonance.

The dissertation concludes with Chapter 6, entitled “Conclusions and Future Work”, in which the Author concisely summarises the results of the research, identifies the limitations of the study, and indicates directions for further work. The Author states that the dissertation develops a method for the dynamic balancing of reciprocating mechanisms that uses mechanical resonance to compensate for inertia forces and reduce drive-energy demand. It is also demonstrated that appropriately selected elastic elements make it possible not only to reduce dynamic forces but also to decrease loads in individual mechanism parts, provided that the requirements concerning the equilibrium position and compensation for the influence of gravity are satisfied. The numerical analyses and experimental studies confirmed the effectiveness of the proposed solution in reducing dynamic loads, structural deformation, and energy consumption in real mechanisms.

3. Major Substantive Comments

1. Application of Lagrange’s equations of the second kind with Lagrange multipliers. In the theoretical part of the dissertation, comprising Chapters 2 and 3, the Author declares that the dynamic equations of motion of the systems under consideration are formulated using Lagrange’s equations of the second kind extended by Lagrange multipliers. In the reviewer’s opinion, this issue requires clarification. The Author formulates the Lagrange function after incorporating the relations resulting from the geometric constraints and then uses this function directly to derive the equations of motion. This means that the constraints have already been taken into account at the stage of defining the Lagrange function, rather than through additional terms representing constraint reactions expressed by means of Lagrange multipliers. Consequently, the mechanisms have been reduced to systems with one degree of freedom associated with the rotational motion of the crank, which eliminates the need to apply the Lagrange-multiplier formalism. In the form presented, the derivations correspond rather to the classical form of Lagrange’s equations of the second kind than to their extended form. This matter should be explained unambiguously during the public defence of the dissertation.

2. Determination of reaction forces in the kinematic pairs. Directly related to the preceding comment is the method used to determine the reaction forces in the kinematic pairs, particularly the forces shown in Figures 3.3 and 3.4. The dissertation does not present complete analytical relations describing these forces as functions of the physical parameters of the systems, the motion coordinates, and their derivatives. The text refers to the use of Lagrange multipliers in determining the reaction forces, but the explanations provided are incomplete and do not permit an unambiguous reconstruction of the computational procedure. The Author presents a Jacobian matrix obtained from the gradients of the constraint equations, but does not formulate clear relations connecting the Lagrange multipliers with the reactions in the kinematic pairs. Particular doubts arise from the interpretation of Equation (3.14), which appears to suggest direct equality between the components of the reaction forces and the Lagrange multipliers. In the general case, such equality does not hold. For ideal constraints, Lagrange multipliers may indeed be used to determine reaction forces, but those forces are related to the gradients of the constraint functions through a proportionality relation rather than a direct equality. Since Lagrange multipliers are unknown scalar quantities dependent on time or on the motion parameters, the manner in which they are used should be presented in considerably greater detail. This issue requires clarification by the Candidate.

3. Formula for the natural frequency of the resonant system. Equation (3.7), which defines the natural frequency of the system after the addition of the supplementary spring, also requires discussion. In the reviewer's opinion, this is one of the key equations in the dissertation, because it forms the basis of the entire concept of using mechanical resonance in the mechanisms under consideration. The omission of mass m_1 from this formula is not explained. Figures 2.8(a) and 2.8(b) show a system described by three masses, m_1, m_2, and m_3, and by the stiffness parameter k. All these quantities influence the dynamic properties of the system and therefore its natural frequency. The omission of mass m_1, corresponding to the part undergoing rotational motion, has neither been justified nor discussed. It should be noted that in single degree of freedom systems it is possible to reduce masses and forces to a selected reduction point, thereby accounting for the influence of all system components on its dynamic properties. This naturally leads to more complex relations, because the reduced mass becomes a function of the generalised coordinate and the equation itself becomes nonlinear. Nevertheless, at least an analysis of the influence of mass m_1 on the final form of the formula appears justified in this case.

4. Inclusion of the dynamic properties of the drive motor. The next comment concerns the expression "a system without friction and external forces", used by the Author in Section 3.9. Although the mathematical model contains no explicitly formulated dissipative elements, the system under consideration includes a drive motor, which may be a source of damping effects. In the case of direct-current (DC) motors, the mechanical characteristic defining the relationship between torque and rotational speed has a finite slope. When this characteristic is included in the equation of motion, a term analogous to viscous damping is obtained. This means that the real system is not ideally lossless, and the drive characteristic may influence both the resonance parameters and the effectiveness of the proposed method of reducing the driving torque. Additional simulation studies accounting for the dynamic properties of the motor would therefore be valuable in order to verify the influence of this phenomenon on the results obtained.

5. The Candidate's contribution to the preparation of the test rigs. The experimental investigations presented in the dissertation were conducted using four laboratory test rigs. However, the Author does not clearly specify his own contribution to their design, construction, and preparation for the experiments. In view of the important role of the experimental part in achieving the objective of the dissertation, it would be advisable to define in detail the scope of work performed independently by the Candidate, including the design of the rigs, their commissioning, calibration of the measuring equipment, and development of the experimental procedures. I would ask the Candidate to address this issue during the public defence.

4. Detailed and Discussion-Oriented Comments

1. Formulation of the research problem and the thesis of the dissertation. The research problem is not formulated in a sufficiently explicit and formal manner. Although the principal content of the dissertation focuses on the use of mechanical resonance generated by introducing an additional elastic element into the slider-crank and Scotch-yoke mechanisms, and although on page 11 (the eleventh line from the bottom) the Author refers to the originality of the proposed solution, this information is dispersed and does not constitute an unambiguous definition of the research problem. As a consequence, the reader, and particularly the reviewer, is required to reconstruct the principal objective of the dissertation and infer the research thesis or hypothesis. From the perspective of research methodology, the research problem, the objective of the dissertation, and - if this was the Author's intention - the research hypothesis should have been stated explicitly and unambiguously in the introductory part of the dissertation.

2. Justification of the practical significance of reducing driving-torque variability. On page 44, the Author states that in many technical applications it is necessary to maintain variations in angular velocity and driving torque within specified limits. This statement is fully justified; however, the discussion could be supplemented with more specific practical examples. It would be useful to present actual industrial applications together with requirements concerning permissible amplitudes of rotational-speed or driving-torque variations. Such examples could include reciprocating compressors, production machinery, punching or blanking equipment, or drive systems subject to stringent requirements regarding uniformity of operation. This would provide a stronger justification for the practical significance of the solution presented in the dissertation.

3. Presentation of simulation results. The simulation results are presented mainly in the form of numerous plots and time histories. This graphical form makes it possible to observe the influence of individual parameters on the behaviour of the systems, but in many cases it makes direct comparison of the results difficult. The presentation should be supplemented with appropriate quantitative indicators, such as root-mean-square values, peak-to-peak amplitudes, reduction factors, or measures of deviation from harmonic waveforms. Introducing such quantities would permit a more objective assessment of the effectiveness of the proposed method and facilitate comparison of different design variants and operating parameters.

4. Lack of development of Equations (3.15) and (3.16). Equations (3.15) and (3.16) are presented without further development and are practically not used in the subsequent part of the dissertation. This makes it difficult to assess their significance for the analyses and to verify the method by which the results presented in Section 3.4 were obtained. Since the Author uses ADAMS for part of the analyses, a more detailed description of the computational procedure should have been provided, or the relevant input files and computational models should have been included as appendices to the dissertation. The present form of presentation leaves some uncertainty as to which results were obtained from analytical relations and which were generated directly by numerical calculations in ADAMS. The Candidate should address this matter during the defence.

5. Coupling of the dynamic model with a deformable-structure model. The dissertation provides only a general discussion of coupling the dynamic equations of motion of the mechanisms with models of deformable structural components based on modal coordinates. This issue is important both for modelling accuracy and for the interpretation of the strength-analysis results. A more detailed description of the method of data exchange between the models, the number of vibration modes included, and the influence of the adopted modelling assumptions on the calculation results would have increased the scientific value of this part of the dissertation.

6. Inconsistency in equation numbering. The dissertation contains an inconsistency in the numbering of Equations (2.58) and (2.59) on page 51. This numbering does not correspond to the system previously adopted, in which equation numbers are associated with chapter numbers. This is a purely editorial comment and does not affect the substantive value of the dissertation.

7. Interpretation of differences between theoretical and experimental results. In the chapter devoted to the experimental investigations, relatively little attention is paid to analysing the causes of discrepancies between the simulation and experimental results. The results indicate that the reduction in the amplitude of the connecting-rod forces and the driving torque obtained in the laboratory tests is clearly smaller than that predicted by the mathematical model. A more detailed analysis of the potential sources of these differences - including friction, clearances in the kinematic pairs, manufacturing inaccuracies, nonlinearities, and measurement uncertainty - would make it possible to assess more reliably the range of applicability of the proposed method in real engineering systems.

5. Evaluation of the Dissertation

The dissertation submitted for review is devoted to the analysis of two groups of mechanisms: the slider-crank mechanism and the Scotch-yoke mechanism. These are single degree of freedom systems whose mathematical description can be formulated within the framework of classical dynamics of mechanical systems under the assumption of planar motion. The subject of the dissertation falls within the field of machine dynamics, reduction of dynamic loads, and improvement of the energy efficiency of drive systems.

I assess the theoretical part of the dissertation positively. The Author has correctly formulated mathematical models of the mechanisms and applied an appropriate research methodology based on Lagrange's equations of the second kind. It should be emphasised that the complete geometric relations resulting from the constraints imposed by the mechanism structures were included, without employing commonly used kinematic simplifications. Consequently, the equations of motion obtained remain fully consistent with the mechanical systems under consideration. I have no fundamental objections regarding the formulation of the dynamic models or their use in the simulation analyses.

Some doubts remain, however, with respect to the part of the dissertation concerning the determination of reaction forces in the kinematic pairs. The use of Lagrange multipliers and the interpretation of the resulting relations have not been presented with sufficient clarity, which makes it difficult to assess the correctness of the computational procedure unambiguously. In my opinion, this issue requires additional explanation by the Candidate during the public defence. My final assessment of this part of the dissertation will depend on the quality of the answers provided.

I also assess the scope of the simulation studies positively. They include analyses of the kinematics and dynamics of motion, as well as an evaluation of the influence of additional elastic elements on the driving torque, dynamic forces, and energy demand of the systems. The parametric studies were conducted over broad ranges of physical parameters, enabling the conditions conducive to achieving the greatest reduction in dynamic loads to be identified. The results are presented clearly and make it possible to assess the influence of individual parameters on the behaviour of the mechanisms.

I likewise have no major objections to the strength analyses carried out using ADAMS. It should be noted, however, that these analyses are primarily engineering in nature and serve to verify the structural correctness of the proposed solutions. Their importance for achieving the principal scientific objective of the dissertation is limited, and they do not constitute a fundamental element of the Candidate's original research contribution.

I consider the experimental investigations to be the most valuable part of the dissertation. The Author prepared test rigs corresponding to the systems analysed in the theoretical part and performed a comprehensive verification of the results obtained by calculation. The measurement systems enabled dynamic forces to be recorded and the power drawn by the drive to be measured, allowing the effectiveness of using mechanical resonance to be assessed directly. Particularly valuable is the demonstration that the effects of reducing dynamic loads observed in the simulations are confirmed by laboratory tests. This constitutes an important argument in favour of the practical usefulness of the proposed concept.

Chapter 5, devoted to practical applications of the research results, may also be assessed positively. The Author did not limit the study to academic models of mechanisms, but demonstrated the possibility of applying the developed concept in real technical devices. The investigations of a reciprocating compressor and a punch press machine are particularly valuable, since these systems represent important groups of industrial machinery. The demonstrated reduction in driving torque and energy demand confirms the application potential of the proposed approach.

A certain shortcoming is the absence of a more in-depth analysis of the reasons for the considerable variation in the effectiveness of the proposed method between different system configurations and between theoretical and experimental results. In many cases, the Author limits the discussion to the presentation of simulation results without attempting to formulate simplified analytical relations that would explain the physical phenomena observed. This creates the impression that some of the analyses are more engineering-oriented than cognitive in nature.

Despite the critical comments presented above, I consider the dissertation to be of a good substantive standard and conclude that its principal scientific objective has been achieved. The Author has demonstrated the ability to conduct theoretical, simulation, and experimental research independently, and the results obtained provide new information concerning the possibility of using mechanical resonance to reduce dynamic loads and energy demand in reciprocating mechanisms.

6. Assessment of the Originality of the Scientific Achievement and of the Candidate's Research Competence in Mechanical Engineering

An analysis of the available literature leads to the conclusion that the use of mechanical resonance as a deliberately induced phenomenon for balancing slider-crank and Scotch-yoke mechanisms is currently a relatively underexplored field of research. Previous work on balancing such mechanisms has focused primarily on the use of counterweights, appropriate mass redistribution, elastic elements serving compensating functions, and optimisation of design parameters. Mechanical resonance, by contrast, has most often been regarded as an undesirable phenomenon whose occurrence should be limited or eliminated.

Against this background, the approach proposed by the Candidate should be regarded as original. The essence of the concept is the deliberate use of mechanical resonance as a means of reducing the influence of inertia forces and the variable components of the driving torque. This represents a qualitative change in the way resonance is perceived in the mechanisms under consideration and a departure from the classical design approach. The Author proposes the use of energy exchange between the mass undergoing reciprocating motion and an additional elastic element operating under conditions close to mechanical resonance. In the proposed concept, the elastic element stores energy during the deceleration phase of the moving mass and releases it during the acceleration phase. As a result, a substantial part of the energy required for periodic changes in the velocity of the mass is exchanged locally between the mass and the spring, without the need for it to be supplied each time by the drive system. According to the interpretation presented by the Author, this leads to a substantial reduction in the amplitude of the driving torque and in the dynamic loads acting on the mechanism.

An important scientific achievement is not only the formulation of the concept itself, but also its comprehensive verification through theoretical analyses, simulation studies, and experiments. The dissertation demonstrates that appropriate selection of the stiffness of the additional spring may result in a substantial reduction in dynamic forces and in the power demand of the system. The Author also presents examples of practical applications of the proposed method, which further enhances the application value of the results obtained.

The Candidate has demonstrated competence corresponding to the requirements for applicants for the doctoral degree in mechanical engineering. Particular emphasis should be placed on his ability to formulate mathematical models of mechanical systems, derive dynamic equations of motion using the

Lagrangian formalism, perform parametric analyses, and interpret the results obtained. The Candidate has also demonstrated knowledge of modern computational tools used in mechanical engineering, including ADAMS and Simcenter Amesim, as well as the ability to integrate multidomain models describing the interaction between a mechanical structure and a working medium.

The scope of the experimental investigations also deserves a positive assessment. The Author has demonstrated the ability to plan and conduct experiments, select measurement equipment, process the results, and analyse them. Particularly valuable is the performance of experiments enabling direct verification of the results obtained through computer modelling and simulation.

Although the systems investigated are relatively uncomplicated in structural terms, the scope of the work performed, its methodological coherence, and the successful demonstration of the validity of the proposed research concept justify the conclusion that the Candidate possesses the substantive knowledge required of applicants for the doctoral degree in mechanical engineering, the ability to conduct scientific research independently, and the capacity to solve original research problems. The results obtained constitute a valuable contribution to the development of knowledge concerning the dynamics and balancing of reciprocating mechanisms.

7. Final Conclusion

On the basis of the analysis of the doctoral dissertation, I conclude that the scientific objective set by the Author has been achieved to a degree corresponding to the requirements applicable to doctoral dissertations in the field of engineering and technology, in the discipline of mechanical engineering. The research presented in the dissertation was conducted using an appropriate scientific methodology comprising mathematical modelling, simulation analyses, and experimental verification of the results.

The Candidate has demonstrated the ability to conduct scientific research independently, formulate and solve research problems, and critically analyse the results obtained. Particular emphasis should be placed on the combination of theoretical research with extensive experimental verification and on the presentation of practical possibilities for applying the proposed solutions in real technical devices.

The analysis conducted leads to the conclusion that the results obtained constitute an original solution to the scientific problem of using mechanical resonance to reduce dynamic loads and energy demand in slider-crank and Scotch-yoke mechanisms. The proposed research concept introduces a new perspective into the field of mechanism balancing and constitutes a valuable contribution to the development of knowledge in machine and mechanism dynamics.

The critical comments presented in this review are primarily of a discussion-oriented, explanatory, or supplementary nature. They do not undermine the fundamental scientific value of the dissertation or the correctness of the principal conclusions formulated by the Candidate. In my opinion, the issues identified may form the subject of substantive discussion during the public defence, without affecting the positive final assessment of the dissertation.

Taking into account the substantive standard of the dissertation, the originality of the research problem addressed, the correctness of the research methods applied, and the significance of the results for the development of the discipline of mechanical engineering, I conclude that the doctoral dissertation submitted by Willy Prastiyo, MSc, Eng., entitled "Balancing of Slider-Crank and Scotch-Yoke Mechanisms by the Use of Mechanical Resonance", meets the requirements for doctoral dissertations set out in Article 187 of the Act of 20 July 2018 - Law on Higher Education and Science.

I therefore recommend that the doctoral dissertation be accepted and that Willy Prastiyo, MSc, Eng., be admitted to the public defence of the doctoral dissertation.

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Signed by / Podpisano przez:

Grzegorz Cieplok
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33