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Management of a Communication Network: A Cooperative Game Approach.

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ROZPRAWA DOKTORSKA

Zarządzanie siecią komunikacyjną: podejście oparte na grze kooperacyjnej

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Dedication

In the name of ALLAH SWT, the Most Gracious, the Most Merciful.

This work is lovingly dedicated to my **beloved parents and younger sister in heaven**, whose love, prayers, and sacrifices continue to guide and strengthen me each day. Though they are no longer here, their presence lives on in every achievement and every step I take.

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ABSTRACT

Communication networks have become increasingly complex due to the involvement of multiple service providers that control different parts of the infrastructure. In such environments, efficient network operation requires not only optimal routing and resource utilization but also fair allocation of the benefits generated by cooperation among providers. Traditional network optimization approaches primarily focus on efficiency and do not adequately address fair profit distribution in multi-provider systems. This dissertation proposes a cooperative game-theoretic framework for the management of communication networks. The network is modeled as a weighted graph, where nodes represent communication points and edges represent transmission links controlled by different service providers. Coalitions are formed by groups of providers that combine their resources to serve communication demands. A characteristic function is defined to evaluate the value of each coalition based on feasible and economically viable routes, integrating network connectivity, transmission costs, and demand.

The study first analyzes a basic model with linear transmission costs and no capacity constraints to provide a clear understanding of coalition behavior, route control, and profit generation. It then extends the model by incorporating capacity constraints, allowing the analysis of congestion effects and limited network resources. The research also develops allocation mechanisms based on cooperative game theory, including the Shapley value, Myerson value, and S-value, to ensure a fair and stable distribution of profits among service providers. In addition, economic centrality measures are introduced to assess the importance of providers by considering flows, costs, and revenues under optimal control without having to consider the network structure in detail. The results show that cooperative game theory is a robust tool for analyzing multi-provider communication networks. The proposed model addresses both structural and economic aspects. It enables a deeper understanding of cooperation, efficiency, and fairness. Key contributions include a coalition-based modeling framework for these networks, an extension to capacity-constrained environments, the development of fair allocation mechanisms using game theory, and new economic measures of centrality. This research helps develop network management tools and lays a foundation for future studies on advanced, dynamic systems.

STRESZCZENIE

Sieci komunikacyjne stają się coraz bardziej złożone ze względu na udział wielu dostawców usług, którzy kontrolują różne części infrastruktury. W takich warunkach efektywne funkcjonowanie sieci wymaga nie tylko optymalnego routingu i wykorzystania zasobów, lecz także sprawiedliwego podziału korzyści wynikających ze współpracy między dostawcami. Tradycyjne podejścia do optymalizacji sieci koncentrują się głównie na efektywności i nie uwzględniają w wystarczającym stopniu sprawiedliwej dystrybucji zysków w systemach wielodostawczych. Niniejsza rozprawa doktorska proponuje ramy zarządzania sieciami komunikacyjnymi oparte na teorii gier kooperacyjnych. Sieć modelowana jest jako graf ważony, w którym węzły reprezentują punkty komunikacyjne, a krawędzie oznaczają łącza transmisyjne kontrolowane przez różnych dostawców usług. Koalicje tworzone są przez grupy dostawców, które łączą swoje zasoby w celu realizacji zapotrzebowania na transmisję danych. Funkcja charakterystyczna definiuje wartość każdej koalicji na podstawie wykonalnych i ekonomicznie uzasadnionych tras, integrując strukturę sieci, koszty transmisji oraz popyt.

W pierwszej części pracy analizowany jest model podstawowy z liniowymi kosztami transmisji i bez ograniczeń przepustowości, co pozwala na przejrzyste badanie zachowania koalicji, kontroli tras oraz generowania zysków. Następnie model zostaje rozszerzony o ograniczenia przepustowości, co umożliwia analizę efektów przeciążenia oraz ograniczonych zasobów sieciowych. W pracy opracowano także mechanizmy alokacji oparte na teorii gier kooperacyjnych, takie jak wartość Shapleya, wartość Myersona oraz S-value, które zapewniają sprawiedliwy i stabilny podział zysków pomiędzy dostawcami usług. Ponadto wprowadzono ekonomiczne miary centralności, które oceniają znaczenie dostawców na podstawie przepływów, kosztów i przychodów przy optymalnej kontroli, bez szerszej analizy struktur sieci. Uzyskane wyniki pokazują, że teoria gier kooperacyjnych stanowi skuteczne narzędzie analizy sieci komunikacyjnych z wieloma dostawcami. Zaproponowany model uwzględnia zarówno aspekty strukturalne, jak i ekonomiczne, umożliwiając głębsze zrozumienie współpracy, efektywności i sprawiedliwości. Główne wkłady pracy obejmują opracowanie modelu opartego na koalicjach, rozszerzenie go o ograniczenia przepustowości, rozwój sprawiedliwych mechanizmów alokacji oraz wprowadzenie nowych ekonomicznych miar centralności. Wyniki te wspierają rozwój narzędzi zarządzania sieciami oraz stanowią podstawę dla przyszłych badań nad zaawansowanymi i dynamicznymi systemami komunikacyjnymi.

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ABBREVIATIONS

SDN – Software-Defined Networking
GPU – Graphic Processing Unit
OSPF – Open Shortest Path First
IS-IS – Intermediate System to Intermediate System
D2D – Device to Device
TU – Transferable Utility
ISP – Internet Service Providers
AI – Artificial Intelligence

NOTATION/SYMBOLS

Chapter 2

Notation/Symbols	**:**	**Descriptions**
$V(G)$	:	Set of vertices in a given Graph G
$E(G)$	:	Set of edges in a graph
(u,v)	:	Edge between vertices u and v
$d_i(v)$	:	In-degree of vertex v
$d_o(v)$	:	Out-degree of vertex v
A	:	Adjacency matrix (set of weights assigned to the connection from vertex i to j)
$\{a_{i,j} \mid i, j \in \{1,2, \ldots, \lvert V\rvert\}\}$	:	Set of elements in an adjacency matrix
$\lvert V\rvert$	:	Total number of vertices
$v_i\ (1 \leq i \leq \lvert V\rvert)$	:	List of vertices attached to vertex
$O(\lvert E\rvert)$	:	Memory space of order equal to the numer of edges
B	:	Incidence matrix (between vertices and edges)
I	:	Identity matrix
$P(k)$	:	Relative frequency of degree k vertices
$\kappa(G)$	:	Vertex connectivity
$\lambda(G)$	:	Edge connectivity
C_v	:	Local clustering coefficient
e_v	:	Set of edges between vertex v's neighbors
$\deg(v)$	:	Number of edges incident to node v
$C_D(v)$	:	The degree centrality of a vertex v
$N(v)$	:	Set of vertices directly connected to v
$\mathrm{C}_{D,w}(v)$	:	Weighted degree centrality
$w_{v,u}$	:	Weight of edge (v, u)
(s, d)	:	Source–destination vertex pairs
$\sigma_{s,d}$	:	The number of shortest paths from s to d
$\sigma_{s,d}(i)$	:	The number of those shortest paths that pass-through vertex (or edge) i.
$B'_G(e)$	:	The edge betweenness centrality of an edge e in a graph G
$v(S)$	:	The value/profit of a coalition of players S
N	:	The set of all players
$\phi = (\phi_1, \ldots, \phi_{\lvert N\rvert})$	:	The payoff distribution
ϕ_i	:	payoff (share of profit/utility) assigned to player i
λ_S	:	A set of weights over coalitions
$v^L(S)$	:	How much value coalition S can actually achieve given the network connections.
$\mu(v, L)$	:	The Myerson value
$Sh(v_L)$	:	The Shapley value
(N, C)	:	An admissibility structure

$C \subseteq 2^N$	:	The set of allowed coalitions.
$c = (S_0, S_1, \ldots, S_k)$	:	A valid chain of coalitions
$p(\pi)$	:	The probability of a chain.

Chapter 3

Notation/Symbols	**:**	**Descriptions**
$G = (V, E)$	:	Represents the network as a graph where V is the set of nodes and E is the set of edges.
$P = \{P_1, P_2, \ldots, P_K\}$	:	The set of players in the game
E_k	:	The set of edges owned by provider P_k.
$A \subseteq P$	:	A coalition of players
$d_{i,j}$	:	Demand for flow between nodes V_i and V_j
$r_{i,j}$	:	The maximum revenue for a unit of flow between nodes V_i and V_j
$U = \{(V_i, V_j)\}_{i<j}$	:	The set of all routes
$S(A)$	:	The set of routes served by coalition A
$C(A)$	:	The set of routes strongly controlled by coalition A
$D \backslash E$	:	The elements of D that are not in E
$\sigma_{i,j}$	:	The numer of shortest paths from node V_i to node V_j in a weighted graph
$\sigma_{i,j}\ (e)$	:	The numer of those shortest paths that pass through edge e
f_p	:	The flow through path p
c_e	:	The unit cost for using the edge e
w_p	:	The profit made by transferring information along the path p.
$w_p^U(e)$	:	The profit from the path p ascribed to the edge e in that path under a uniform split
n_p	:	The numer of edges in path p
$C^U(e)$	:	The economic centrality of the edge e based on the uniform splitting of profits along paths.
$S_{i,j}$	:	The set of paths whose ends are the nodes V_i and V_j
$w_p^R(e)$	:	The profit from the path p ascribed to the edge e under a cost-proportional split
$C^R(e)$	:	The economic centrality of the edge e based on proportional splitting of profits along paths.
$G_R = (P, F)$	:	Relational graph in which each node represents a player and each edge represents a direct bilateral relationship between two players.

Chapter 4

Notation/Symbols	**:**	**Descriptions**
$V = \{V_1, V_2, \ldots, V_J\}$	:	Nodes in a network.
M	:	The total number of edges in the network
$w_{i,j}$	:	The unit transmission cost for a direct edge between V_i and V_j.

$r_{i,j}$	:	The maximum unit revenue for transmission between V_i and V_j.
$d_{i,j}$	:	The demand for information transmission between V_i and V_j.
$R = \{r_{i,j}\}$	:	Matrix of maximum unit revenues
$D = \{d_{i,j}\}$	:	Demand matrix
$c_{i,j}^{A}$	:	The minimum transmission cost between V_i and V_j using only edges controlled by coalition A.
$S_E(A)$	:	The set of routes that coalition A can economically serve.
$C_E(A)$	:	The set of routes economically controlled by coalition A.
$C_S(A)$	:	The set of routes strongly controlled by coalition A.
$\sigma_{i,j}(v)$	:	The number of these shortest paths that pass through node v
ω_e	:	The weight of edge e
n_y	:	The number of edges in a path y
$Y_{i,j}$	:	The set of shortest paths between nodes i and j.
$1\ (e \in y)$	:	Indicator function: 1 if edge e is in shortest path y, and 0 if not.
$\tilde{r}_{i,j}^{A}$	:	The effective reward per unit for coalition A to transmit between V_i and V_j

Chapter 5

Notation/Symbols	**:**	**Descriptions**
$(w_{1,(e)}, w_{2,(e)})$	:	Weights describing the edge e (unit transmission cost and capacity)
f_q	:	The flow assigned to path q
k_q	:	The unit cost of the path q.
$C^R(e)$	:	The proportional economic centrality of an edge e
$C^U(e)$	:	The uniform economic centrality of edge e
$k_{i,j}$	:	The cost of the cheapest path between nodes V_i and V_j.
c_e	:	The unit cost of edge e
$Q_{i,j}$	:	The set of shortest paths between nodes V_i and V_j
n_q	:	The number of edges in path q
$\tilde{C}_P(e\|q)$	:	The proportional economic centrality of edge e with respect to path q
$\tilde{C}_P(e)$	:	The proportional economic centrality of edge e in the entire network
$\tilde{C}_U(e\|q)$	:	The uniform economic centrality of edge e with respect to path q
$\tilde{C}_U(e)$	:	The overall uniform economic centrality of edge e

Chapter 1

Introduction

This chapter explores how cooperative game theory can improve the management of modern communication networks, which are becoming more complex and feature multiple providers. It explains the motivation for this research: combining network optimization with fairness in allocation to address the challenges posed by strategic interactions among service providers. The chapter clearly states the main goal: to create a unified framework using graph theory, network flow analysis, and cooperative game theory for managing communication networks. It outlines the objectives, research questions, and hypotheses, describes the study's scope and limitations, and emphasizes the framework as its key contribution. The chapter concludes by presenting the dissertation's structure, with this introduction serving as the foundation for the theoretical and modeling advancements in later chapters.

1.1 Background and Motivation

Communication networks are a key part of modern digital infrastructure **(Okardi & Arubere, 2021; Hunukumbure et al., 2022; OECD, 2023).** They support many applications, including Internet services, mobile communication, cloud computing, and smart city systems **(Beshley et al., 2023).** These networks consist of interconnected nodes and transmission links that enable information to be shared across different locations. As digital transformation is advancing, the need for efficient, reliable, and scalable communication networks has increased significantly **(Irmak et al., 2023; Brunetti et al., 2020).**

A communication network can be modeled as a graph where nodes stand for communication points and edges represent transmission links. This approach has been extensively used in network flow theory and optimization models. **(Ahuja et al., 1993; Bertsekas, 1998).** In real-world situations, these networks are usually operated by multiple service providers. Each provider manages a part of the network infrastructure, such as optical fiber links, routers, or switching systems. As a result, the network's overall performance relies on interactions and coordination among these providers **(Uniyal et al., 2020).** In multi-provider communication networks, data transfer between two nodes often requires cooperation among several providers. A single provider may lack sufficient infrastructure to establish an end-to-end connection on its own. Therefore, providers need to collaborate by sharing their resources to enable data transmission across the network. This interdependence creates a complex environment where cooperation, competition, and strategic decision-making coexist. **(Secci, 2009; Vincenzi, 2022).**

Traditional approaches to managing communication networks mainly focus on optimization techniques such as shortest-path algorithms, minimum-cost flow models, and congestion control mechanisms **(Ahuja et al., 1993; Ford & Fulkerson, 1962).** These methods aim to enhance efficiency by reducing transmission costs and maximizing resource use. However, they do not address how the benefits or revenues should be shared among the participating providers. This limitation is especially important in decentralized network environments with multiple stakeholders **(Ogryczak et al., 2014; Amigo et al., 2016).** Without fair and effective allocation methods, conflicts can arise among providers. Some providers contribute more critical infrastructure, like links that connect different parts of the network. These links are vital for maintaining connectivity and enabling communication between nodes. Providers who control such critical components often hold higher strategic importance, but traditional models do not explicitly measure or reward this contribution **(Freeman, 1977; Newman, 2018).**

Cooperative game theory offers a strong framework for analyzing these situations. It examines how groups of players can form coalitions and share the resulting benefits. In communication networks, service providers are considered players, and the network itself defines the structure where cooperation occurs **(Shams & Luise, 2013; Singh et al., 2022).** A cooperative game is characterized by a set of players and a characteristic function that assigns a value to each coalition **(Shapley, 1953).** In communication networks, a coalition's value depends on its capacity to meet communication demands using the infrastructure it controls. This value is affected by network connectivity, transmission costs, and demand patterns **(Saad et al., 2009; Cerulli et al., 2019).** Recent research extends this idea by including communication constraints and directed interactions within networks, resulting in more realistic models of coalition formation **(Bistaffa et al., 2017; Manuel et al., 2020; Gavilán et al., 2023).**

Allocation methods such as the Shapley value provide a systematic way to share the total profit among providers based on their marginal contributions **(Shapley, 1953).** The Shapley value has been extensively used in network analysis to assess the importance of nodes and edges, as well as to distribute costs and revenues in networked systems **(Aadithya et al., 2010; Michalak et al., 2013).** Recent advances have further expanded its usefulness through approximation methods and machine learning techniques, enabling efficient computation in large-scale networks **(Bistaffa et al., 2017; Becker et al., 2020; Passacantando et al., 2025).** Another key development is the integration of network structure into cooperative games, such as graph-restricted games and the Myerson value. These models account for the fact that cooperation is limited by the underlying network topology **(Myerson, 1977).** Recent research provides axiomatic and structural extensions of these models, enhancing the understanding of fair link contributions and coalition structures within communication networks **(Gavilán et al., 2023; Ramsey et al., 2024; Chakrabarti et al., 2024; Li & Shan, 2026).**

Alongside structural and economic factors, real communication networks face capacity limits. Each transmission link has a finite capacity, restricting the amount of data it can transmit. When demand exceeds capacity, congestion occurs, resulting in lower performance and degraded service **(Kelly, 1997; Low & Lapsley, 1999).** Capacity constraints significantly influence routing decisions, coalition viability, and profitability. Network flow models with capacity constraints, such as maximum flow and multicommodity flow problems, have been widely studied to address these challenges **(Khanal et al., 2021).** However, these models primarily focus on optimal flow distribution and do not consider cooperative behavior among multiple providers. Emerging research is beginning to fill this gap by combining cooperative game theory with resource allocation and routing strategies in communication networks **(Jorswieck et al., 2009; Shi et al., 2012; Ramsey et al., 2024).**

Traditional centrality measures, such as degree centrality and betweenness centrality, have been used to assess the importance of nodes and edges in a network **(Freeman, 1977; Brandes, 2001).** However, these measures are purely structural and do not account for economic factors such as flows, costs, and revenues. Recent studies highlight the need for economic and game-theoretic centrality measures that better capture real network usage and value creation **(Szczepanski et al., 2015; Manuel & Ortega, 2023; Ramsey et al., 2024).** Despite increasing interest in cooperative network models, several gaps still exist. Many existing studies depend on simplified assumptions, such as unlimited capacity or purely structural evaluation of network elements. These assumptions limit the applicability of the models to real-world communication systems. There is a demand for a unified framework that combines network structure, economic factors, and capacity constraints within a cooperative game context. Recent research also underscores the importance of integrating cooperative game theory with advanced computational techniques, such as artificial intelligence and network learning, to enhance scalability and decision-making **(Han et al., 2019; Norbu et al., 2021; González et al., 2024).**

The motivation for this research comes from these challenges. There is a clear need to develop a comprehensive model that captures both the physical and economic aspects of communication networks. Such a model should provide tools to evaluate each provider's contribution, design fair allocation mechanisms, and support efficient network management. This study aims to address these issues by modeling communication networks as cooperative games where providers control network edges and form coalitions to serve communication routes. By combining graph theory, network flow optimization, and cooperative game theory, this research seeks to deepen understanding of how network structure influences cooperation, efficiency, and profit-sharing. The results are expected to help improve decision-making in multi-provider communication networks and to contribute to the development of fair and efficient resource allocation mechanisms.

1.2 Objective of the Study

1.2.1 General Objective

The general objective of this study is to develop a comprehensive framework for the management of communication networks based on a cooperative game-theoretic approach. The study aims to integrate network structure, flow optimization, and economic considerations to analyze cooperation among service providers and to design fair and efficient mechanisms for profit allocation.

1.2.2 Specific Objectives

To achieve the general objective, this study defines the following specific objectives:

- To model a communication network as a cooperative game in which service providers control subsets of the set of network edges
- To define a characteristic function that captures both the physical connectivity and economic feasibility of communication routes
- To analyze coalition formation among providers based on their ability to serve network demands

- To study the impact of network structure on provider importance using centrality and game-theoretic measures
- To incorporate transmission costs and capacity constraints into the cooperative network model
- To develop and evaluate allocation rules such as the Shapley value, Myerson value, and S-value for the fair distribution of network profits
- To compare structural centrality measures with economic and game-theoretic centrality in evaluating provider contributions
- To examine how cooperative behavior influences network efficiency, stability, and resource utilization

1.3 Research Questions and Hypotheses

This study examines the management of communication networks within a cooperative game-theoretic framework. With reference to the objectives outlined in Section 1.2, the following research questions are proposed, along with corresponding hypotheses that predict outcomes of the proposed approach:

- How can a communication network be modeled as a cooperative game among service providers?
  Hypothesis: We hypothesize that representing the network as a cooperative game, where providers control subsets of edges, effectively captures the interdependence among providers and their joint contribution to communication services.
- How should the characteristic function be defined to reflect both connectivity and economic feasibility?
  Hypothesis: We hypothesize that a characteristic function incorporating both physical connectivity and economic factors, such as transmission costs and revenues, provides an accurate and realistic valuation of coalitions.
- How do coalitions of providers form in communication networks? **Hypothesis:** Providers form coalitions based on their ability to collectively establish feasible, profitable communication routes within the network.
- How does network structure influence the importance of service providers? **Hypothesis:** Providers controlling structurally critical edges, such as bridges or high-centrality links, have greater influence and receive higher payoffs in cooperative game solutions.
- How do transmission costs and capacity constraints affect the value of a coalition and network performance?

Hypothesis: We hypothesize that incorporating transmission costs and capacity constraints significantly alters the feasibility of coalitions and alters the payoff distributions among providers.

- Which allocation methods provide a fair and stable distribution of network profits? **Hypothesis:** We hypothesize that cooperative game-theoretic allocation methods, such as the Shapley value, Myerson value, and S-value, ensure a fair and stable distribution of profits among providers.
- How do game-theoretic measures compare with traditional structural centrality measures?

  Hypothesis: We hypothesize that game-theoretic measures better capture the true contribution of providers by considering both network structure and economic value, compared to purely structural measures.
- How does cooperation among providers impact overall network efficiency? **Hypothesis:** We hypothesize that cooperation among providers improves network efficiency, enhances resource utilization, and increases the total value generated by the network.

1.4 Scope of the Study

This research focuses on the management of communication networks using a cooperative game-theoretic framework to analyze interactions among multiple service providers and to design fair and efficient mechanisms for resource allocation and profit distribution. The study is restricted to the following areas:

- Network modeling: The research models communication networks as weighted graphs, where nodes represent communication points and edges represent transmission links controlled by multiple service providers.
- Formulation of a cooperative game: The study develops a cooperative game model in which service providers act as players controlling subsets of network edges, and coalitions are formed to serve communication demands.
- Derivation of the characteristic function: The research defines the value of coalitions based on feasible and economically viable communication routes, incorporating both connectivity and costs.
- Network flow and routing: The study analyzes how communication demands are served using shortest-path and flow-based approaches, linking network optimization with the capabilities of coalitions.

- Capacity constraints: The research incorporates capacity constraints on network edges to reflect realistic constraints and evaluate their impact on the feasibility of coalitions and network performance.
- Allocation of network profit: The study applies cooperative game-theoretic allocation methods, including the Shapley value, Myerson value, and S-value, to distribute profits among service providers.
- Centrality analysis: The research also evaluates the importance of providers using structural centrality measures.

This study focuses on developing a theoretical and analytical framework for cooperative communication networks. The scope is restricted to cooperative game theory and does not extend to non-cooperative models, dynamic network evolution, or other types of network systems beyond communication networks.

1.5 Contribution of the Study

This study develops a cooperative game-theoretic framework for network management that integrates network structure, flow optimization, and economic analysis. The main contribution is a unified model treating communication networks as cooperative games, in which service providers control network edges, thereby linking graph theory, network flow, and cooperative game theory in a single framework.

1. The research defines a characteristic function capturing both network connectivity and economic feasibility. It incorporates transmission costs and demand to reflect actual operations.
2. The study incorporates edge capacity constraints into cooperative network games, enabling analysis of congestion, the feasibility of coalitions, and their impacts on network performance and profit.
3. The research develops a formal framework to determine which coalitions can serve specific communication routes, based on infrastructure and economic conditions. This links coalition behavior to network topology and routing limits.
4. The study applies allocation rules such as the Shapley value, Myerson value, and S-value to distribute profits among providers. It shows how these methods ensure fairness and stability.
5. Introduction of economic centrality measures: The research advances centrality analysis by including flow, cost, and revenue in an assessment of the importance of providers, offering a realistic alternative to traditional centrality measures.

6. The study links network flow optimization and cooperative game theory, enabling the analysis of efficiency and fairness in communication networks.
7. The research provides a theoretical foundation for understanding cooperation, resource allocation, and strategy in decentralized communication networks.
8. These contributions advance communication network modeling and offer practical insights for designing efficient, fair management mechanisms in multi-provider networks.

1.6 Structure of the Dissertation

The dissertation is organized into several chapters, each contributing to the development and understanding of the research. Each chapter addresses a specific aspect of the study, from the introduction of the problem to the development of models, and concludes with findings and recommendations.

Chapter 1 introduces the study. It provides the background and motivation, defines the research objectives, outlines the research questions and hypotheses, describes the scope of the study, highlights the main contributions, and explains the overall structure of the dissertation.

Chapter 2 presents the theoretical foundations required for the study. It defines communication networks and their architectures, introduces graph theory and network representation, analyzes centrality measures, and discusses shortest path problems and routing algorithms. It also introduces cooperative game theory, including concepts of coalition formation and allocation.

Chapter 3 presents a general model for the cooperative games in communication networks considered here. It represents a network as a weighted graph, assigns edges to service providers, and models coalitions as groups of providers that combine their resources. It defines the sets of routes that coalitions can serve, introduces the concept of a relational graph to capture interactions among providers, and formulates the characteristic function based on connectivity and cost efficiency. The chapter provides the foundation for analyzing cooperation, allocation, and stability in subsequent chapters.

Chapter 4 develops a particular version of the cooperative communication network model introduced in Chapter 3. It focuses on a setting with linear transmission costs and no capacity constraints, where the cost of a path equals the sum of its edge weights, and congestion is ignored. This setting allows a clear analysis of coalition behavior, route control, and profit generation. Using the concepts of served, economically served, and controlled routes, the chapter formulates the cooperative network game and provides the basis for allocating profit and analyzing stability.

Chapter 5 extends the model developed in Chapter 4 by incorporating the realistic assumption of capacity constraints. As a result, a coalition may not be able to serve all of the demand for a particular route even if it controls a feasible path. This chapter analyzes how capacity constraints affect the feasibility of routes, the behavior of coalitions, and the total profit generated in a network.

Chapter 6 concludes the dissertation by summarizing the main findings, discussing theoretical and practical implications, and providing recommendations for future research.

Chapter 2

Preliminaries

This chapter outlines the theoretical concepts essential for the analysis of communication networks using a cooperative game approach. We begin the study by defining communication networks, explaining their features, and classifying them into centralized, decentralized, and distributed architectures. We discuss the management implications for each architecture type. Then, we discuss the basic concepts of graph theory, explaining various types of graphs, their representations, and, especially, the characteristics of graphs that model networks. We present centrality measures to identify central nodes, the focal points of communication and collaboration, and those that drive coalition formation. We discuss the significance of the shortest-path problem and Dijkstra's algorithm in network routing and performance. We conclude with an exposition of cooperative game theory, the core concepts of connectivity and coalition structures, and the Shapley Value, Myerson Value, and S-value for fair and stable distributions, demonstrating collaboration among the agents of the network and closing the chapter. These concepts provide the basis for the analytical modeling and solution frameworks in Chapter 3 – General Model of Communication Network.

2.1 Foundations of Communication Network Management

2.1.1 Definition and Characteristics of Communication Networks

Communication networks are structured systems in which multiple nodes, such as devices, servers, or access points, are connected via communication links that enable data exchange. These nodes may belong to different administrative domains and operate under diverse technical constraints, reflecting the heterogeneous nature of practical network environments **(Ramsey et al., 2025).** Their scope extends from local-scale configurations, such as local area networks using multicasting for efficient team communication, to large-scale, international, and heterogeneous architectures integrating cellular, satellite, and sensor-based subsystems. Fundamentally, a communication network operates by transmitting data packets from one endpoint to another across a sequence of intermediate nodes. The operational challenge lies in ensuring that this flow of information remains efficient and reliable while accounting for the economic and strategic motivations of the various stakeholders managing different segments of the infrastructure **(Ibrahimov et al., 2024).** The topology of a network embodies more than its physical layout; it reflects logical relationships that define feasible routes for data transmission. Each edge in such a topology corresponds to a communication link whose characteristics, such as available bandwidth, latency, and reliability, can vary widely. In multi-operator contexts, these edges might also represent contractual agreements governing interconnection policies **(Shah et al., 2019).** The resulting graphical representation provides a natural mathematical abstraction for exploring structural properties relevant to both cooperative and competitive interactions among operators. When competitive interactions predominate, decentralization emerges naturally, with each agent making independent decisions based on local information about its neighbors and the quality of its links. This autonomy can enhance fault tolerance but also create complex coordination problems when large-scale cooperation is sought **(Telmanov et al., 2025).**

From an operational standpoint, specific characteristics stand out as particularly relevant for analysis. First, connectivity: whether every pair of nodes has at least one viable communication path between them. The second aspect is capacity, which refers to the amount of data that can flow through edges without breaching performance limits. The third is adaptability, the network's ability to maintain service quality under changing traffic patterns or adverse conditions, such as link failures or congestion peaks **(Riahi et al., 2025).** Another key feature is fairness in resource allocation across users or organizational boundaries, a point closely tied to cooperative game-theoretic considerations in which agents may strategically share infrastructure resources. The interplay between physical constraints (e.g., signal attenuation or interference) and higher-layer protocols means that network performance cannot be attributed solely to hardware capabilities. Instead, policy decisions, such as routing strategies, influence end-to-end behavior. This development has produced two main approaches: cooperative models that ensure a fair distribution of benefits and noncooperative models that determine stable outcomes, such as Nash equilibria, in competitive settings. In modern infrastructures that achieve global coverage by linking operator-managed domains across continents, network characteristics extend beyond technical specifications to include factors of economic feasibility. For instance, an operator controlling a segment of an international path must decide whether forwarding third-party traffic increases its long-term utility relative to potential costs or risks **(Ramsey et al., 2025).** This adds strategic dynamics that shape connectivity patterns. The diversity of network components increases management complexity. Devices might differ in supported communication protocols or energy constraints; some links achieve high throughput, but incur substantial variability in delay due to contention-based issues.

In wireless systems, environmental factors such as mobility and channel fading dynamically affect link availability, necessitating adaptive policies to maintain connectivity **(Riahi et al., 2025).** Given these influences, many architectures benefit from distributed decision-making models in which each participating node reacts in real time to local observations. Distinct from these physical and protocol-level features are structural properties directly linked to agents' potential for coalition formation. Graph-theoretic measures, such as centrality indices, indicate which nodes serve as essential intermediaries within the network topology. This factor affects both cooperative valuation schemes, such as Myerson's value, and resilience analysis under targeted attacks **(Skibski et al., 2019).** An operator whose controlled nodes exhibit high betweenness centrality may have greater bargaining power when negotiating coalition terms because removing or restricting those nodes reduces overall reachability. Such characteristics highlight why modeling these systems through mathematical frameworks that incorporate strategic decision-making provides richer insights than purely engineering-oriented evaluations. Coalitional games offer tools for mapping collaborative scenarios in which value is determined by pooling diverse communication resources into shared service offerings **(Ramsey et al., 2025).**

In contrast, noncooperative games help predict behavior when coordination fails or competition predominates **(Telmanov et al., 2025).** Both perspectives illuminate essential features of communication networks: their innate interdependence among agents, their sensitivity to local actions that propagate system-wide effects, and their dependence on stable yet adaptable arrangements for sustaining performance over time. Ultimately, defining a communication network extends beyond listing its tangible components; it encompasses recognizing the operational context in which those components function together, or sometimes against each other, to achieve overlapping objectives while respecting individual constraints and incentives.

2.1.2 Types of Communication Networks

2.1.2.1 Centralized Networks

A single authoritative node, often called the data server or controller, builds and manages all network information in a centralized network (see Figure 2.1). This architectural choice imposes a clear hierarchy: peripheral nodes interact indirectly through the central entity rather than directly with each other. Such a configuration simplifies certain aspects of management, as global decision-making can be streamlined through oversight by a single coordinating element **(Smed et al., 2002).** However, it also concentrates risk. Should the primary node fail, the entire system may experience disruptions or complete service cessation, unless redundancy mechanisms are in place.

From an operational standpoint, centralized structures provide predictable data consistency and resource allocation because all nodes obtain their instructions and data from a single source. The centralized design enables efficient policy enforcement across the network without requiring distributed consensus among numerous autonomous nodes. In software-defined networking (SDN), for example, a centralized controller maintains a global view of topology and traffic flow, enabling dynamic reconfiguration of routing policies to meet quality-of-service requirements. By decoupling the control plane from the data plane, this approach supports coherent network-wide optimization **(Bennaceur et al., 2021).** However, such cohesion comes at the potential cost of scalability: as the number of connected devices grows, so too does the burden on the controller's processing and communication capacity. In hierarchical variants of centralized systems, an upper-level root controller supervises subordinate local controllers. These local controllers manage specific domains directly while periodically feeding aggregated state information up to the root. The design aims to balance scalability with oversight: local controllers reduce the immediate load on the root by handling intra-domain tasks independently. Still, the necessity for continuous coordination means communication overheads can become significant if domain boundaries are numerous or dynamic. The persistence property of centralized architectures, in which user and system states remain consistent when entities enter or leave, can be advantageous for maintaining collaborative environments **(Smed et al., 2002).** Players or agents reconnecting to such a system can quickly retrieve their last known state from the focal node without complex peer-to-peer resynchronization. The centralized architecture makes such setups particularly suitable for virtual environments and platforms that rely heavily on shared context integrity. Challenges in these networks often stem from bottlenecks at the central node. High volumes of real-time communication may saturate both computational processing and link bandwidth capacities **(Bennaceur et al., 2021).**

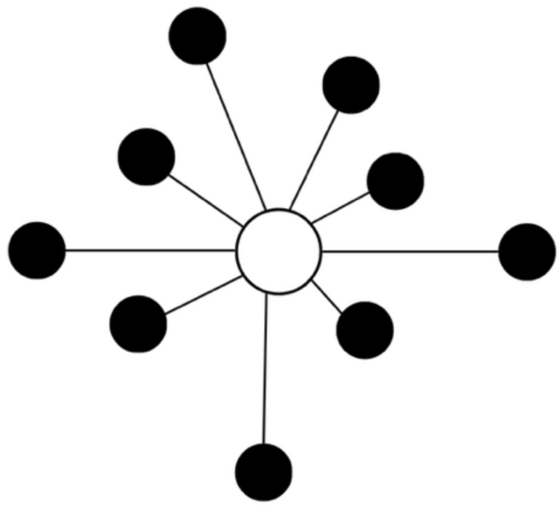


Figure 2.1 Centralized Network

In multidomain deployments that employ a centralized model at higher administrative layers and distribute lower-tier tasks locally, cooperation between controllers is crucial **(Hua et al., 2018).** Efficient inter-controller protocols prevent mismatches between global routing strategies and localized traffic optimization efforts. Here lies an operational tension: too frequent state synchronization increases overheads; too infrequent updates risk stale information guiding decisions that no longer reflect network reality. Ultimately, centralized network design reflects trade-offs between administrative simplicity and flexibility. Its ability to enforce structure across independent components makes it appealing for networks that demand rigorous coordination, such as financial transaction systems. However, its vulnerability to single-point failures necessitates careful redundancy planning. Introducing cooperative game-theoretic mechanisms could further enhance stability by aligning participant incentives with dependable engagement in maintaining central operations under varying load scenarios.

2.1.2.2 Decentralized Networks

Unlike architectures with a singular control point, decentralized networks operate without a central governing node, distributing authority and decision-making across all participants (see Figure 2.2). Such a structure inherently shifts operational dynamics, making each node both a consumer and a contributor of information, thereby promoting resilience against failures in individual network components **(Telmanov et**

al., 2025). The absence of a hierarchical core allows decisions to be taken locally based on partial information, yet ideally coordinated through shared protocols or cooperative frameworks that maintain overall efficiency. The resulting topology tends toward a more uniform distribution of connectivity, with nodes potentially acting as intermediaries for others in multi-hop routes, especially within wireless configurations where direct links may not exist between all communicating parties **(Hadzic et al., 2013).** One interesting aspect is how cooperation emerges without central enforcement.

Degree centrality in decentralized systems often exhibits wide dispersion, revealing hubs that emerge naturally through repeated use rather than by design. Distributed algorithms help identify optimal coalition structures without requiring a global view, a property necessary when the overhead of gathering system-wide state is prohibitive **(Hadzic et al., 2013).** These algorithms aim to increase aggregate utility while ensuring that no participant has an incentive to defect from cooperative arrangements given the current allocation rule. Decentralized operation also affects routing strategy choices. Without a master controller assigning paths, nodes rely on local metrics and limited information from neighbors to select routes.

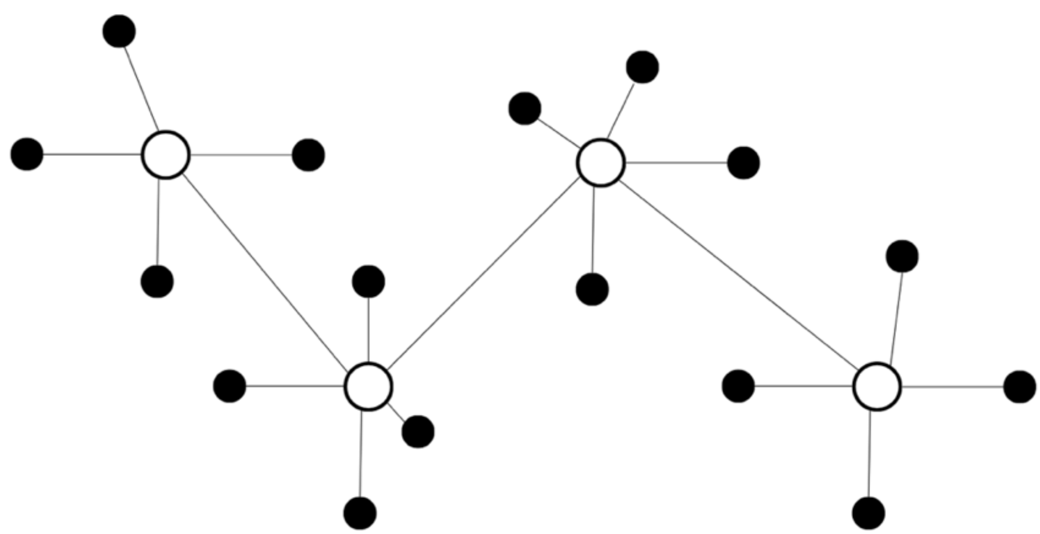


Figure 2.2. Decentralized Network in the Form of a Backbone Network

In Figure 2.2, because each white node represents a node with equal control and routing potential, the network can be interpreted as decentralized. The connections among the white nodes indicate that there is no single point of control over traffic flow, as every core peer directly exchanges data with the rest. The local black nodes represent participants that are attached to the closest peer, and each peer acts as a local source and a conduit. Data can traverse multiple peers, and decisions can be made based on local information shared by neighbors. The system avoids a rigid hierarchy and encourages multi-hop routing because no node manages the entire system. This creates an environment in which local cooperation, devoid of a central authority, is enabled by local collaboration and shared protocols. But it can also be thought of as a backbone network with three core routers at the center of all the system flows. These interconnected core routers handle long-haul traffic on the network's primary route. Each core router manages a number of leaf nodes, which represent local access points or groups of customers that route through the nearby core. Traffic from these leaf nodes, via the upward movement, circulates vertically to the core and traverses the backbone to other regions. This configuration consolidates high-volume flows in the backbone, streamlines path distances in the local area, and quarantines local outages. It also clarifies the basis for cooperative rational analysis, as each core router manages a pivotal section of the transit route.

However, fluctuations in efficiency are common; the absence of central oversight makes rapid convergence toward optimal load balancing harder under volatile conditions, such as sudden traffic spikes. Game-theoretic planning anticipates these shifts by embedding adaptive decision rules that respond locally while aligning with system-wide objectives **(Hadzic et al., 2013; Trestian & Muntean, 2019)**.

2.1.2.3 Distributed Networks

Distributed networks operate under an architecture in which control, data processing, and decision-making are spread across multiple nodes, eliminating any dependency on a single point (see Figure 2.3). Such a configuration differs from decentralization primarily in the degree of autonomy and independence granted to each node. In distributed setups, there is no predefined hierarchy, and tasks can be executed simultaneously across multiple locations with minimal coordination overheads. These architectures support scalable and robust environments by dynamically reallocating workloads to unaffected parts of the network during regional failures **(Telmanov et al., 2025).** One defining trait of distributed networks is their reliance on cooperative protocols that allow disparate nodes to work toward shared objectives without needing a central reference state. These protocols can embed solution concepts from cooperative game theory, enabling rational agents within the network to form dynamic coalitions. In multi-operator or multidomain contexts, this approach becomes particularly relevant; each administrative segment may pursue its own benefit while still entering coalitions that yield greater total utility **(Lu et al., 2015).** The Shapley value is frequently discussed here as a means of proportionally distributing surplus gains based on each node's measurable contributions. The interaction between distributed architectures and graph-theoretic modeling adds significant analytical depth. An undirected or directed communication graph represents nodes as vertices and uses edges to indicate connectivity potential **(Algaba et al., 2021).**

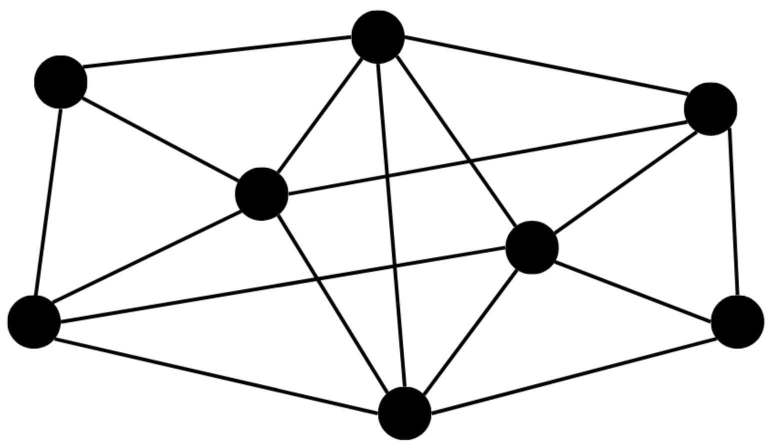


Figure 2.3 Distributed Network

In distributed networks, routing decisions might use variants of Dijkstra's algorithm for shortest-path calculations without assuming global topological information. Instead, each node processes local link-weight data, congestion estimates, and interference levels, and exchanges condensed path information with immediate neighbors **(Liu & Liu, 2020).** This method allows routing updates to occur asynchronously while gradually converging toward efficient network-wide flows. When combined with cooperative decision rules, such as Myerson's value for restricted cooperation scenarios, in which specific routes are unavailable or expensive, the resulting allocation respects both constraints on the infrastructure and principles of fairness. Failure management deserves separate attention in distributed configurations. Because the system diffuses responsibility, designers embed redundancy into communication pathways and keep alternative paths operational, so packets continue to be transmitted even when individual links fail. Cooperative incentives can reinforce this resilience by rewarding nodes that maintain backup routes or contribute to traffic rerouting during disruptions. This reward system discourages selfish inactivity, which might otherwise reduce global quality of service if participants ignore fault-handling obligations. In operational deployment, heterogeneity among nodes is inevitable; processing capabilities, energy reserves, and link quality all vary considerably. Fair resource-sharing mechanisms grounded in cooperative game theory help navigate these disparities. Symmetric negotiation rules produce parity between equals; asymmetric versions weigh allocations by capacity or demonstrated reliability over time **(Telmanov et al., 2025).**

Furthermore, payoff-based reinforcement mechanisms keep coalition-linked routing sufficiently beneficial that agents prefer cooperation even during high-load episodes. An understated but essential aspect concerns how distributed architectures handle incomplete or outdated information, given their asynchronous nature. Delays in propagating topology changes across large networks risk misaligned decisions, as some agents may optimize based on obsolete metrics.

2.2 Graph Theory in Network Management

2.2.1 Fundamentals of Graph Theory

2.2.1.1 Vertices and Edges

In the context of graph theory applied to communication networks, vertices (also known as nodes) and edges form the foundational abstractions from which more complex analytical structures emerge. A vertex, often denoted as an element of *V (G)* for a given graph *G*, represents an object or entity within the network; this could be a physical node, such as a router, sensor, or base station, or a logical node representing a virtualized service endpoint **(Michalak et al., 2013).** Edges, drawn from the set *E (G),* model connections between vertices; these may take the form of physical cabling, radio frequency links in wireless systems, or logical channels in overlay networks. Each edge (*u, v*) joins two vertices *u* and *v,* encapsulating not merely connectivity but potentially attributes such as capacity, latency, cost, or reliability. The interpretation of these elements depends heavily on both network type and analytical purpose. The situations can be illustrated using points and lines, as shown in Figure 2.4. Points *A*, *B*, *C*, *D*, and *E* are vertices, while the lines are edges. The complete diagram forms a graph. The intersection of lines *AD* and *BE* is not a vertex because it does not represent an actual junction or connection point. The graph in Figure 2.4 can represent various situations. If *A*, *B*, *C*, *D*, and *E* denote football teams, an edge between two teams indicates that they have played a match. In this case, team *A* has played against teams *B*, *D*, and *E*, but not against team *C*. The degree of a vertex, the number of edges emanating from that vertex, shows how many matches the corresponding team has played.

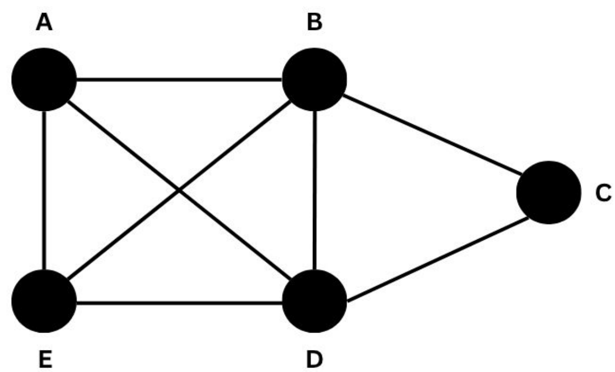


Figure 2.4 A Graph

In many communication graphs, edges are undirected when representing mutual connections, as in peer-to-peer systems where data can flow in either direction between devices, or directed when modeling asymmetric relationships, such as one-way upstream links or follower relationships in social platforms **(Pandey, 2025).** Vertices themselves exhibit varying degrees of centrality depending on how edges connect them across the graph. Edges take on further strategic significance in settings where their existence is mutable, temporary connections are established through opportunistic wireless contact or dynamic virtual channels in overlay networks. The possibility of manipulating the structure of edges introduces another layer to cooperative modeling; agents might form or drop links strategically to increase their bargaining power within coalitions **(Chessa & Loiseau, 2017).** In such scenarios, functions describing the value of a coalition must adapt dynamically as graph topologies evolve: any calculation of an edge's marginal contribution must incorporate not only the utility derived from existing paths but also potential gains from newly forged edges. Self-loops, edges connecting a vertex to itself, occasionally serve a functional role in weighted-graph games, as discussed in localization algorithms for sensor networks **(Hadzic et al., 2013).** While abstract in physical terms for most communication systems, self-loops can represent internal processing gains or recirculated information that partially contributes to a coalition's goals. Accounting for these contributions correctly (often at half their nominal weight) avoids overstating their utility when summing the values of edges held by a coalition. The definition of a neighbor emerges naturally from vertex-edge relationships: neighboring vertices share a direct edge connection.

2.2.1.2 Directed and Undirected Graphs

Directed and undirected graphs are two primary categories in graph theory, distinguished fundamentally by whether edges impose directionality on connectivity. In an undirected graph, as you can see in Figure 2.5, each edge represents a bidirectional link between two vertices, meaning communication or association can occur in both directions with equal validity **(Xu, 2025).** Such representation is common in physical point-to-point connections, like fiber cables or symmetrical radio links, where the underlying medium supports reciprocal transmission without alteration. The absence of directional constraints simplifies specific analyses: determining connectivity becomes straightforward because any path between two nodes exists in both directions. In terms of centrality measures, degree centrality for an undirected graph is based on counting the number of edges attached to a vertex, without having to consider the orientation of those edges **(Pandey, 2025).**

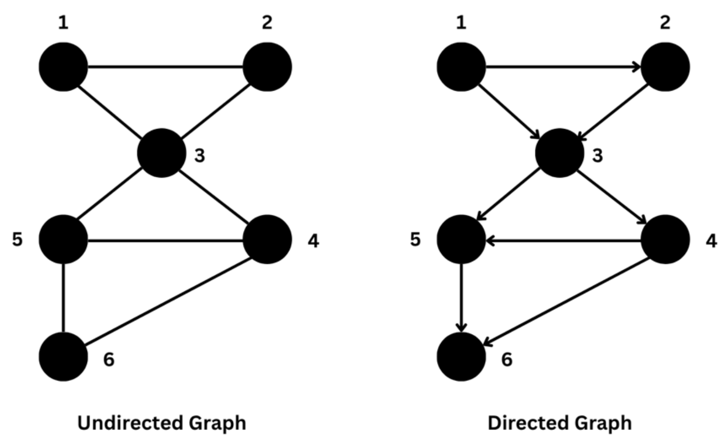


Figure 2.5 A Directed and Undirected Graph

Directed graphs (digraphs), as you can see in Figure 2.5, in contrast, impose a specific orientation on edges. Here, the edge (*u, v*) denotes that flow from *u* can reach *v* along this connection, but not necessarily vice versa unless another distinct edge (*v, u*) exists **(Magzhan & Jani, 2013).** This directional property models asymmetric relations such as master-slave control links, one-way broadcast channels, or dependency flows where data or influence travels in a defined direction. Complexities arise because the existence of a path depends not only on vertex adjacency but also on the allowed direction of travel along successive edges. Such a distinction introduces nuanced concepts of connectivity. Strong connectivity ensures that every vertex is reachable from every other vertex via directed paths in both directions.

In contrast, weak connectivity treats all directed edges as undirected, ignoring direction **(Pandey, 2025).** Degree centrality bifurcates naturally in directed graphs into in-degree and out-degree measures: $d_i(v)$ and $d_o(v)$ denote the in-degree and out-degree of vertex v, respectively **(Pandey, 2025).** These reflect different operational roles: high out-degree nodes act as sources or distributors of information, while high in-degree nodes act as sinks or aggregators of traffic. Coalition reward schemes might weight these differently depending on whether network objectives prioritize the capacity to disseminate over the ability to collect. Combining these into ordered degree pairs $d_i(v)$ and $d_o(v)$ enables simultaneous evaluation of bidirectional capabilities, where applicable. When modeling real-world scenarios such as routing protocols or load distribution algorithms, incorporating directionality means algorithms must carefully evaluate admissible paths. Adaptations of shortest-path methods like Dijkstra's must respect edge orientation; attempting to traverse an edge against its specified direction immediately disqualifies it from consideration **(Magzhan & Jani, 2013).** In the context of distributed routing, maintaining accurate local knowledge of incoming versus outgoing links is essential; mismatches lead to failed deliveries even if logical proximity suggests otherwise. Weighted directed graphs extend the concept further by assigning link weights that differ according to the direction of travel, reflecting asymmetric capacities or costs. This variation produces different optimal routes that a simple undirected model would fail to capture. Directionality also refines how graph components are defined.

2.2.1.3 Weighted and Unweighted Graphs

In graph-theoretic models of communication networks, one primary distinction concerns whether edges carry weights or remain unweighted. Choosing between these representations directly determines the analytical methods available and the realism of network performance evaluation. An unweighted graph (see Figure 2.6) embodies edges as binary relationships, either a link exists between vertices or it does not, without quantifying properties such as costs, capacity, delay, or reliability. Such an approach can simplify computational procedures when the focus is solely on structural connectivity. Pathfinding in such contexts often reduces to counting the number of edges between nodes, predicting shortest routes according to the minimal number of vertex traversals rather than the accumulated costs of links. Weighted graphs (see Figure 2.6) incorporate numerical edge weights to represent measurable attributes that influence network behavior. These might correspond to cost factors such as monetary expenses for using certain links, probability measures reflecting expected availability, or distance metrics related to physical separation **(Xu, 2025).**

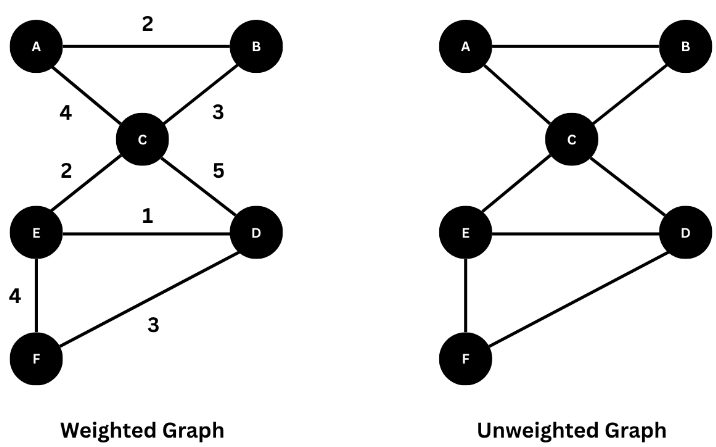


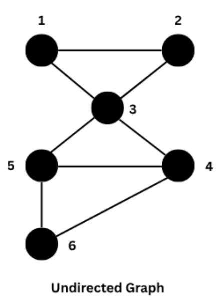
Figure 2.6 A Weighted and Unweighted Graph

In operational network management, edge weights frequently represent capacity constraints or quality-of-service parameters such as average latency or packet loss rates. Mathematically, given a set of vertices V and edges E, a weighted adjacency matrix A of size $|V| \times |V|$ can store these values directly, where $A_{i,j}$ captures the weight assigned to the connection from vertex i to vertex j **(Xu, 2025).** Such representation allows algorithms to compute optimal paths or coalition structures by summing weights along candidate routes and favoring those with minimal aggregate cost. In practical terms, the choice of a model critically influences routing strategies. Weights describing the length or cost of an edge integrate naturally with optimization algorithms such as Dijkstra's for deriving shortest paths when the weights are non-negative **(Arul et al., 2023).** When negative edge weights appear, for instance, representing credit or favorable transmission conditions, the Bellman-Ford method is applicable, as it can accommodate such weights while still detecting cycles that would undermine optimality **(Arul et al., 2023).** Unweighted graphs yield more straightforward computation because each edge contributes equally to the total path length, yet they ignore potentially significant heterogeneity among links in real systems. From the perspective of cooperative game theory, edge weighting has profound consequences for evaluating marginal contributions within coalitions. Suppose two nodes collaborate to relay packets across multiple hops; if their connecting edges have higher capacities and lower delays than alternatives, their inclusion dramatically increases the value of a coalition under resource allocation rules like the Shapley value **(Yamada et al., 2025).** The calculation of marginal contributions must then account for quality-adjusted path advantages rather than assuming all connections are equally beneficial. Likewise, Myerson's value extends the evaluation of a coalition by considering feasible connected subgraphs weighted by link properties; removing a high-weight bridge can collapse both connectivity and performance far more severely than dropping a low-weight peripheral edge **(Algaba, 2021.; Kakoty et al., 2024).** Unweighted models retain analytical merit for exploring high-level structural properties, such as the robustness of connectivity and identification of components, without regard for traffic-specific measures. For example, determining whether a social network remains connected after the removal of nodes may rely solely on the presence or absence of connections, regardless of their intensity or frequency **(Saxena & Iyengar, 2020).** Weighted versions refine that view by signaling not just positional importance but also how well a node can sustain its bridging role under operational loads. Planarity analysis presents another subtle difference: while planarity depends only on topology, incorporating edge weights into layout decisions could prioritize embedding certain high-importance links in more accessible locations for maintenance or optimization. Large-scale networks are subject to computational limitations when operating on heavily weighted graphs, due to the increased complexity of calculating optimal coalitions or shortest-path sets across billions of nodes **(Xu, 2025).**

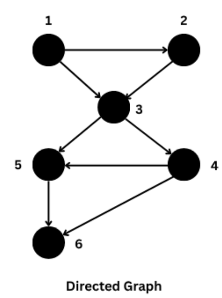
2.2.2 Representation of Graphs

2.2.2.1 Adjacency Matrix

The adjacency matrix is a widely employed representation for capturing the structure of graphs in communication networks. It provides a tabular mapping in which both rows and columns correspond to vertices, and the cell entries describe the presence or characteristics of edges between those vertices **(Pérez-Sechi et al., 2025).** The adjacency matrix, also known as the connection matrix, represents a simple labeled graph using rows and columns that correspond to its vertices. The Adjacency graph A of an unweighted graph is a 2D square matrix of size $|V| \times |V|$, where each element $\{a_{i,j} \mid i, j \in \{1,2..., |V|\}\}$ indicates whether two nodes in the graph are connected. Here, $|V|$ denotes the total number of vertices. In a binary graph (i.e., there is maximally one edge from any vertex i to any vertex j), $a_{i,j} = 1$ if vertex i is connected to vertex j, and $a_{i,j} = 0$ otherwise. In graphs without self-loops, all diagonal entries are 0. For undirected graphs, the matrix is symmetric **(Sauras-Altuzarra et al., 2025).** This symmetric property simplifies specific computational tasks, such as connectivity checks, because analysis can exploit redundancies to reduce processing load **(Gąsior & Drwal, 2013).** For non-binary, non-weighted graphs, $a_{i,j}$ can be defined as the number of edges from vertex i to vertex j. The adjacency matrix of directed and undirected graphs is shown in Figure 2.7.



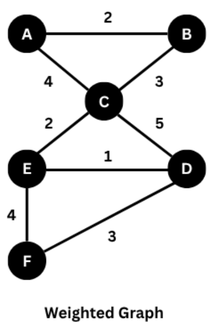
	1	2	3	4	5	6
1	0	1	1	0	0	0
2	1	0	1	0	0	0
3	1	1	0	1	1	0
4	0	0	1	0	1	1
5	0	0	1	1	0	1
6	0	0	0	1	1	0



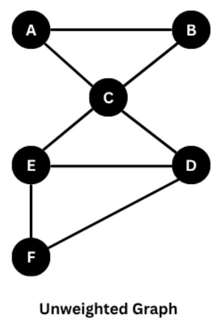
	1	2	3	4	5	6
1	0	1	1	0	0	0
2	0	0	1	0	0	0
3	0	0	0	1	1	0
4	0	0	0	0	1	1
5	0	0	0	0	0	1
6	0	0	0	0	0	0

Figure 2.7 The Adjacency Matrix of Directed and Undirected Graphs

In weighted networks, these entries of the adjacency matrix take numeric values representing metrics such as link capacity, latency, or other relevant performance costs **(Michalak et al., 2013).** In a singly weighted binary graph, $a_{i,j}$ takes the value of the weight of the edge between i and j **(Xu, 2020).** When edges have k weights, each element of the adjacency matrix can be represented by a k-dimensional vector. When there is no edge between two nodes, then the definition of $a_{i,j}$ depends on the interpretation of the weight. When the weight defines the strength of a relationship, and there is no edge between nodes, the weight can be set to zero. When the weight determines the cost of a link, it can be set to ∞ (or sufficiently large to ensure that such a link is not used). Rather than merely indicating adjacency, a weight encodes additional quantitative information that guides optimization algorithms. Weighted adjacency matrices can even handle asymmetric scenarios by allowing $V_{i,j} \neq V_{j,i,}$ when forward and reverse transmissions differ in performance characteristics, a common occurrence in wireless or satellite systems. The adjacency matrices of weighted and unweighted graphs are illustrated in Figure 2.8.



	A	B	C	D	E	F
A	0	2	4	0	0	0
B	2	0	3	0	0	0
C	4	3	0	5	2	0
D	0	0	5	0	1	3
E	0	0	2	1	0	4
F	0	0	0	3	4	0



	A	B	C	D	E	F
A	0	1	1	0	0	0
B	1	0	1	0	0	0
C	1	1	0	1	1	0
D	0	0	1	0	1	1
E	0	0	1	1	0	1
F	0	0	0	1	1	0

Figure 2.8 The Adjacency Matrix of Weighted and Unweighted Graphs

2.2.2.2 Adjacency List

The adjacency list representation of a graph contains n separate lists, one for each vertex v_i $(1 \leq i \leq n)$, where each list identifies the vertices directly connected to v_i and, if necessary, the weights of the corresponding edges **(Skiena, 2003),** as illustrated in Figure 2.9 and Figure 2.10. Such an approach contrasts with matrix-based storage by offering more memory-efficient handling of sparse networks, such as large sensor deployments of specific wireless mesh configurations, as it records only existing connections rather than placeholders for non-existent ones **(Michalak et al., 2013; Arul et al., 2023).** It requires memory space of order $O(|E|)$, making it efficient for representing large sparse graphs where $|E|$ is much smaller than $|V| \times |V|$ **(Xu, 2020).** This structure simplifies operations such as node insertion and deletion. Operationally, adjacency lists align well with algorithms that iterate over a node's neighbors. For example, in pathfinding routines like Dijkstra's algorithm adapted for weighted graphs, retrieving all outgoing edges from a specific vertex requires traversing its list entries directly rather than scanning an entire row in a matrix. Such a method often yields time savings proportional to the sparsity level, since computational effort scales with the actual incidence count rather than the total number of vertices.

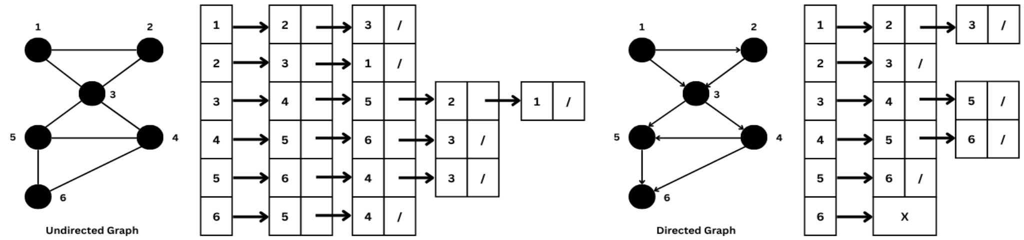


Figure 2.9 The Adjacency List of Directed and Undirected Graphs

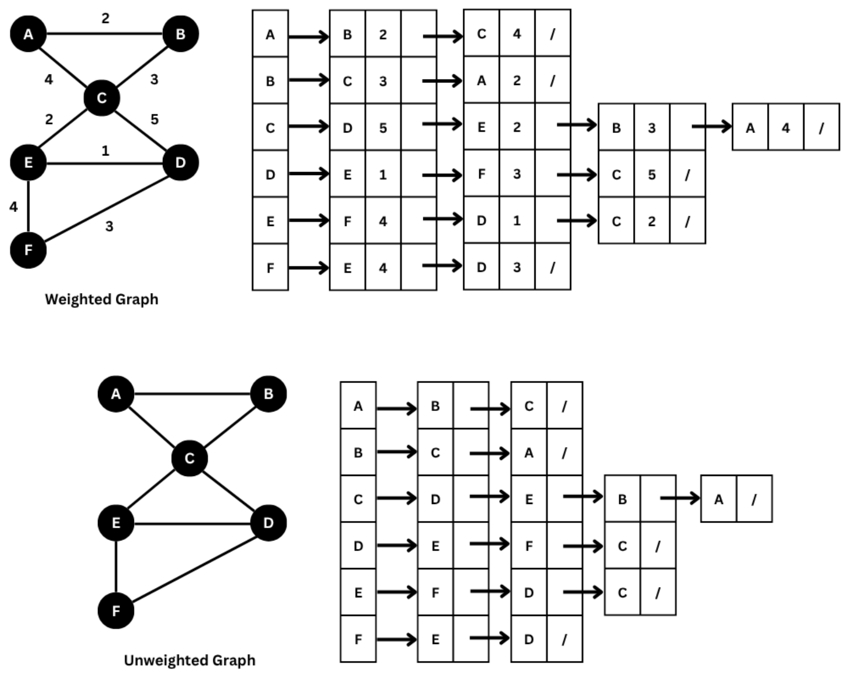


Figure 2.10 The Adjacency List of Weighted and Unweighted Graphs

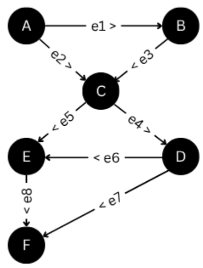
Suppose two agents form a new edge; updating both corresponding adjacency lists instantly informs local routing and connectedness checks. This agility matters when modelling mobile agents in wireless contexts where connectivity changes frequently due to movement or interference effects **(Lin et al., 2018).** When evaluating the utility of a particular node to a coalition, say, by enabling lower-cost routes, the algorithm queries its adjacency list to compile relevant performance metrics for the links it controls. Lists also readily accommodate multi-attribute edges by embedding structured records: Beyond simple weights, entries might contain reliability scores or encryption status flags relevant to determining the value of a coalition. The locality of adjacency lists significantly improves the efficiency of distributed algorithms. By maintaining their own adjacency lists representing direct peer connections and capacities, they can make informed routing or coalition decisions without synchronizing with every other network participant **(Telmanov et al., 2025).** Adjacency lists also exhibit adaptable performance in computing centrality measures, important to both analytical planning and operational governance **(Arul et al., 2023).**

2.2.2.3 Incidence Matrix

An incidence matrix offers another formalism for representing the structure of a graph, focusing on the relationship between vertices and edges rather than solely on vertex-to-vertex adjacency. In its standard form for a simple undirected graph, the incidence matrix B is defined as an $|V|$ x $|E|$ matrix, where $|V|$ denotes the number of vertices and $|E|$ the number of edges. The entry $b_{v,e}$ records whether vertex v is incident to edge e. For unweighted undirected networks, this is typically expressed as one if vertex v is one of the two endpoints of edge e, and zero otherwise **(Skiena, 1990; Arul et al., 2023).** Some researchers instead define the incidence matrix as the transpose of the matrix given above, with columns corresponding to vertices and rows to edges. The physicist Kirchhoff first introduced the concept of the incidence matrix in 1847. The incidence matrix B of a graph and the adjacency matrix A of its line graph are connected through the relation.

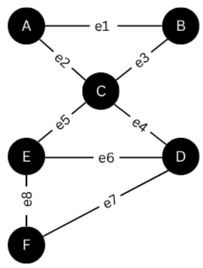
$$A = B^T B - 2I, \tag{2.1}$$

where I denotes the identity matrix **(Skiena, 1990),** this contrasts with adjacency matrices, which describe vertex-to-vertex connectivity directly; here, the focus shifts to which edges connect which vertices, providing a bipartite perspective between the sets. In directed graphs (see Figure 2.11), incidence matrices incorporate orientation by employing signed entries. A standard convention assigns +1 to indicate that vertex v is the head (terminal) of edge e and −1 to denote that it is the tail (origin), with zero if there is no incidence. This directional encoding explicitly captures flow patterns: in communication networks carrying asymmetric traffic, such as upstream/downstream channels, the signs in each column reveal permissible traversal directions **(Pandey, 2025).**



Directed Graph

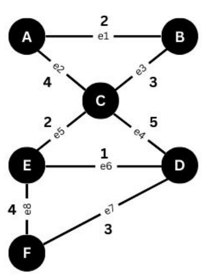
	e1	e2	e3	e4	e5	e6	e7	e8
A	1	1	0	0	0	0	0	0
B	-1	0	1	0	0	0	0	0
C	0	-1	-1	1	1	0	0	0
D	0	0	0	-1	0	1	1	0
E	0	0	0	0	-1	-1	0	1
F	0	0	0	0	0	0	-1	-1



Undirected Graph

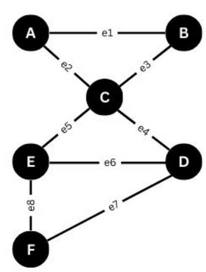
	e1	e2	e3	e4	e5	e6	e7	e8
A	1	1	0	0	0	0	0	0
B	1	0	1	0	0	0	0	0
C	0	1	1	1	1	0	0	0
D	0	0	0	1	0	1	1	0
E	0	0	0	0	1	1	0	1
F	0	0	0	0	0	0	1	1

2.11 The Incidence Matrix of Directed and Undirected Graphs



Weighted Graph

	e1	e2	e3	e4	e5	e6	e7	e8
A	2	4	0	0	0	0	0	0
B	2	0	3	0	0	0	0	0
C	0	4	3	5	2	0	0	0
D	0	0	0	5	0	1	3	0
E	0	0	0	0	2	1	0	4
F	0	0	0	0	0	0	3	4



Unweighted Graph

	A	B	C	D	E	F
A	0	1	1	0	0	0
B	1	0	1	0	0	0
C	1	1	0	1	1	0
D	0	0	1	0	1	1
E	0	0	1	1	0	1
F	0	0	0	1	1	0

Figure 2.12 The Incidence Matrix of Weighted and Unweighted Graphs

For weighted graphs (see Figure 2.12), additional matrices or augmented incidence structures store link-specific metrics, such as capacity or delay, alongside these binary or signed patterns. From an algorithmic standpoint, incidence matrices integrate well with linear algebraic techniques for network flow optimization. In multicommodity flow models relevant to network management, constraints expressing flow conservation at each node are concisely formulated by multiplying the incidence matrix by a vector of edge flows, yielding net inflows or outflows per vertex. This connection enables the direct application of numerical solvers based on sparse matrix methods, since most large-scale communication networks produce highly sparse incidence structures **(Arul et al., 2023).** The sparsity has computational implications: memory scales with $O(|E| + |V|)$ rather than quadratically in $|V|$, making it more tractable for networks where the edge count grows linearly with the node count, as in road-like or tree-like topologies.

2.2.3 Properties of Graphs

2.2.3.1 Degree Distribution

The degree of a vertex in an undirected graph is simply the number of edges incident to it. Degree distribution has a statistical description, often encapsulated by a probability distribution $P(k)$, where $P(k)$ denotes the fraction of nodes in the network with degree k **(Arul et al., 2023).** In contrast, directed graphs partition this into in-degree $d_i(v)$ and out-degree $d_o(v)$, highlighting asymmetric roles such as packet source or sink **(Woźniak, 2025).** This distinction has substantial implications: in-degree distributions can reveal aggregation points where traffic converges, while out-degree distributions can expose dissemination hubs that spread information or control signals. In many large-scale networks, degree distributions deviate from uniformity or random (Poissonian) patterns. For instance, scale-free networks exhibit heavy-tailed distributions in which a relatively small number of nodes have extremely high degree values **(Saxena & Iyengar, 2020).** Such hubs play disproportionate structural and functional roles; their removal or failure can lead to pronounced drops in connectivity or efficiency. Conversely, random geometric graphs or specific sensor network layouts may have more narrowly concentrated degree profiles due to physical constraints on link formation. The underlying spatial, economic, or technological constraints determine whether degree heterogeneity arises naturally or whether designers must artificially create it to achieve the desired resilience characteristics.

Degree distribution also informs robustness analysis via targeted attack simulations: removing nodes with the highest degrees tends to fragment networks faster than removing low-degree nodes **(Pandey, 2025).** Authors exploit this characteristic when comparing the effectiveness of centrality measures. In directed graphs, separate examination of in-degree and out-degree distributions exposes nuanced structural biases. Communication settings in which information predominantly flows outward from a few sources yield skewed out-degree profiles; conversely, aggregation-centric architectures yield skewed in-degree profiles centered on sink nodes. Hybrid bias occurs when some vertices serve dual roles as major broadcasters and aggregators, an arrangement prone to bottlenecks unless cooperative scheduling enforces equitable channel allocation across both incoming and outgoing flows **(Saad, et al., 2009).**

2.2.3.2 Connectivity

Connectivity in graph-theoretic terms refers to the extent to which vertices in a network are reachable from one another via paths composed of available edges. It can be expressed in qualitative terms, e.g., whether the graph forms a single connected component, and quantitatively, through metrics such as vertex connectivity $\kappa(G)$ and edge connectivity $\lambda(G)$, which measure the minimal number of vertices or edges, respectively, whose removal results in a disconnected structure **(Arul et al., 2023).** In undirected communication networks, high connectivity implies redundancy in pathways that prevents node isolation when failures occur. Vertex connectivity evaluates how node removal affects a network's robustness. *If* $\kappa(G) = k$, the removal of at least k vertices is required to disconnect the graph. Edge connectivity is defined analogously. These measures are essential for evaluating resilience, particularly under targeted attack scenarios where adversaries may remove high-centrality nodes or critical links. In directed structures, the concept extends to strong and weak connectivity. Strong connectivity demands that every node be reachable from every other node via an edge traveled in the appropriate direction; weak connectivity treats all edges as undirected when checking reachability. This difference matters for systems with asymmetric capacities or functional roles; such asymmetry may lower effective strong connectivity even when weakly connected pathways remain plentiful. From an operational viewpoint, reductions in strong connectivity result in a loss of direct communication between certain participants, unless intermediate relays re-establish directional paths; such relay formation often emerges as a cooperative phenomenon driven by local incentives. Graph theory enables the formal evaluation of how connectivity shifts under dynamic conditions. For example, time-varying graphs model mobile ad hoc networks in which link availability fluctuates due to mobility and interference **(Lin et al., 2018).**

In these settings, maintaining a desirable level of connectivity requires adaptive topology control: nodes may intentionally add redundant links or adjust transmission power to preserve paths between coalition partners deemed to be valuable under cooperative payoffs. When a model incorporates weighted edges, one can set thresholds on allowable link costs to evaluate connectivity. Two nodes may remain technically connected, yet prohibitively expensive routes can make cooperation infeasible **(Saad et al., 2009; Georgiou et al., 2016; Fan et al., 2019).** Connectivity directly affects the feasibility of a coalition between operators in a network.

2.2.3.3 Clustering Coefficient

The clustering coefficient gives a numerical measure of how strongly nodes in a graph group together. In communication networks, this reflects the presence of tightly connected, localized substructures. Here, neighbors are highly interconnected. There are two notions: the local clustering coefficient, which measures, for an individual node, the links among its neighbors divided by all possible links, and the global clustering coefficient (or transitivity), which summarizes this tendency across the entire network. This formalizes the intuitive “triangle-closure” property: if node A links to both B and C, then B and C are more likely to be directly connected. In operational situations such as social or collaborative systems, a high local clustering coefficient shows redundancy in information channels. Neighbors can still exchange data even if a direct link fails, supporting resilience. Structurally, some domains naturally exhibit high clustering because functional modules rely on mutual connections. In social-type graphs for service coordination, friend-of-a-friend effects create many triads. In biological communication, analogous systems such as protein interaction networks and functional complexes give rise to dense subnetworks. Technological setups, such as autonomous system peering, connect routers fully to minimize latency **(Pandey, 2025).** Communication performance benefits from these clusters due to shorter local traffic paths, but bottlenecks may occur when cross-cluster links are sparse.

Payoff rules that combine centrality and clustering produce more nuanced distributions than those based solely on degree. For an undirected graph, a vertex v with degree k_v has a local clustering coefficient C_v, given by

$$C_v = \frac{2e_v}{k_v(k_v - 1)}, \tag{2.2}$$

Here, e_v counts the existing edges among v's neighbors. For directed graphs, adaptations exist. They either consider ordered pairs separately or collapse directionality, depending on the application **(Arul et al., 2023).** Weighted variants use edge-weight functions (e.g., bandwidth or reliability) to capture the strength of connections. They do not just reflect existence. These weighted coefficients help assess coalition strength as link qualities vary, not just for idealized links. High average clustering changes diffusion dynamics in coalitions. Localized message bursts travel quickly among neighbors, but may take longer to leave clusters if bridging ties are rare. In resource allocation, such as joint spectrum leasing, understanding clustering helps identify where agreements will see rapid internal adoption versus where bridging is necessary. Graph mining tools, such as spectral clustering and community detection, often reveal modules with higher clustering coefficients than expected under null models. These modules frequently coincide with natural coalitions, as their dense links cut transaction costs **(Platje, 2013).**

Game frameworks may treat each community as a super player in higher-level negotiations among communities. This simplifies multilateral bargaining and preserves fair division based on intra-community values. In dynamic networks, tracking changes in the clustering coefficient over time signals shifts in coalition stability. A steep drop in average local clustering could precede group fragmentation due to mobility or failure. Proactive steps may include reinforcing vulnerable links or changing payoffs to keep engagement as the structure erodes. Analysts can compare clustering to expectations from random graphs (configuration models) to learn whether closed triads stem from intrinsic design or random chance **(Zhang et al., 2021).** Exceeding the baseline suggests intentional closed triangles, useful for coalition protocols that rely on reliable indirect reachability. Near-baseline values suggest little benefit from indirect ties, so one should be cautious about assuming coalition stability from degree alone. Cluster structure and resource redundancy are especially relevant in multidomain hybrid networks. Clustering at domain boundaries, such as within wireless access points linked by various backhaul paths, increases redundancy for load-balancing without harming quality of service when particular links fail.

Physical-layer redundancy then mirrors graph-theoretic triangle closure. Cooperative payoff formulas can reflect this by raising characteristic function values for reduced risk. Clustering also affects security: an adversary infiltrating a cluster member may gain quick access to others if defense depends on perimeter measures rather than internal trust **(Han et al., 2019).** Security clauses must weigh the benefits of mutual connectivity in communication against the increased surface area for lateral attacks. Creating sub-coalitions with limited cross-links can balance connectivity and risk under threats. Computing exact coefficients in large graphs is expensive because it requires enumerating neighbor connections. Adjacency lists speed this process by checking only existing incident edges **(Valiente, 2022).** Approximations and sampling methods scale up calculations, enabling real-time evaluation of coalitions. Overall, the clustering coefficient links network structure to the feasibility of cooperation, quantifying how dense neighborhoods impact feasibility, resilience, diffusion, bargaining, and vulnerability. It turns small-scale link arrangements into practical indicators that guide allocation and stability decisions grounded in both theory and real-world needs.

2.3 Centrality Measures in Network Management

2.3.1 Overview of Centrality Measures

Centrality, within the analytical framework of graph-based communication networks, describes the extent to which a node or edge holds a position of structural or functional importance. This importance can manifest in various ways, such as prominence in connectivity, advantage in facilitating flows, or the ability to reach other vertices quickly; each formal measure captures a different aspect of this concept. A key nuance is that, in some cases, these measures can be derived solely from local network information, while others require complete knowledge of the entire topology. Degree centrality and semi-local variants are examples of the former, quantifying influence by counting the number of immediate connections or aggregating limited neighborhood data. Conversely, betweenness, closeness, eigenvector-based metrics, Katz centrality, and PageRank depend on calculations that use the spectral properties of the entire graph. **(Saxena & Iyengar, 2020).** In operational environments where acquiring or maintaining an accurate global state is costly or impractical, such as large-scale wireless ad hoc systems, distinguishing between local and global measures becomes essential. Beyond their definitions, centrality reflects how a network's topology influences performance.

A node with high betweenness acts as a bridge between otherwise sparsely connected regions; removing it could force traffic onto longer routes or fragment the network. However, different formulations reveal subtle differences: shortest-path betweenness counts only routes along minimal geodesics, whereas random-walk betweenness considers stochastic transit patterns through all possible pathways **(Gómez, 2019).**

Nodes that have zero importance under shortest-path analysis might still rank highly under the random-walk approach if they often mediate flows between loosely connected zones. This divergence demonstrates that authors should view centrality as a family of metrics, each defined by its assumed routing behavior, rather than as a single property. The usefulness of centrality extends into resource allocation modeling within cooperative game theory. However, equating high centrality with the right to high payoff without context can be misleading: peripheral nodes handling niche traffic can also deliver significant utility, even if their generic positional metrics seem modest. In network management, degree centrality provides intuitive insight by quantifying raw connectivity:

$$C_D(v) = \deg(v), \tag{2.3}$$

where deg(v) is the number of edges incident to node v **(Meshcheryakova & Shvydun, 2024),** this measure is simple to compute and often correlates with potential influence. Still, it ignores the broader distribution of connections across the network. In such cases, researchers have developed trail-based adaptations emphasizing movement along physical paths rather than ideal geodesics. This highlights that the characteristics of a domain should guide metric selection. Without aligning assumptions with actual dynamics, authors risk producing centrality scores that misstate true influence.

Additionally, analyzing how network changes affect centrality provides practical insight into network stability. Studies show that degree and betweenness rankings can shift with the addition or removal of nodes and edges, driven by evolving data or topology changes **(Dörpinghaus et al., 2025)**. This sensitivity affects the calculation of cooperative payoffs: the rating of a node deemed to be critical based on high betweenness may decline if slight network reconfigurations redirect routing. Such volatility warns against relying solely on static centrality measures for long-term agreements. The method of implementation, such as normalization schemes, also influences interpretability across diverse networks. For example, raw degree values are not directly comparable between networks of different sizes unless scaled by the maximum possible degree **(Smith & Escudero, 2020).** In communication networks, similar metrics could highlight backup routes or dormant gateways that become more important during failures; aligning reward schemes with this conditional influence could encourage maintaining these backups even when idle. Algorithmically, calculating precise rankings for some centrality measures is challenging at large scales; however, fast approximation methods for *k*-path variants correlated with betweenness run much more quickly on large graphs **(Saxena & Iyengar, 2020).** Approximations with acceptable error bounds allow timely use in adaptive approaches where payoffs depend on ongoing network position data. Ultimately, the conceptions of centrality combine topological analysis with functional understanding. They link the description of a structure, the connections that exist, to prescriptive management, how to reward contributors so that incentives promote both direct connectivity and nuanced flow capacities, accommodating diverse network conditions.

2.3.2 Types of Centrality Measures

2.3.2.1 Degree Centrality

Degree centrality captures an intuitively straightforward yet structurally critical aspect of a node's role in a network: the number of direct connections it possesses. In formal terms, for an undirected graph, the degree centrality $C_D(v)$ of a vertex v is given simply by the cardinality of its adjacency set. If the set $N(v)$ contains all the vertices directly connected to v, then

$$C_D(v) = |\, N(v)|, \tag{2.4}$$

This measure counts incident edges without regard to their weight or cost, representing a purely structural view of prominence **(Gómez, 2019).** It suits contexts where raw connectivity yields operational influence, for example, in broadcast situations where more direct neighbors translate into wider immediate reach. In directed graphs, degree centrality splits naturally into two forms: in-degree and out-degree, corresponding respectively to the number of incoming and outgoing edges for each vertex **(Michalak et al., 2013).** These separate counts illuminate asymmetric capacities such as aggregation points versus dissemination hubs. A node with high out-degree but low in-degree often functions as a source or distributor, while the converse may represent a sink or collection point; both roles can be strategically important depending on network objectives. High degree values frequently correlate with potential influence over information flow and coalition feasibility. Regarding models of cooperative games featuring communication graphs, nodes with higher degree centrality can connect more coalition members directly **(Meshcheryakova & Shvydun, 2024).** Nodes with degrees far above average are often termed hubs; they occupy structurally dominant positions within many topologies **(Gómez, 2019).**

Degree centrality remains a local metric that ignores indirect paths and global arrangements. Because it does not capture positional effects beyond immediate neighbors, it can misrepresent importance when mediation between distant nodes defines functional significance. Accordingly, multi-metric assessment helps align cooperative resource allocation with actual performance potential rather than relying solely on the number of connections. From the perspective of large-scale computation, one advantage of degree centrality is its ease of calculation; it requires only counting the lengths of adjacency lists or the nonzero entries in adjacency matrix rows **(Michalak et al., 2013; Meshcheryakova & Shvydun, 2024).** This simplicity allows rapid updates under dynamic conditions: adding or removing an edge affects only two vertices' scores in undirected cases (and one count per vertex per direction in directed instances). This property makes it practical for adaptive frameworks that recalculate incentives based on up-to-date topology snapshots without incurring high computational cost. Weighted extensions introduce nuance by factoring link quality into the measure. Instead of counting edges equally, weighted degree (sometimes termed strength) sums the weights of edges incident to the vertex:

$$\mathrm{C}_{D,w}(\, v\,) = \sum_{u \in N(v)} w_{w,u}, \tag{2.5}$$

where $w_{v,u}$ denotes the assigned weight for edge (v, u), such as capacity or reliability index **(Michalak et al., 2013)**. In cooperative allocations, this can better reflect a node's substantive contribution. Two nodes of equal unweighted degree could differ significantly if one's links are consistently more capable routes for coalition traffic. Comparative studies show that degree centrality satisfies axioms such as anonymity and monotonicity. Degree centrality assigns equal value to identically connected vertices and rises when new connections form. However, this formulation may not satisfy all fairness principles in cooperative game theory. It can violate axioms such as endpoint increase or balanced contribution, depending on the precise formal definitions applied. The endpoint increase axiom ensures that a player's payoff does not decrease when their marginal contribution rises. This maintains consistency between effort and reward. If these conditions are not met, the allocation may produce asymmetric or non-monotonic outcomes. Such violations often occur when the value function is designed to prioritize structural connectivity or flow efficiency within the network, thereby unintentionally distorting fairness among participants **(Meshcheryakova & Shvydun,**

2024). This focus can distort fairness relationships among participants. Awareness of these axiomatic properties is essential when embedding this metric into fair division schemes; reliance on measures that violate fairness principles can misalign participants' incentives.

2.3.2.2 Closeness Centrality

Closeness centrality measures a node's accessibility to all other nodes in the network by considering the shortest paths from it to every other vertex. In its classical form, authors calculate it as the reciprocal of the mean distance between a node and all reachable vertices. Thus, nodes with higher closeness centrality can disseminate information or exert influence more rapidly across the graph because they reside closer to others in terms of path length. This measure inherently depends on a global view of the network topology: computing exact closeness requires determining shortest paths from a given vertex to all the others before averaging their distances. In practice, such calculations can be computationally intensive for large graphs, prompting research into approximations and scalable algorithms. Variants have emerged to handle the complexities of real-world systems where the exact global state may be unavailable or expensive to maintain. One practical approach involves sampling nodes and computing distances only within a sample set; if a node under analysis is part of the sample, its distance to other nodes in the set is directly calculated; otherwise, an estimation algorithm approximates them **(Saxena & Iyengar, 2020).** Such an approach enables linear-time methods to estimate closeness on massive networks. Researchers have linked approximations to GPU-based frameworks that accelerate shortest-path computations when large-scale datasets require parallel processing.

In conceptual terms, closeness centrality captures ease of access: a node with a high score typically lies near the “center” of a network, meaning that the mean distance to other nodes is relatively low. Such positioning translates operationally into the capability of rapidly delivering information, making these nodes advantageous coalition members in cooperative game settings where prompt interaction improves utility. Adjustments are necessary for disconnected graphs because standard closeness becomes undefined for vertices unreachable from certain parts of the network. Modified definitions compute closeness only for vertices within the same connected component as the focal node. They may be normalized by the component's size rather than by the total number of vertices **(Meshcheryakova & Shvydun, 2024).** This refinement avoids penalizing nodes embedded in smaller subgraphs while still measuring their relative reach inside those components. In weighted networks, edge weights reflecting capacities or latencies further dictate effective closeness: weights alter shortest-path calculations so that a physically distant but high-capacity link could yield higher effective closeness than a short but congested route. From the perspective of game-theoretic allocation, marginal contributions related to closeness highlight potential bargaining power. Nodes that can quickly connect others can reduce delays in communication and increase shared payoffs under fairness-oriented distribution schemes **(Michalak et al., 2013).**

However, high closeness does not automatically imply broad structural control; unlike betweenness measures, which capture mediation across many flows, closeness privileges positional proximity and may undervalue specialized nodes that act as unique gateways, even if they lie further from most vertices. The sensitivity of closeness centrality to network modifications is noteworthy. Adding shortcuts, new edges connecting distant vertices, can substantially boost scores for specific nodes by reducing overall path lengths **(Meshcheryakova & Shvydun, 2024).** While degree indicates the number of immediate connections, closeness incorporates indirect links via path-length aggregation; hence, degree hubs often, but not always, exhibit high closeness because their many direct links produce short paths to many nodes **(Saxena & Iyengar, 2020).** Exceptions arise when edges connect primarily to peripheral or sparsely linked nodes: degree remains high, but average distances stay large due to poor connectivity with the central regions. Finally, integrating closeness measures into multi-layer hybrid networks requires careful tier-wise interpretation.

2.3.2.3 Betweenness Centrality

Betweenness centrality assigns a quantitative measure of the extent to which a node lies on the shortest paths between other pairs of vertices, thereby capturing its potential control over information or resource flows throughout the network. The formal definition considers all ordered source–destination vertex pairs (s,d) in the graph, excluding cases where either s or d equals the node under consideration i. For each such pair, one computes $\sigma_{s,d}$, the number of shortest paths from s to d, and $\sigma_{s,d}(i)$, the number of those shortest paths that pass-through i. The betweenness centrality for vertex i is then calculated as

$$\mathrm{C}_B(i) = \frac{1}{(N-1)(N-2)} \sum_{s \neq d \neq i} \frac{\sigma_{sd}(i)}{\sigma_{sd}}, \tag{2.6}$$

where N is the total number of vertices **(Gómez, 2019),** the normalization factor ensures that values are comparable across graphs of differing sizes. In operational terms, nodes attaining high scores under this metric are often intermediaries that bridge otherwise weakly connected regions of the network and can exert disproportionate influence on flow efficiency and coalition connectivity. An essential nuance in interpreting betweenness lies in the definition of a path. In its classical form, betweenness centrality focuses solely on shortest paths; however, variations exist that consider all possible paths, weighted inversely by their length or by probability in a random-walk model. This distinction can cause substantial differences in ranking. Nodes ignored entirely in strict shortest-path computations might emerge as key facilitators in stochastic routing contexts, typical in wireless ad hoc communications, because flows can deviate from globally shortest routes to avoid congestion or to opportunistically use links **(Gómez, 2019).**

Betweenness also has a sensitivity dimension worth acknowledging. Small topological changes, for example, by introducing shortcut links between regions, can sharply reduce the betweenness of specific vertices by creating alternative routes that bypass them. Conversely, removing peripheral redundancies can increase betweenness when traffic must funnel through fewer intermediaries **(Gómez, 2019; Saxena & Iyengar, 2020).** This volatility suggests careful handling when integrating betweenness into long-term contractual resource allocations; otherwise, participants may contest revised rewards if role shifts appear abruptly due to structural changes unrelated to individual effort. From a computational perspective, exact calculation requires enumerating all the shortest paths between all possible vertex pairs, a task that scales poorly with large graphs when implemented naively. Optimized algorithms exploit breadth-first search for unweighted graphs or Dijkstra's algorithm for weighted graphs to identify shortest paths more efficiently, while maintaining data structures that track path counts and intermediary appearances **(Szcześniak et al., 2019).** Despite optimization, very large-scale dynamic systems may favor approximation schemes that estimate betweenness by sampling a subset of vertices as source–destination pairs rather than exhaustively evaluating all possibilities. Applied examples underscore how betweenness links directly to operational leverage overflows. In communication infrastructures modeled by transportation-like graphs, such as air traffic networks or metropolitan fiber backbones, a node (airport or router) with high betweenness effectively controls many indirect transit relations **(Saxena & Iyengar, 2020).** Disruption here has ripple effects well beyond the loss of direct neighbors; geographically separated regions experience increased latency or degraded quality of service since alternative paths must traverse longer routes.

2.3.2.4 Eigenvector Centrality

Eigenvector centrality offers a more nuanced approach to assessing the importance of nodes in communication networks than purely local measures such as degree centrality. Rather than counting connections, it assigns relative scores based on the premise that links to high-scoring nodes contribute more to a node's own score than links to low-scoring ones. This recursive definition captures both direct and indirect paths of influence because a node's status depends on its neighbors' status in proportion to their own. Mathematically, given an adjacency matrix A for a network, the centrality vector c is defined by the eigenvector equation.

$$Ac = \lambda c. \tag{2.6}$$

where λ is the principal eigenvalue and c is its corresponding eigenvector **(Meshcheryakova & Shvydun, 2024).** Under the Perron-Frobenius theorem, the principal eigenvector contains non-negative entries for connected graphs, allowing consistent vertex ranking. This formulation implies that a node's eigenvector centrality is high if it connects to other nodes that themselves have high centrality. In operational terms, in a communication network, authors interpret this as evidence that links to well-connected substructures increase a node's importance even when it has few direct neighbors. The metric identifies actors embedded in influential regions of the topology. Such embedding effects often reveal gateways between major communities or relay stations in a backbone infrastructure. Degree or closeness measures may underestimate the strategic value of such nodes.

In directed graphs, authors must carefully choose an appropriate normalization or apply variants like PageRank, which introduce damping factors for random walks **(Gómez, 2019).** PageRank can be interpreted as a modified eigenvector centrality with stochastic paths added to ensure convergence even on reducible graphs. The implication for cooperative game modeling is that while raw eigenvector values capture potential connectedness, directed link structures may require adaptations to avoid distorted rankings for sinks or disconnected components. In weighted networks, authors incorporate edge weights by using a weighted adjacency matrix, giving stronger links greater influence on the score.

2.3.2.5 Edge Centrality

Edge centrality measures the importance or influence of edges within a network structure. It extends traditional centrality metrics, which typically focus on vertices, to consider the roles that edges play in connecting vertices and facilitating information flow or other dynamics across the network. One prominent formulation of edge centrality is edge betweenness centrality, which quantifies the number of shortest paths that pass through a given edge. This reflects how much the edge helps control communication between pairs of vertices. When an edge lies within many least-cost routes, it often becomes a critical component in maintaining the network's connectivity, reliability, and economic performance. The edge betweenness centrality of an edge *e* in a graph *G*, denoted $B'_G(e)$ (or simply $B'(e)$ when *G* is clear), measures the frequency at which *e* appears on a shortest path between two distinct vertices *i* and *j* **(Newman et al., 2017).** Let $\sigma_{i,j}$ denote how many shortest paths connect two different vertices *i* and *j,* and let $\sigma_{i,j}(e)$ represent how many of those shortest paths go through the edge *e*. Then

$$B'_G(e) = \sum_{i,j} \frac{\sigma_{i,j}(e)}{\sigma_{i,j}}, \tag{2.7}$$

This formulation highlights how certain edges can be strategically important in communication networks, mainly when routing relies on shortest-path selection. Edges with high $B'_G(e)$ values often control large portions of profitable routes or attract traffic demand. Their removal or failure may sharply reduce the network's performance **(Bröhl & Lehnertz, 2019).**

In communication networks managed by multiple service providers, edge centrality aligns with economic outcomes. The value of an edge grows when it lies on routes that serve high demand and provide positive marginal profits. Such centrality approximates the share of revenue a provider deserves when multiple parties cooperate to transmit data **(Ramsey et al., 2024).** Their model weights shortest paths by both demand and profit because traffic flows generate revenue only when the reward from a route exceeds its cost. This idea produces a measure of economic centrality that sums the relative contributions of each shortest path multiplied by the profit it enables. In the uniform version, each edge on a shortest path receives an equal share of the profit. In the proportional version, profits are weighted according to edge costs that define the route. Both formulas treat an edge as valuable only when the coalition that owns it can economically serve the associated demand. This reflects a realistic market condition in which revenues arise only from cost-efficient operations.

Other approaches have extended the concept of edge centrality. Nearest-neighbor edge centrality captures the influence of local neighborhoods, similar to traditional degree metrics that assess how many links connect to influential nodes. Edge closeness centrality ranks edges by how easily other edges can be reached from them, using a structure analogous to vertex closeness. These variants can identify beneficial connections that do not necessarily lie on global shortest paths but still accelerate communication across local subnetworks **(Bröhl & Lehnertz, 2022).** Weighted versions of edge centrality help capture the characteristics of real networks. Edges may differ in bandwidth, reliability, latency, or operational cost. Adapting centrality scores to these parameters helps rank edges more precisely by their contribution to total network performance. Doing so improves network management and supports practical decisions such as maintenance planning, profit sharing, infrastructure investment, and coalition building among service providers. In models of cooperative networks, edge centrality therefore offers more than a structural measure. It predicts bargaining power. A provider holding a key edge can demand a greater share of profits in coalition agreements. In this sense, centrality becomes an indicator of economic value. An edge with high centrality anchors cooperation by guaranteeing access to profitable routes. Low centrality edges may not justify large payoffs in coalition formation, even if they participate in a coalition's routing structure. Understanding these dynamics helps ensure that profit allocation mechanisms remain both efficient and stable across multi-operator communication systems.

2.4 Importance of Shortest Paths in a Network

Shortest (least cost) path computation plays a central role in network optimization because it directly affects latency, resource utilization, and overall throughput. At its core, the shortest path problem involves determining an optimal route between two vertices in a graph, where vertices often correspond to nodes or devices in the network and edges represent available links endowed with weights reflecting metrics such as distance, cost, delay, or reliability **(Conlan et al., 2021).** Selecting paths that minimize cumulative weight enables the system to deliver data more efficiently while avoiding unnecessary congestion on longer or less-capable links. Researchers have developed algorithms such as Dijkstra's, Bellman-Ford, A*, Floyd-Warshall, and Johnson's to address this problem under various constraints **(Iqbal et al., 2018).** Among these, Dijkstra's standard algorithm remains highly favored for road-network-inspired routing models due to its efficiency in computing single-source shortest paths in graphs with non-negative weights **(Khan et al., 2012).** However, when scaling to large networks, whether terrestrial backbones or multi-hop wireless configurations, the original algorithm may require adaptations to reduce computational overheads and account for network-specific constraints, such as dynamic edge weights driven by time-varying congestion. Systems require the rapid determination of a path in scenarios where routing decisions must occur within seconds, such as emergency response coordination in ad hoc deployments or rerouting during core infrastructure failures **(Iqbal et al., 2018).**

From an operational management perspective rooted in cooperative game theory, shortest-path outcomes feed directly into performance metrics for the coalition. The feasibility of cooperation depends on the existence of low-cost routes between members **(Saad, et al., 2009; Ramsey et al., 2024).** Technological advances have broadened shortest-path computation beyond purely static models. Temporal load-aware approaches now integrate predicted congestion patterns derived from historic travel-time heatmaps into routing decisions **(Conlan et al., 2021).** Networks whose traffic profiles vary diurnally or seasonally can benefit from this integration; it yields routes that are not only shortest in a static sense but also least likely to suffer performance degradation at intended transmission times. This adaptability aligns closely with incentives for cooperation: agents adjusting their forwarding behavior to select temporally efficient paths contribute sustained benefits to coalition partners by reducing end-to-end delay unpredictability. Challenges arise when computing exact shortest paths across massive graphs. Even algorithms with polynomial complexity can become bottlenecks when invoked frequently on high-dimensional datasets or during rapid topological changes. Such a need has driven the development of hybrid methodologies that combine algorithms specifically applied to graphs with optimization techniques, minimum spanning trees, and spectral clustering to narrow search spaces before applying pathfinding procedures **(Xu, 2025).** Such pre-processing can partition networks into manageable domains, where local shortest paths are computed independently and stitched together via inter-domain gateway optimization an architecture reminiscent of multidomain cooperative routing.

2.5 Dijkstra's Algorithm

2.5.1 Principles of Dijkstra's Algorithm

Dijkstra's algorithm is one of the most well-known methods for computing the shortest path in a graph with non-negative edge weights, offering deterministic, optimal results through a systematic traversal strategy. Its underlying principle is to progressively extend a set of visited nodes from a designated source, ensuring that at each iteration, the added node represents the shortest currently achievable distance from the source. The algorithm maintains a distance label d_j for each node j, representing the length of the currently shortest path found from the source to j, and a predecessor label p_j that records the previous vertex along that path. Operationally, the process begins by initializing $d_s = 0$ for the source s, assigning $d_i = \infty$ for every other vertex i, and marking all the nodes except s as unvisited. From this starting point, the algorithm compares the distances from s to the nodes directly connected to it. It selects the closest node to s and marks it as visited. The paths to the other neighbors are stored (it suffices to remember the path length and the previous node; here, the starting node). At each subsequent step, the algorithm extends the path from the last node marked as visited to its neighbors. If a new shortest route to a node is found, it replaces the previous one in memory (it suffices to remember only the total length and the previous node visited in the path). The unvisited node with the currently smallest distance from s is then marked as visited. This step is repeated until all the nodes have been marked as visited. This requires n-1 steps, where n is the number of nodes. This selection criterion is a greedy choice: always commit to exploring from the vertex with the minimal known distance from among the previously unvisited vertices. For each neighbor of this chosen node, an update rule is applied: if traversing through this node leads to a shorter route to a neighbor than previously recorded, then

$$d_j = \min(d_j, d_k + l_{k,j}), \tag{2.8}$$

where $l_{k,j}$ denotes the length of the edge between node k (the currently processed vertex) and a neighbor j, then both distance and predecessor records are updated accordingly. This relaxation step ensures accurate propagation from already-optimized parts of the network toward unexplored vertices. An essential aspect of Dijkstra's method is that it is restricted to non-negative lengths of edges. Negative lengths violate the assumptions underlying greedy expansion because they can retroactively shorten paths after the algorithm marks a vertex as visited **(Khan, 2023).** In dense graphs with many edges, efficiency can become problematic; while standard implementations using adjacency matrices have complexity $O(n^2)$ for n vertices, optimizations via priority queues (binary heaps or Fibonacci heaps) reduce complexity to $O(|E| + |V| \log |V|)$ when adjacency lists are employed **(R. Chen, 2022).** This adaptation illustrates how data structure directly influences run-time performance. Dijkstra's operational principle implicitly constructs an optimal path tree rooted at the source **(Khan, 2023).** As the algorithm integrates each new node into the tree and determines its shortest route, it generates routes that collectively span all the reachable vertices with minimal cumulative cost from the root based on the current graph weights.

2.5.2 Application in Communication Network Routing

Applying Dijkstra's algorithm to communication network routing provides a systematic mechanism for determining efficient paths between communicating nodes, leveraging the graph-theoretic abstraction in which vertices denote network elements and edges represent communication links with associated costs. When implemented in routing contexts, the algorithm operates on up-to-date topology and link-state information to compute shortest paths that minimize the aggregate length of edges traversed. Authors can derive these weights from various performance metrics such as latency, hop count, energy use, or congestion levels, depending on the system's operational priorities **(Khan, 2023).** In a typical deployment scenario, network routers or control entities maintain an adjacency structure describing the current connectivity graph. Dijkstra's algorithm is then applied to this structure. Modern communication networks that adapt routes dynamically to changing conditions may execute Dijkstra's algorithm reactively when they detect topology changes, such as link failures or restorations, or proactively at regular intervals. For example, in link-state routing protocols such as OSPF or IS-IS, which disseminate complete link-state advertisements to all routers within a domain, each router maintains the full topology locally. Such coordination enables routers to run synchronous computations that produce consistent choices of path and prevent routing loops **(Magzhan & Jani, 2013).** One advantage under these circumstances is deterministic convergence: given identical link metrics, all routers will construct congruent shortest-path trees. The operational scope extends further when edge weights incorporate dynamic attributes, such as predicted congestion or residual energy reserves, in battery-driven wireless nodes.

2.6 Cooperative Game Theory in Network Management

2.6.1 Overview of Cooperative Game Theory

Cooperative game theory offers a structured mathematical framework for analyzing scenarios in which rational agents form coalitions to improve collective outcomes beyond what they could achieve independently. In the context of communication network management, the "players" are typically nodes, or administrators of sets of nodes or edges capable of both autonomous action and strategic collaboration. The foundation lies in defining a characteristic function $v(S)$ that quantifies the total utility obtainable when a subset S of agents cooperates, subject to connectivity or other operational constraints **(Bekolli & Pagria, 2025).** Coalitions can increase efficiency by pooling resources such as bandwidth, energy reserves, or spectrum allocations, turning potential individual inefficiencies into collective gains. However, strategic considerations about fairness, stability, and incentive compatibility shape this pooling rather than purely technical factors **(Bekolli & Pagria, 2025).** One of the analytical strengths here is that cooperative game theory does not require all players to act selfishly; it explicitly allows distributional negotiations over the surplus generated. In settings where some participants hold disproportionately favorable positions, such as high-degree or high-centrality nodes in a network, the challenge is to ensure an equitable distribution without undermining the viability of a coalition **(Bekolli & Pagria, 2025).**

The Shapley value allocates payoffs proportionally to average marginal contributions across all possible coalition orderings. This method satisfies certain fairness axioms but requires accurate computation of each agent's contribution, considering both direct and indirect connectivity effects. For convex games, which are essential in many resource-sharing models, the Shapley value lies within the core, a set of allocations in which no subgroup has an incentive to defect, thereby indicating both coalition stability and individual fairness **(Wang et al., 2023).** Allocation rules can also adapt to structural restrictions. Suppose specific agents can only cooperate along existing communication links, as captured in graph-theoretic representations. In this case, Myerson's value modifies payoff computations by restricting feasible coalitions to connected subgraphs **(Saad et al., 2009).** This modification prevents overvaluing disconnected groupings that cannot realize cooperative benefits operationally.

Beyond allocation stability, cooperative strategies must address robustness under varying environmental conditions **(Bekolli & Pagria, 2025).** Scenario analysis and simulation offer ways to probe performance when assumptions shift. For example, changes in link reliability or node failure probabilities can affect the overall value of a coalition. These simulations test whether chosen sharing rules continue to align incentives with collective performance targets under stress. Operational complexity remains a practical concern; many theoretical solutions require exhaustive enumeration of permutations of coalitions or connected subgraph structures. In real systems with hundreds or thousands of agents, exact computation becomes infeasible without methods for decomposition or approximation heuristics that preserve key fairness properties while reducing overheads. Applications such as grid construction planning under demand-response regimes employ hybrid methodologies that combine kernel-based allocations with Shapley computations to both quantify marginal contributions and enforce policy constraints **(Wang et al., 2023; T. Chen et al., 2024).** In risk management for distribution networks, game-theoretic modeling enables the optimization of risk-sharing arrangements among stakeholders, accounting for the reliability and failure likelihoods of assets **(T. Chen et al., 2024).** While cooperative game frameworks promote efficiency and coordination in principle, imbalances from uneven bargaining power can introduce inequities if not explicitly countered. In some economic interpretations, coalitional arrangements exacerbate inequalities unless carefully mediated by rules that satisfy justice and proportionality principles, allocating gains according to either contribution magnitude or participant needs **(Bekolli & Pagria, 2025).**

2.6.2 Players and Coalitions

In applying cooperative game theory to communication network management, the notion of players encompasses all decision-making entities capable of contributing to collective utility. These entities can be individual physical nodes, such as routers, base stations, or sensors; they may also be logical agents, such as virtualized network functions or administrative controllers representing entire subdomains. Each player possesses a defined set of strategies and objectives, which, in mathematical terms, are captured by their strategy space S_i and the corresponding payoff function u_i, which maps strategic choices to numerical outcomes reflective of operational benefit **(Singh et al., 2012).** Players differ in their capabilities, positions within the network topology, and available resources. This heterogeneity is central to coalition dynamics because the marginal contribution of a player to the value of a coalition, $v(S)$, depends not only on the available resources but also on how those resources fit within the communication network. Position matters greatly: nodes occupying high-centrality positions can act as bridges between otherwise disconnected components, thereby enabling cooperation among members that lack direct links.

Coalitions are subsets of players formed with the explicit intent of coordinating strategies to maximize joint payoffs rather than acting independently. In practical terms, for communications networks, coalitions entail formal or protocol-enforced agreements about the shared use of resources such as spectrum segments, routing capacity, or data caching and storage. The ability to form binding contracts stabilizes cooperative arrangements: when participants commit contractually or via algorithmic enforcement to perform agreed-upon actions (forward specific traffic flows, maintain certain links), coalition stability increases because deviation becomes less attractive relative to sustained cooperation **(Chessa & Loiseau, 2017).** Models of cooperative games with transferable utility interpret sum-rate, throughput gains, and other aggregated metrics as divisible benefits that authors can allocate freely among members **(Y. Chen et al., 2018).** Transferable utility simplifies analysis by allowing redistribution in monetary terms or equivalent credits, even if operational contributions differ; non-transferable setups must instead partition indivisible benefits directly. Coalition formation in networks is rarely static. Dynamic algorithms based on merge-and-split rules have shown efficacy when initial configurations start as noncooperative sets and evolve through iterative decisions that compare potential group performance with current arrangements **(Saad et al., 2009).** Neighboring players or coalitions cooperate (merge) if doing so improves total utility according to predefined orderings, such as utilitarian preferences; players act independently when the decomposition of coalitions yields higher aggregate payoffs for subsets of players than maintaining larger aggregates. Such dynamics reflect real-world behavior in wireless D2D environments, where local gains from forming small groups may at times outweigh broader, but less immediate, benefits from remaining in larger coalitions **(Y. Chen et al., 2018).** Concepts of stability here include Nash-stable coalition structures, in which no player can unilaterally improve outcomes by moving to another coalition, given the existing allocations, a property desirable for minimizing instability in cooperative arrangements. Hierarchies can naturally emerge within coalitions, especially in multidomain contexts with layered control structures.

A central coordinator may oversee global strategy, while subsets handle domain-specific issues autonomously; these nested coalitions resemble hybrid network control and can be viewed through a game-theoretic lens. Embedding smaller coalitions within larger strategic alliances enables scaling cooperation without overwhelming any single coordinating entity. Allocation within this hierarchy often blends calculations of payoffs made at local levels to ensure fairness in micro-coalitions, with simplified aggregation schemes at the global level to reduce computational overheads. Incentives are critical for sustaining coalition membership over time. Free-riding behaviors, in which some members reap collective benefits without proportional contribution, can erode effectiveness unless prevented through allocation mechanisms that condition payoffs on measurable inputs, such as packet forwarding volume or energy commitment during joint transmission phases **(Li et al., 2011; Juma et al., 2019; V. P. Singh et al., 2023).** Network topology strongly influences the patterns of feasible coalitions.

2.6.3 Payoff Distribution

The distribution of payoffs describes how a coalition allocates its total generated utility (profit) among its members. A coalition may deliver increased utility from throughput gains, cost savings, spectrum access rights, or other resource advantages achievable through collective action. The underpinning mathematical formulation considers a transferable utility (TU) game (N, v) where N denotes the set of players and $v(S)$ the value of coalition S. The function v is called the characteristic function of the game. The definition of this function for the games considered here will be presented in the following chapter. The payoff distribution is a vector $\phi = (\phi_1, \ldots, \phi_{|N|})$ such that $\sum_{i \in N} \phi_i = v(N)$ in TU settings **(Kakoty et al., 2024),** i.e., all of the utility achievable from coalition formation is distributed between the players. In addition to this efficiency condition, a fair division among the players should ensure that no set of players wishes to leave the coalition of all players (the grand coalition).

When links have weights representing quality-of-service attributes or costs, these weights directly affect $v(S)$ and, hence, each member's value to a coalition. Players providing high-capacity or low-latency edges to a coalition generally receive larger allocations. Here, distribution rules can also compute appropriate distributions of cost within coalitions based on metrics analogous to gain allocation, resulting in net payoffs that balance collected benefits against assumed costs. This symmetry in the gain–loss treatment reflects realistic operational conditions in which participation imposes simultaneous resource commitments and returns. In contexts such as wireless network service provisioning via multiple operators as cooperative players, centralized coordination may distribute profits based on contributions measured by flow-optimization outcomes **(Aram et al., 2009; Lu et al., 2014; Ramsey et al., 2025).**

Suppose one provider facilitates critical traffic paths, enabling others to meet demand targets. In that case, the provider's share increases not only from raw capacity but also from allowing feasible traffic distribution across the coalition. Conversely, nodes acting as secondary relays might earn smaller amounts unless their roles become pivotal under altered load distributions, for instance, during congestion mitigation events. **(Saad et al., 2011; Lu et al., 2014).** Graph-theoretic elements feed directly into payoffs: centrality metrics such as betweenness and degree can serve as proxies for potential influence in a coalition when detailed throughput data is unavailable.

In finite-attenuation network games derived from bipartite structures with worker teams **(Allouch et al., 2024),** the distribution of productivity follows coalition-stability principles; payoff allocations converge, depending on attenuation factors that determine how individual effort translates into retained output at the group level. Such models show applicability beyond conceptual fairness; they can produce predictable shares that respond to structural modifications, such as adding redundancy links or adjusting the strength of connections between clusters. Several allocation methods have been proposed for shortest path games. Cost allocation among players controlling network edges was studied using cooperative game-theoretic approaches, while the nucleolus was later investigated as an alternative solution concept for allocating the value generated by cooperation in shortest-path networks **(Voorneveld & Grahn, 2002; Baïou & Barahona, 2019).**

2.6.4 Superadditivity

Superadditivity describes how cooperation increases total benefits in a communication network. When independent network operators or service providers share resources, they often achieve higher throughput, cover more users, reduce transmission delay, and improve reliability **(Alahmed & Tong, 2023).** Cooperative game theory uses superadditivity to justify the formation of a grand coalition. Cooperation yields more total value than each player acting alone. Superadditivity states that the value achieved by a group of cooperating players, called a coalition, is at least as large as the sum of the values of those same players working in separate groups. Let A and B be two disjoint sets of players (i.e., they have no members in common). The characteristic function, labeled $v(\cdot)$, assigns a payoff or value to each possible coalition. A cooperative game is superadditive if:

$$v\,(A\,\cup B) \geq v(A) + v(B), \tag{2.9}$$

meaning that the value created when A and B merge is no less than the sum of their separate values. This inequality captures the benefit of aggregating network resources. When two network operators combine their routing capacity or spectrum, they generate more usable bandwidth or reduce costs. The system benefits from shorter paths, balanced loads, and expanded coverage. Players in a communication network have shared goals. These include packet delivery, maintaining connectivity, and ensuring service quality. These goals benefit from cooperation. Superadditivity reflects this structure. **(Alahmed & Tong, 2023).**

Network management often faces fragmentation. Each independent operator controls only a part of the infrastructure. Without coordination, traffic may take longer routes. Some links may become congested while spare capacity elsewhere remains idle. Cooperation solves this inefficiency. Joint routing uses the best paths. Shared load balancing prevents bottlenecks. The combined network becomes more efficient. This creates a positive gain for the entire coalition. Superadditivity also gives players strong incentives to form the grand coalition when feasible. A player who joins a larger group increases total capacity. The coalition can then allocate extra value using a fair rule, such as the Shapley value or the S-value. These allocation mechanisms reward players who contribute key links or nodes. This prevents free riding. A player earns value only if their managed edges or services create measurable improvements in routing or performance.

Nevertheless, in practice, superadditivity may not hold automatically. Cooperation can require coordination costs, communication overheads, or incompatible technology. If the cost of integrating the infrastructure of two sets of players is too high, then:

$$v\,(A\ \cup B) < v(A) + v(B), \tag{2.10}$$

This violates superadditivity. Cooperative strategies must keep integration costs low. Using shared protocols, standard interfaces, or automated network configuration helps preserve superadditivity. When cooperation is cost-effective and straightforward, the game structure favors unified coalitions. In real communication systems, larger coalitions often gain access to more efficient routing. The marginal benefit of adding new players grows as the network expands. A link controlled by a small Internet provider may have little value on its own. However, once connected to major backbone providers, the same link can provide significant routing shortcuts. This shows how superadditivity emerges from network effects. **(Saad et al., 2009; Lu et al., 2014a).**

Superadditivity underpins the stability of a coalition. If no coalition of players can attain a total payoff exceeding their ascribed payoff under the grand coalition, no group prefers to leave the grand coalition. The grand coalition remains optimal. Network managers can use this property to plan cooperative agreements and joint operations. They can model different coalition structures and check when merging improves performance. If the model shows strict superadditivity for many coalition pairs, cooperation should be enforced as a management strategy. Superadditivity supports network policies that encourage the use of shared infrastructure. It provides a clear mathematical argument. Cooperation increases total economic and operational value. Fair allocation rules can then distribute this value so every player benefits **(Alahmed & Tong, 2023).**

2.6.5 Core of the Game

The core is one of the most important stability concepts in cooperative game theory. An allocation belongs to the core if no coalition can obtain a higher payoff by leaving the grand coalition and acting independently. Relatedly, the core and bargaining set have been studied extensively in shortest path games, and conditions guaranteeing stable cooperative outcomes have been established for several classes of network games **(Grahn, 2001; Baïou & Barahona, 2019).**

The core represents a set of payoff allocations in which no subset of players (a coalition) can achieve greater utility by defecting from the grand coalition and forming their own agreement. Formally, for a transferable utility game (N, v), an allocation vector ϕ lies in the core if it satisfies two conditions: efficiency $\sum_{i \in N} \phi_i = v(N)$, and coalitional rationality, $\sum_{i \in S} \phi_i \geq v(S)$ for all $S \subseteq N$. Efficiency ensures that agents thoroughly distribute the total worth generated through cooperation without creating any surplus or deficit. Coalitional rationality guarantees that no smaller coalition finds its allocated share inferior to the share that it could generate independently based on the game's characteristic function. In the context of communication networks, these properties embody stability. If an allocation is in the core, no subset of players has an incentive to reorganize outside of the agreed cooperative framework **(Madiman, 2008; Gemp et al., 2024).** The graph's topology naturally has a strong influence on the form of $v(S)$. This will be considered in the following chapters.

Operational challenges arise when translating this theoretical construct into live network management systems. Computing the core, especially in large-scale networks with numerous feasible coalitions shaped by topological constraints, becomes computationally intensive. The size of candidate allocation sets can explode combinatorially with $|N|$, forcing reliance on approximation schemes or structural simplifications. Methods leveraging linear programming formulations of the constraints defining the core can be practical for moderate N **(Gemp et al., 2024).**

2.6.6 Balanced Game and Relation to The Core

Balancedness plays an important role in cooperative game theory because every balanced game possesses a non-empty core. Shortest path games are not necessarily balanced, and conditions ensuring balancedness have been derived for specific classes of such games. Other shortest path formulations have been shown to be balanced, thereby guaranteeing the existence of stable allocations within the core **(Fragnelli et al., 2000; Grahn, 2001; Voorneveld & Grahn, 2002).**

A balanced game ensures stability in cooperative game theory. A game is balanced when every feasible group of players can only create value that is limited by the value of the grand coalition. A balanced collection assigns non-negative weights to coalitions. Each player must contribute exactly one unit of effort across all the coalitions they join **(Madiman, 2008; Alahmed & Tong, 2023).** Formally, a set of weights λ_S is balanced if:

$$\sum_{S\subseteq N: i\in S} \lambda_s = 1, \forall\, i \in N, \tag{2.11}$$

A game is balanced when for any such set of weights:

$$\sum_{S\subseteq N} \lambda_s v(S) \leq v(N), \tag{2.12}$$

This condition states that the total value created by combining coalitions over time cannot exceed the value of the grand coalition. A balanced game prevents small groups from generating an advantage that undermines the global agreement. The core contains all stable allocations. An allocation is in the core when two conditions hold. First, efficiency requires that the entire value of the grand coalition is distributed:

$$\sum_{i \in N} \phi_i = v(N), \tag{2.13}$$

Second, coalitional rationality requires that no coalition can obtain more on its own:

$$\sum_{i \in S} \phi_i \geq v(S) \quad \forall S \subseteq N, \tag{2.14}$$

If a payoff vector satisfies both rules, no group benefits by leaving the agreement. This makes network operations allocation stable. Balancedness and the core are strongly linked. A cooperative game has a non-empty core if and only if it is balanced. This is known as the Bondareva Shapley theorem. It guarantees that if cooperation is efficient and internal group incentives are controlled, then at least one stable payoff distribution exists. Communication network managers can use this condition to check whether cooperative routing or capacity sharing will be stable when deployed in real systems. In network management, balance ensures that resource-sharing agreements do not create profitable breakaway groups. The core is a set of outcomes in which all players remain committed to cooperative policies, and no subset has an incentive to defect **(Madiman, 2008).**

2.6.7 An Example of an Unbalanced Game

We consider a cooperative game arising from a simple communication network among three players, with the set of three players denoted $N = \{1, 2, 3\}$. The network supports one unit of traffic that must be routed from node A to node B, generating a revenue of three. Each edge in the route incurs a cost of 1. No single player controls enough infrastructure to create a complete A–B path, so any singleton coalition earns zero profit. Any pair of players can jointly route traffic over exactly two edges, yielding a total cost of 2 and a profit of 1. The grand coalition can also earn at most one because the system supports only a single unit of demand. This setup shows that each coalition's productive capacity depends on structural feasibility rather than geometric layout. The key issues are whether a coalition has enough combined edges to form a valid path and how routing costs affect the profit it can achieve, see Figure 2.13

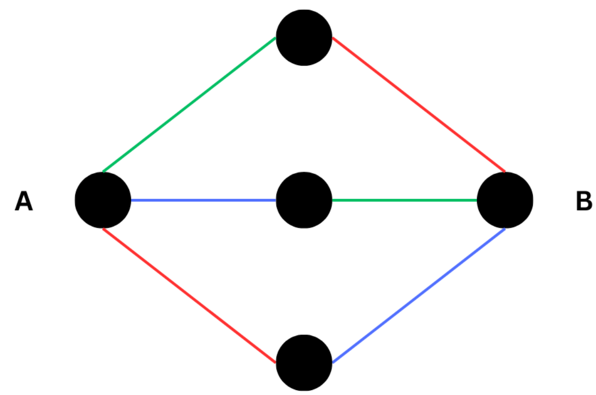


Figure 2.13. An Example Showing that Superadditive Games do not always have a Core

Let $v(S)$ denote the profit earned by coalition S in the network. Given the system's structure, the empty coalition yields no value; thus, $v(\emptyset) = 0$. Each singleton coalition also yields zero, since no individual player can construct a complete A–B path, implying $v(\{1\}) = v(\{2\}) = v(\{3\}) = 0$. Any pair of players can combine their edges to form a feasible route that uses two edges, yielding revenue of 3 and total cost of 2, so every two-player coalition earns a profit of 1. The grand coalition earns the same profit because the network supports only a single unit of demand, so $v(\{1, 2, 3\}) = 1$. The characteristic function, therefore, takes a simple form: coalitions of size zero or one earn zero profit, while coalitions of size two or three earn one. This creates a classic three-player majority-type game derived from the underlying routing constraints of the communication network.

Superadditivity requires that, for any two disjoint coalitions S and T, the value created by their merger must be at least as large as the sum of their individual values; see Formula 2.8. In this three-player network, given the symmetry between the players, the only nontrivial cases occur when S and T are both singletons, or when one coalition is a singleton and the other contains the other two players. For example, if $S = \{1\}$ and $T = \{2\}$, then each coalition earns zero on its own, but together they form a feasible A–B path that yields a profit of one, so $v(\{1, 2\}) = 1 \geq 0 + 0$. When $S = \{1\}$ and $T = \{2,3\}$, then $v(\{1\}) = 0$, $v(\{2,3\}) = 1$, and $v(\{1,2,3\}) = 1 = v(\{1\}) + v(\{2,3\})$. Cases involving the empty coalition are straightforward because $v(\emptyset) = 0$ and $v(S \cup \emptyset) = v(S)$. No other disjoint coalitions exist in a three-player set, so these checks are complete. The result confirms that the game is superadditive, meaning that cooperation never reduces total value and merging coalitions always weakly increases the achievable profit.

The core is the set of efficient and stable allocations in a transferable utility game. An allocation lies in the core when it distributes the entire value of the grand coalition without loss and ensures that every coalition receives at least what it can generate on its own. In this network game, efficiency requires that $x_1 + x_2 + x_3 = 1$, since the grand coalition earns a profit of 1. Coalitional rationality adds the conditions $x_1 \geq 0$, $x_2 \geq 0$, $x_3 \geq 0$, and $x_1 + x_2 \geq 1$, $x_1 + x_3 \geq 1$, $x_2 + x_3 \geq 1$, since each two-player coalition can earn one. Combining the three pairwise constraints yields $2(x_1 + x_2 + x_3) \geq 3$, which contradicts the efficiency requirement that the sum of payoffs equals one. No allocation can satisfy all these inequalities simultaneously. This proves that the game's core is empty. The result shows that, even though the game is superadditive, the incentives of the two-player coalitions are too strong relative to those of the grand coalition, and superadditivity alone cannot guarantee the existence of a stable allocation in communication network settings.

In general, whether a cooperative game is balanced can be investigated by solving a linear programming problem in which the problem is to maximize an objective function whose coefficients correspond to the payoffs of a proper subset of the players, and the decision variables are the weights ascribed to those coalitions. (**Atay & Núñez, 2019; Zou, 2023).** When there are *K* players, the *K* constraints state that the sum of the weights ascribed to the coalitions in which a given player is a member cannot exceed 1. If the optimal solution to such a problem does not exceed the value of the grand coalition, then the game is balanced.

Balancedness provides a structural condition that determines whether the core of a cooperative game is nonempty. The Bondareva-Shapley theorem states that a transferable utility game has a nonempty core if and only if it is balanced. In this three-player network game, we can construct a balanced collection by assigning weights of 1/2 to the three two-player coalitions and 0 to all others. Each player appears in exactly two of these coalitions, so the sum of weights for each player equals one, which satisfies the balancedness requirement. However, the weighted sum of coalition values is three halves, which exceeds the value of the grand coalition. This violates the balancedness condition, indicating that the game is not balanced. By the Bondareva-Shapley theorem, this violation establishes that the game's core is empty. When there are multiple allocations within the core or the core does not exist, a number of methods have been proposed for proposing fair allocations. The Shapley value, the Myerson value, and the S-value are three of these propositions.

2.7 Shapley Value

2.7.1 Definition and Mathematical Formulation

The Shapley value is a formal solution concept for cooperative games, assigning to each player a uniquely determined payoff that reflects their average marginal contribution to all of the possible coalitions that they could join **(Shapley, 1953).** The formulation is based on the characteristic function $v(S)$, which denotes the utility achievable by coalition $S \subseteq N$, where N represents the set of all players. For a given player $i \in N$, the Shapley value $\phi_i(v)$ is defined as

$$\phi_i\,(v) = \sum_{S \subseteq N \setminus \{i\}} \frac{|S|!\ (n - |S| - 1)!}{n!}\,[v(S \cup \{i\}) - v(S)].$$

(2.15)

In this expression, $n = |N|$ is the total number of players, and the combinatorial term $\frac{|S|!\,(n-|S|-1)!}{n!}$ serves as a weighting factor representing the probability that coalition S precedes player i in a random ordering of all the players **(Madani et al., 2022).** The difference term $v(S \cup \{i\}) - v(S)$ quantifies player i's marginal contribution: how much additional worth they bring when joining coalition S. Aggregating over all possible coalitions produces an allocation satisfying several axioms central to cooperative game theory. An important property highlighted by Shapley's original work is uniqueness: within TU games, $\phi(v)$ is the only payoff distribution rule that satisfies the efficiency, symmetry, dummy player, and additivity axioms simultaneously **(Saad, et al., 2009; Amigo, 2013).** Efficiency implies that $\sum_{i \in N} \phi_i = v(N)$, meaning that the grand coalition distributes its total value among participants, with no surplus or deficit. Symmetry means that if two players contribute equally to every coalition, they must receive identical shares; the dummy player condition states that any participant adding no extra value to any coalition obtains a zero payoff; the additivity condition implies linearity across games, if the characteristic function of two games with the same set of players is summed, i.e., $v = v^{(1)} + v^{(2)}$, then the Shapley values for the game based on v equal the sum of values computed for each scenario separately.

2.7.2 Interpretation in Network Context

The practical meaning of the Shapley value within communication networks emerges from how the specific operational and structural environment shapes marginal contributions. In a network graph populated by players representing routers, base stations, or administrative controllers, each participant's connection pattern and link quality determine the degree to which they can enhance the performance of a coalition. When applying the formulation in 2.7.1, $v(S)$ reflects not just an abstract utility but a tangible metric, such as aggregate throughput, minimized latency, expanded coverage, or reduced energy cost or profit, that the set S can achieve given current link states and graph topology **(Michalak et al., 2013).** The combinatorial weighting in Equation (2.15) integrates all possible orders of coalition formation. In practice, this assumption may not reflect the spatial relation between the players in a network. In many realistic scenarios, network topology imposes natural orders in which coalitions are formed.

The weighting of edges in a network plays an important role. In capacity-sensitive environments where edge weights encode available bandwidth or reliability metrics, $v(S)$ assigns greater value to coalitions whose internal links meet high-performance thresholds **(Michalak et al., 2013; Gautam et al., 2022).** Marginal contributions under the Shapley formula thus yield allocations that reward nodes that furnish superior links, even if their positional centrality is otherwise modest. A low-degree node attached via an ultra-reliable link to the broader coalition may yield higher payoffs than hubs connected through unreliable channels, because its presence meaningfully elevates the coalition's overall utility. Operational constraints also influence how these theoretical payoffs map onto incentive schemes. In networks subject to physical cost structures, energy consumption per transmission and spectrum leasing fees, $v(S)$ naturally incorporates these expenditures alongside gains, so that net marginal contributions measure surplus rather than gross benefit **(Kakoty et al., 2024).** When authors adopt this net perspective, Shapley allocations discourage free riding, giving participants whose engagement yields negative net benefits low or zero payoffs. Such reasoning matches intuitive strategic design in communication systems, where stable cooperation depends on rewarding actions that genuinely improve net efficiency. From an engineering standpoint, interpreting Shapley outputs means translating them into an actionable allotment of resource or policy privileges. In centrally managed domains with comprehensive topology awareness, payoff vectors derived from live network states can serve as input for bandwidth-sharing assignments or routing path priorities **(Ramsey et al., 2024).** Players with higher Shapley shares might gain greater control over scheduling decisions or receive advantageous time-division-multiplexing slot positions to reflect their cooperative importance.

2.8 Myerson Value

The Myerson value extends the Shapley value to cooperative games in which an underlying relational graph constrains the set of fruitful coalitions. Synergistic benefits of cooperation occur only when coalition members are connected via a relational graph. Hence, the value of the union of two disjoint coalitions whose members are not connected in this relational graph is defined to be the sum of the values of these coalitions **(Saad et al., 2009)**. Formally, given a transferable utility game (N, v) and a relational structure L represented as a graph over N, one constructs a restricted game v^L in which for any coalition $S \subseteq N$, $v^L(S)$ equals the sum of $v(T)$ over connected components T within S as defined by L **(Algaba et al., 2021).** The Myerson value $\mu(v, L)$ is then obtained by computing the Shapley value of this restricted game:

$$\mu(v, L) = Sh(v_L), \tag{2.16}$$

This formulation ensures that marginal contributions reflect both resource endowments and the structural role in enabling connectivity under the given network topology. A player's worth increases not only when they bring high intrinsic utility but also when their presence links disconnected subcomponents of the relational graph into productive coalitions. Operationalizing this definition requires careful mapping between network topology and the characteristic function. In practice, $v(S)$ depends on the relative positions occupied by the players in a real network **(Algaba et al., 2021; Ramsey et al., 2024).** It is assumed that when two coalitions combine, then synergetic benefits can only occur when the coalitions are connected on the basis of a graph describing the relationship between the players. In other words, when two unconnected coalitions, according to the relationship graph, join forces, the value of the resulting coalition is simply the sum of the values of the two constituent coalitions. The formation of other coalitions can bring additional benefits. Consequently, $\mu(v, L)$ tends to assign higher payoffs to coalitions whose members occupy neighboring positions in the relational network. Extensions modify the values of coalition further by incorporating edge-specific attributes, such as processing capability or priority class, into calculations of marginal contribution **(Kakoty et al., 2024).** The weighted Myerson value adjusts the equal treatment of equivalent positions in favor of proportionality to such predefined weights. Decomposing Myerson's approach for weighted networks reveals an interplay between a graph's topology and payoff profiles. For example, given a network where each node has a weight signifying operational capacity or contractual priority, applying weighted variants of the Myerson value ensures the allocation respects both connectivity and heterogeneous levels of influence **(Kakoty et al., 2024).** This method may dramatically alter allocations compared with uniform weighting; two structurally similar nodes can receive different shares if one maintains higher-quality pathways essential to sustaining connected sub-coalitions. A further refinement arises in proportional loss-sharing schemes when cooperative efforts involve not only gains but also shared liabilities, such as energy costs or maintenance downtime **(Alkhaled & Petrosyan, 2022).**

2.9 S-Values

2.9.1 Theory of the S-Value

The S-value measures each player's contribution in a cooperative game when not all coalitions are possible. While traditional analysis assumes every subset of players can form a coalition, this view is unrealistic in many real systems **(Hellman & Peretz, 2018).** For example, consider communication networks and service providers, which are constrained by infrastructure. Here, coalitions must adhere to structural and feasibility rules, meaning some coalitions cannot form. These infeasible coalitions, therefore, should not influence the allocation of value **(Hellman & Peretz, 2018).**

The S-value extends the Shapley value to games where players form only allowed coalitions. A coalition of players is admissible if the system's structure will enable it to form, such as through a connected relational graph or a set of possible contracts. As a result, the characteristic function is defined only on admissible coalitions, not all subsets. The S-value keeps the four classical axioms of Shapley's rule: symmetry, additivity, null player, and efficiency. The difference is the allocation space. Instead of averaging a player's marginal contribution over all orderings, the S-value looks only at allowed sequences of coalition formation. The system's rules limit these sequences. For example, a player can join a coalition only if it is a neighbor of an existing member in a graph describing the relationships between the players. A chain is used to formalize this idea. It represents a step-by-step sequence of allowed coalitions from the empty coalition to the grand coalition. A player's value comes from the sum of the marginal contributions along these chains, averaged over all the possible chains **(Hellman & Peretz, 2018).**

2.9.2 Formula for the S-Value

Let (N, C) be an admissibility structure, where N is the set of players and $C \subseteq 2^N$ is the set of allowed coalitions. For a valid chain $c = (S_0, S_1, ..., S_k)$, with $S_0 = \emptyset$, $S_k = N$, and each subsequent coalition in a chain is obtained by either adding or removing one member, the marginal value at step l is:

$$v(S_l) - v(S_{l-1}), \tag{2.17}$$

The contribution assigned to player I along the chain is **(Hellman & Peretz, 2018):**

$$\psi_i^c(v) = \sum_{l=1}^{k} 1(i \in (S_l \setminus S_{l-1}) \cup (S_{l-1} \setminus S_l))\,[v(S_l) - v(S_{l-1})], \tag{2.18}$$

where 1 (D) is the value of the indicator function for condition D, 1(D) is equal to 1 when the condition D is satisfied, otherwise it is equal to zero. The S-value comes from averaging over all permissible chains (automorphisms):

$$\phi = \sum_{\pi \in \mathrm{Aut}(N,A)} p(\pi)\, \psi^{\pi}(v), \tag{2.19}$$

where $p(\pi)$ is the probability of a chain, and $\psi^{\pi}(v)$ is the vector of the corresponding contributions of the players. This approach preserves the properties: efficiency, symmetry, and feasibility. The result gives each player a value based on coalitions that can actually form. In networks, this means players connected to key links or resources may have more bargaining power, even if their individual worth is the same as others.

It should be noted that in the above definition, the chain of coalitions from the null to the grand coalition can include individual players leaving a coalition. This thesis considers values in which an initial player is chosen at random, and at each successive stage, a player who is a neighbor of a current member of the coalition in the relationship structure is chosen at random.

2.10 Properties of These Values (Symmetry, Efficiency, Null Player)

The three fundamental properties that these values satisfy are symmetry, efficiency, and the null player condition **(Hellman & Peretz, 2018).**

2.10.1 Symmetry

Symmetry means that if two players always produce the same marginal contribution when joining every admissible coalition, then they receive the same allocation. For all feasible coalitions S not containing players i and j:

$$v(S \cup \{i\}) = v(S \cup \{j\}) \Rightarrow \phi_i(v) = \phi_j(v), \tag{2.20}$$

This ensures equal treatment of players who are indistinguishable according to the system's structure **(Hellman & Peretz, 2018).**

2.10.2 Efficiency

Efficiency requires that the total allocated value equals the worth of the grand coalition, which is the set of all players. A value allocation must distribute the entire payoff of the grand coalition across players:

$$\sum_{i \in N} \phi_i(v) = v(N), \tag{2.21}$$

Even when only feasible coalitions are used in computation, the full value of the grand coalition is allocated with no loss or excess **(Hellman & Peretz, 2018).**

2.10.3 Null Player

A null player contributes no value to any admissible coalition. If adding player i to any feasible coalition does not change the value of the coalition, then:

$$v(S \cup \{i\}) = v(S), \forall S \in A. \tag{2.22}$$

The values defined above assign:

$$\phi_i(v) = 0. \tag{2.23}$$

This prevents allocating value to players who make no real contribution to the system **(Hellman & Peretz, 2018).**

2.11 A Simple Example Illustrating the Differences Between the Values

This section shows how different allocation rules yield distinct outcomes in a simple cooperative game with three players. The characteristic function assigns 10 to any single player, 30 to any pair, and 48 to the grand coalition. Consider the player set $N = \{1, 2, 3\}$.

Figure 2.14. A Simple Example of a Relationship Graph.

The characteristic function satisfies $v(\phi) = 0$, $v(\{i\}) = 10$, $v(\{i, j\}) = 30$, and $v(N) = 48$ $i, j \in N, i \neq j$. This function is superadditive because each merged coalition produces a value at least as large as the sum of the smaller coalitions. For example, $v(\{i\}) + v(\{j\}) = 20$ while $v(\{i, j\}) = 30$. The analogous conditions hold when a single player joins a coalition of the other two players. For each permutation σ of the players, we can compute the marginal contribution of player i as

$$marg(i, \sigma) = v(S \cup \{i\}) - v(S), \tag{2.24}$$

where S is the set of players before i in σ. The Shapley value is computed first. This rule averages each player's marginal contribution across all six permutations of the players. The table of marginal contributions is shown in Table 2.1:

Table 2.1 Marginal Contributions to the Shapley Value

Permutation	**Player 1**	**Player 2**	**Player 3**
1,2,3	10	20	18
1,3,2	10	18	20
2,1,3	20	10	18
2,3,1	18	10	20
3,1,2	20	18	10
3,2,1	18	20	10
Mean	**16**	**16**	**16**

The game is symmetric, since the value depends only on coalition size. Each player produces the same pattern of marginal gains. Hence, each player receives the same reward (according to efficiency, 1/3 of the grand coalition's payoff).

Now consider the game played with a communication structure given by the relational graph with nodes {1, 2, 3} and edges (1, 2) and (2, 3), as shown in Figure 2.14. For any coalition S, the Myerson restricted game $v^L(S)$ equals the sum of the values of the connected components induced by S. The characteristic function under this structure satisfies $v(\phi) = 0$, $v(\{i\}) = 10$, $v(\{1,2\}) = v(\{2,3\}) = 30$, $v(\{1,3\}) = 20$, and $v(N) = 48$, $i \in N$. Players 1 and 3 cannot cooperate directly, and player 2 serves as the link between them. Since there is no synergy between Players 1 and 3 alone, the payoff of the corresponding coalition is simply the sum of the payoffs of the individual players. The Shapley value of v^L corresponds to the Myerson allocation. The marginal contributions computed from v^L are shown in Table 2.2:

Table 2.2 Derivation of Myerson Value

Permutation	**Player 1**	**Player 2**	**Player 3**
1,2,3	10	20	18
1,3,2	10	28	10
2,1,3	20	10	18
2,3,1	18	10	20
3,1,2	10	28	10
3,2,1	18	20	10
Mean	**86/6**	**116/6**	**86/6**
	43/3	**58/3**	**43/3**

The marginal contributions in v_L show that player 2 receives 58/3, while players 1 and 3 receive 43/3. Player 2 is central in the relational graph and thus gets a larger share. Players 1 and 3 play symmetric roles in the game and thus obtain the same value. The difference arises because the communication graph makes player 2 the only bridge between the outer players.

We now consider the probabilistic S-value based on an expanding coalition under the same communication graph. The first player to enter the coalition is chosen at random, and each successive entrant is randomly selected from the neighbors of the current coalition. For example, if player 1 enters first, then player 2 must enter next, and player 3 joins last. If player 2 enters first, then the next entrant is chosen at random from players 1 and 3. This process generates a probability distribution over the possible orders of coalition formation. The S-value reflects the expected marginal contributions under this stochastic process. Entry order matters because, for example, when bridge players join later, they often add a greater value to the coalition that has already formed. It follows that the distribution of the order of coalition formation is shown in Table 2.3:

Table 2.3 The Distribution of the Order of Coalition Formation

Permutation	**Probability**
1,2,3	1/3
3,2,1	1/3
2,1,3	1/6
2,3,1	1/6

The marginal contributions given the ordering are the same as in the original Shapley formulation. The results are shown in Table 2.4.

Table 2.4 Derivation of the S-Value.

Permutation	**Probability**	**Player 1**	**Player 2**	**Player 3**
1,2,3	1/3	10	20	18
3,2,1	1/3	18	20	10
2,1,3	1/6	20	10	18
2,3,1	1/6	18	10	20
Expected value		47/3	50/3	47/3

The result gives players 1 and 3 the value 47/3, while player 2 receives 50/3. The process still rewards the central position of player 2, but less than the Myerson value. Compared to the Shapley value, this formulation favors players who enter a coalition as the second player. If player 2 is not the first player to enter a coalition, then it has to be the second. These three values show apparent differences. The Shapley value treats all players equally. The Myerson value rewards connectivity in the relational graph and gives a premium to the central node. The S-value reflects both connectivity and the probability of entry at each stage. Each rule captures a different behavioral assumption about cooperation, and this example shows that allocation rules can diverge even when they essentially use the same characteristic function.

Several cooperative network games have been proposed in the literature. Early studies considered shortest path games in which players own network nodes and cooperate to establish a path between a source and a sink. Later models assumed ownership of network edges and focused on the allocation of costs or profits generated by cooperation. More recent work has investigated the nucleolus as a solution concept for shortest-path games **(Fragnelli et al., 2000; Grahn, 2001; Voorneveld & Grahn, 2002; Baïou & Barahona, 2019).**

These models differ from the communication network model developed in this dissertation. Existing shortest path games typically focus on a single source-destination pair and determine whether a coalition can establish a feasible path. In contrast, the proposed model considers multiple communication demands among many node pairs, incorporates transmission costs and revenues, and extends to include capacity constraints. Consequently, it provides a more realistic representation of cooperation among service providers in communication networks **(Fragnelli et al., 2000; Grahn, 2001; Voorneveld & Grahn, 2002; Baïou & Barahona, 2019).**

Chapter 3

General Model of Communication Network

This chapter presents the cooperative game model we use to analyze communication networks. We describe the physical network as a weighted graph, where nodes represent communication points and edges represent transmission links. We assign each service provider a set of edges and model coalitions as groups of providers that pool their links. These providers satisfy the demand for communication between pairs of nodes. Each of these pairs of nodes will be referred to as a route. A set of edges that can be used to satisfy demand along a route will be called a path. We define the sets of routes that a coalition can serve and which it controls, based on reachability. We introduce the concept of a relational graph to capture the interaction structure among providers and to identify which coalitions are feasible. We define the characteristic function, v, as the total profit a coalition can obtain by serving routes on the basis of an appropriately defined auction, thereby linking physical connectivity, flow, demand, and price competition into a single framework. This prepares the ground for our later use of allocation rules, such as the Shapley value. In summary, we develop the structural and economic tools needed to study cooperation among service providers and set the foundation for the analysis of allocation rules and stability under various assumptions in the following chapters.

3.1 Description of Problem

In my dissertation, the primary model analyzes Internet Service Providers (ISPs) as cooperative players managing a communication network. Each ISP owns specific links through which data passes, and all ISPs must manage network traffic optimally and distribute the total gains among all ISPs in a cooperative, consistent, and stable manner.

A communication network is abstracted as a graph with nodes and edges **(Kumar, A. et al., 2004).** The nodes represent communication centers, such as data centers and exchanges, while the edges represent links between disparate ISPs. Each ISP controls a subset of network links and collectively contributes to meeting the total demand for data transmission between communication hubs. The network is modeled as a weighted graph, where nodes represent communication centers, such as routers or access points, and edges represent communication links that can be characterized by cost, capacity, and direction **(Kumar et al., 2004).**

This problem concerns the provision of data transmission services to end users by several ISPs in a cooperative framework. Cooperation is needed because ISPs must route data packets through multiple paths to reach their intended recipients **(Cheema & Saqib, 2010).** The total profit generated hinges on the efficiency of data routing and the coalition structures formed among ISPs **(He & Walrand, 2005).** Figure 3.1 illustrates a basic example with two nodes, A and B, connected by two parallel links. Each edge is controlled by an Internet Service Provider (ISP). The customer orders an overall data flow of x units to be sent from node A to node B. The objective here is to split the overall flow across the two routes to minimize total cost or maximize total benefit.

The form of the optimal strategy depends on the cost structure. Suppose that the cost of sending a unit of flow along an edge is constant. When the capacity of the cheaper edge is sufficient to send the required information, then it should all be sent along this edge. When the capacity of the cheaper edge is not sufficient to send the required information, then as much information as possible should be sent along this edge, and the remainder should be sent along the more expensive edge.

In practice, congestion may occur. In this case, the marginal cost of sending an additional unit of information along an edge is increasing. Under such a scenario, it will often be optimal to only partially use the capacity of both edges in such a way to equalize the marginal costs of sending an extra unit of information along the two edges.

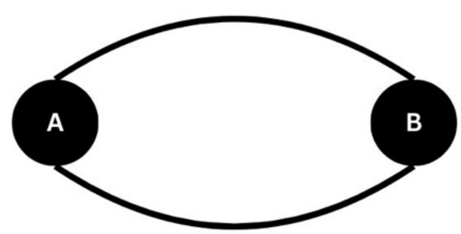


Figure 3.1. A Simple Example of a Communication Network with 2 Players.

This particular framework demonstrates the basic construction of a cooperative distributed network. The core aspects of cooperative network management include fairness, efficiency, and coalition stability **(Saad et al., 2009).**

3.2 Notation

This model outlines the functions of the communication network using a formal notation. It represents the network as a graph $G = (V, E)$, where V is the set of nodes and E is the set of edges. The nodes are the communication points (routers, local servers, and gateways). At the same time, the edges serve as communication channels (fiber lines and wireless links) that interconnect the nodes **(Kumar et al., 2004).** Each edge is associated with a vector of "weights", which define the parameters of the link; e.g., the cost of "directly transmitting" between unconnected nodes is assumed to be infinite, thereby excluding such a link from the optimization procedure **(Srikant & Ying, 2014).**

The players in the game are represented by $P = \{P_1, P_2, ..., P_K\}$. Each player P_k owns a subset of edges, $E_k \subseteq E$, which together form the overall network. The sets $E_1, E_2, ..., E_K$ form a partition of E, meaning that each link belongs to exactly one player. These players can cooperate by combining their subnetworks to create larger coalitions, $A \subseteq P$ **(Saad et al., 2009).** Each coalition A can transmit information only through the edges it controls $\cup_{P_k \in A} E_k$. For each route, corresponding to a pair of nodes (i, j), there is a fixed demand $d_{i,j}$ representing the required amount of information flow, and a maximum revenue rate $r_{i,j}$ indicating the maximum income earned per unit of information flow between nodes i and j. This upper bound reflects real-world regulatory practice. Within the European Union, for instance, wholesale charges that one network operator may levy on another for cross-border traffic are capped by binding EU regulation. This limits the per-unit revenue achievable on any inter-provider route **(Muñoz-Acevedo & Grzybowski, 2023).** Let U denote the set of all routes.

Definition 1.a) *The coalition A serves route (i, j) if there exists at least one path between nodes i and j whose edges are all serviced by players in A. The set of routes served by coalition A is denoted by S(A).*

b) *A coalition A strongly controls route (i, j) if it serves the route and no coalition disjoint from A can serve the same route. The set of routes strongly controlled by* *coalition A is denoted by C(A). In set notation, C(A) = S(A) \ S (A^c), where D\ E denotes the elements of D that are not in E.*

From these definitions, the inclusion $C(A) \subseteq S(A)$ follows immediately. Strong control requires exclusive service of the routes, while service alone requires only reachability. This means every strongly controlled route must also be one the coalition can serve. Monocity with respect to coalition size also holds. If A is a subset of B, then $C(A) \subseteq C(B), S(A) \subseteq S(B)$. By definition, $S(\phi) = \phi$ an empty coalition contains no edges and thus cannot serve any route. By assumption, since the network is described by a connected graph, the grand coalition controls all of the routes, i.e., *C(P) = U.* Note that any route served by at least one of two disjoint coalitions A and B must be served by the union of these coalitions. Thus $S(A \cup B) \supseteq S(A) \cup S(B)$. Suppose that a route is controlled by one of two disjoint coalitions, A or B. It follows that the union of these coalitions serves that route. In addition, either A^c or B^c cannot serve that route. Hence, $C(A \cup B) \supseteq C(A) \cup C(B)$. If both A and A^c can serve the same route, they compete for that route.

To illustrate these ideas, consider the network illustrated in Figure 3.2. This describes a cooperation game on a simple communication network with three players. Player 1 controls the green edges, player 2 the red edges, and player 3 the blue edges. The routes served and controlled by various coalitions are described in Table 3.1.

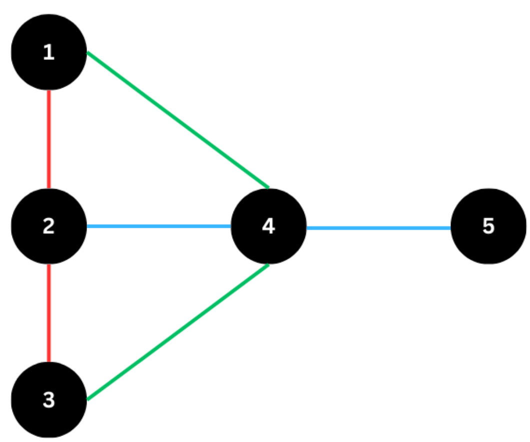


Figure 3.2. A Simple Example of a Communication Network with 3 Players

Table 3.1. Description of the Level of Control of the Network in Figure 3.2 by Various Coalitions

A	***S*(*A*)**	***C*(*A*)**
ϕ	ϕ	ϕ
$\{P_1\}$	{(1,3), (1,4), (3,4)}	ϕ
$\{P_2\}$	{(1,2), (1,3), (2,3)}	ϕ
$\{P_3\}$	{(2,4), (2,5), (4,5)}	{(2,5), (4,5)}
$\{P_1, P_2\}$	{(1,2), (1,3), (1,4), (2,3), (2,4), (3,4)}	{(1,2), (1,3), (1,4), (2,3), (3,4)}
$\{P_1, P_3\}$	U	{(1,4), (1,5), (2,4), (2,5), (3,4), (3,5), (4,5)}
$\{P_2, P_3\}$	U	{(1,2), (2,3), (2,4), (2,5), (4,5)}
$\{P_1, P_2, P_3\} = P$	U	U

3.3 General Approach to Defining the Characteristic Function

To determine how well coalitions function, the model describes a characteristic function $v(A)$. This function describes the maximum guaranteed payoff that the coalition A can secure **(Saad et al., 2009).** In order to define the characteristic function, the following assumptions are made. The value of the grand coalition P is assumed to be the maximum profit that such a coalition can obtain when the price for serving a route is given by $r_{i,j}$. Hence, $v(P)$ can be determined as the solution to an optimal flow problem. It should be noted that we assume that if the marginal cost of sending additional information along route (i, j) is $\geq r_{i,j}$, then such demand is not met, but there is no cost for not satisfying demand.

The value of a coalition A is based on the result of a two-player auction (game) for supplying the routes in which the other player is A^c. Here, we consider the possibility of multiple-round auctions based on the concept of the second-price auction (**Anunrojwong et al., 2024)** with a maximum acceptable price. Second-price auctions are popular, since they give an incentive to firms to declare a price based on their costs. Only coalitions that can serve a route are eligible to bid for that route. In each round of the auction, a price and suppliers are set for each route. The suppliers then declare how much demand they are willing to supply on the routes assigned to them at the current prices. This requires the solution of two independent flow problems. Under the rules of the auction, the suppliers must supply the amounts they have declared, and the current price is guaranteed as the minimum price. It is possible that demand for some routes may remain unsatisfied. In this case, an additional round of the auction is carried out, and the prices of the routes with unsatisfied demand can be adjusted upwards according to the bidding. This procedure is repeated until either all of the demand is satisfied or neither of the coalitions taking part in the auction is able (or willing) to supply the residual demand. A similar procedure is used **in Gąsior and Drwal (2013).** Here, an iterative capacity allocation scheme for self-managed networks converges to a Pareto-optimal Nash equilibrium. This happens through successive adjustments made by self-interested nodes. However, the mechanism developed in this chapter differs. Instead of a non-cooperative approach, it uses a cooperative framework. In this framework, coalitions of providers, not individual players, compete at each round to supply the residual demand. The resulting payoffs are distributed according to the concept of a cooperative solution. The profits obtained by the coalitions from such an auction are assumed to be the values of those coalitions in the characteristic function. The value of the empty coalition is assumed to be equal to zero.

Now we consider the number of two-player games that need to be solved to define the characteristic form of the game. Such games are used to define the value of the $2^{|p|}$ - 2 proper subsets of the grand coalition. Suppose there is an odd number of players, say $|P| = 2n + 1$. It may be assumed that A is a coalition with at most n members and thus A^C has at least $n + 1$ players. Since each two-player game gives the value of two coalitions and there is a one-to-one relation between A and A^C, it can be seen that $2^{|p|-1} - 1$ such games must be solved. For example, when there are three players, it suffices to consider the three two-player games in which one coalition is formed by one player and the other coalition consists of the two remaining players.

Now suppose that there is an even number of players, say $|P| = 2n$. It may be assumed that A is a coalition of at most n players, and that when A consists of n players, then Player 1 belongs to A. Arguing as above, again $2^{|p|-1} - 1$, such two player games must be solved. For example, when there are four players, the following seven two-player games must be solved:

a) Four games in which a coalition of three players bids against the other player.
b) Three games in which one coalition consists of Player 1 and one other player, and the other coalition consists of the remaining two players.

The definition of the auction used in such two-player games will be considered in Section 3.4 and further developed in Chapters 4 and 5. This model captures interacting economic and operational features: the structure of the network, the flow of information, and the bargaining power of coalitions. The framework of a cooperative game models both the benefits of cooperation and cost allocation between network providers **(Verdonck, 2016).** Integrating cooperative game theory and graph theory, these ideas lead to analyzing the stability of coalitions and the equitable distribution of profits and /or costs. **(Ray, 2007; Saad et al., 2009; Fele et al., 2017; Muros, 2019).**

3.4 Example of a problem

This section presents a simple two-node communication network to illustrate the application of the general model and the cooperative interaction among service providers (see Figure 3.3). This network consists of two nodes, A and B, and two parallel links. Each link corresponds to a different provider's transmission link.

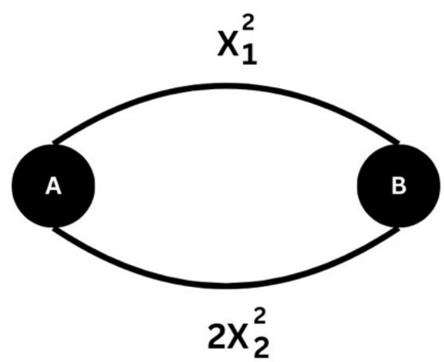


Figure 3.3. A Simple Example of a Communication Network

Let x_i denote the flow along link i. The function x_1^2 gives the cost of the first link, while the second link has a cost function $2x_2^2$. The quadratic cost functions are designed to illustrate the increasing expense of sending information as volume increases, reflecting congestion and overloaded network resources. We assume that the demand for communication between A and B is fixed at 100 units, which must be routed through the two links. It is assumed that the links have unlimited capacity, and the maximum revenue for sending a unit of information is set equal to 200.

First, we consider the problem faced by the coalition of both players. Since there cannot be any rival bids, this coalition is invited to serve this demand at a unit price of 200. Suppose that the coalition should satisfy all of these demands using both links. In this case, the flows should be set to minimize the cost of satisfying this demand. This optimization problem can be written as:

$$\text{Minimize } C = x_1^2 + 2x_2^2.$$

subject to

$$x_1 + x_2 = 100, \qquad x_1 + x_2 \geq 0.$$

By substituting $x_2 = 100 - x_1$, the cost function becomes:

$$C = x_1^2 + 2(100 - x_1)^2 = 3x_1^2 - 400x_1 + 20000$$

Differentiating with respect to x_1 and setting the result equal to zero gives

$$\frac{dC}{dx_1} = 6x_1 - 400 = 0,$$

Which yields the optimal flow $x_1^* \approx 66.67$. Substituting this value into the demand constraint leads to $x_2^* \approx 33.33$. It should be noted that such a flow equalizes the marginal costs of the flows along the two edges. These are $2x_1^* = 4x_2^* \approx 133.33.$ Since this marginal cost is not greater than the revenue obtained from transferring a unit of information, the coalition of two players should satisfy all of this demand. When the revenue obtained from transferring a unit of information is less than $2x_1^* \approx 133.33$, the flows along the edges should be set so that the marginal cost of sending information is equal to this unit revenue. The total profit of the coalition is thus

$$v(\{1{,}2\}) = 200 \times 200 - x_1^2 - 2x_2^2 \approx 13333.33$$

This result shows that the optimal allocation divides the total demand so that two-thirds of the flow travels along the first link and one-third along the second. The first link carries more flow because for any given flow along a link, the first has a lower marginal cost, making it the more efficient path.

Now we consider the case when two providers compete against each other. It should be noted that such competition can come in different forms. We assume that the providers compete in an auction (possibly over several rounds) for the right to supply the demand. For the purpose of defining any such problem as a cooperative game, it suffices to consider auctions with two players. We will use adaptations of the classical second-price auction. In our scenario, the current price is set to be equal to the minimum of the maximum set price and the highest price proposed. In cases where there are two equal price bids p_0, then the current price is set to be $p_0 + \varepsilon$ for a given positive ε. If the auction takes place over several rounds, then the price set in a given round is a guaranteed minimum price for the units sold in that round. The final price is set by the appropriate rules and could be higher than the current price in any given round. Such a procedure is adapted to problems in which the marginal cost of transmission is non-decreasing in the amount of information transmitted. In such a case, the price demanded by the provider in each round of voting should increase according to the units already sold.

Under one scenario, the providers might bid to serve all the demand for a route. Under another scenario, the auctioneer (possibly the government selling rights to transmit) might sell the right to transmit bundles of information until the whole demand is sold. Finally, the auctioneer might design an auction to elicit the bidders to declare their marginal costs in appropriate scenarios and appropriately assign demand to various bidders. Such procedures are often used in energy markets, where independent system operators clear wholesale electricity through marginal-cost-based auctions in which generators submit price–quantity offers and the system operator dispatches units in merit order until demand is met **(Hogan, 2022).**

Suppose that the two providers in the scenario illustrated by Figure 3.3 bid to supply the whole demand. Under such a second-price auction, called an auction of type 1, the providers should bid a price that covers their average transmission cost. Under such a strategy, this would ensure that a provider does not make a loss and makes a profit when the provider wins the auction. The average cost of the providers are $\frac{x_1^2}{x_1} = x_1$, and $\frac{2x_2^2}{x_2} = 2x_2$, respectively. Hence, in an auction to supply 100 units, Provider 1 should bid 100, and Provider 2 should bid 200 (which is also the maximum price). Hence, such an auction assigns the route to Provider 1 at a price of 200. It follows that $v(\{2\}) = 0$. The profit of Provider 1 is given by

$$v(\{1\}) = 200 \times 100 - 100 \times 100 = 10000.$$

Suppose now that the providers bid for successive bundles of twenty units of demand. This will be called an auction of type 2. Arguing as above, in the first round, Provider 1 should bid 20 and Provider 2 should bid 40. Hence, such an auction assigns this demand to Provider 1 at a current price of 40. The position of the winning bidder changes, since its average costs for supplying a second bundle are given by the increase in costs for supplying this bundle divided by the size of the bundle, i.e., $\frac{40^2 - 20^2}{20} = 60$. Provider 2 makes the same bid as in the first round. Hence, the second round of the auction assigns a bundle to Provider 2 at a current price of 60. The average costs incurred by Provider 2 for supplying a second bundle are $\frac{2(40^2 - 20^2)}{20} = 120$. Hence, the third round of the auction assigns a bundle to Provider 1 at a current price of 120. The average costs incurred by Provider 1 for supplying a third bundle are $\frac{60^2 - 40^2}{20} = 100$. Hence, the fourth round of the auction again assigns a bundle to Provider 1 at the current price of 120. The average costs incurred by Provider 1 for supplying a fourth bundle are $\frac{80^2 - 60^2}{20} = 140$. Hence, the final round of the auction assigns a bundle to Provider 2 at a current price of 140. We will assume that under the rules of the auction, the final price for any unit equals the price set in the final round. It follows that such an auction procedure assigns 60 and 40 units to Provider 1 and Provider 2, respectively, at a price of 140 per unit. The profits obtained by the two providers are

$$v(\{1\}) = 140 \times 60 - 60 \times 60 = 4800$$
$$v(\{2\}) = 140 \times 40 - 2 \times 40 \times 40 = 2400.$$

It should be noted that such an auction gives a supply profile that is relatively close to the profile that minimizes the supply cost. As the size of the bundles decreases, the bids of the providers will tend to their marginal costs, given the amount already assigned to them. It follows that the assignment of demand will tend to the one that minimizes the supply costs. Obviously, this comes at the cost of increasing the number of rounds needed in the auction.

Finally, we consider an auction in which the providers are asked in each round their current price for a "marginal unit" (a type 3 auction). The price is set to be the minimum of the maximum set price and either the highest price given or $p_0 + \varepsilon$ when both firms offer the same price. Given the results of the current bidding, the providers then state how much they wish to supply at the current price. If the total supply from this round covers the residual demand, then the residual demand is split between the players proportionally to their declared level of supply in the current round. If the residual demand is not covered, then the providers are assigned their declared levels of supply. For illustration, we assume that $\varepsilon = 40$. In the first round, the two providers announce their marginal costs given that neither has satisfied any demand. This is zero for both providers. Hence, the price is set to be 40. At this price, Provider 1 is willing to 20 supply units and Provider 2 is willing to supply 10 units. These are the levels of supply satisfying the condition that the providers' marginal costs are equal to the current price. There are 70 units of residual demand. In the second round, both providers announced marginal costs of 40. The price is set to be 80. At this price, the providers are willing to supply a total of 40 units and 20 units, respectively. There are 40 units of residual demand. In the third round, both providers announce marginal costs of 80. The price is set to be 120. At this price, the providers are willing to supply a total of 60 units and 30 units, respectively. The residual demand is 10. In the fourth round, both providers announce marginal costs of 120. The price is set to be 160. The providers are willing to supply an additional 20 units and 10 units, respectively. These bids are split in proportion to the residual demand, i.e., Provider 1 obtains two-thirds of the residual demand. Hence, Provider 1 is assigned a total of $\frac{200}{3}$ units of demand, and Provider 2 is assigned $\frac{100}{3}$ units of demand. Assuming that the price in the final round is the price given for all the units, it follows that the profits of the providers are given by

$$v(\{1\}) = 160 \times \frac{200}{3} - \left(\frac{200}{3}\right)^2 \approx 6222.22$$

$$v(\{2\}) = 160 \times \frac{100}{3} - 2\left(\frac{100}{3}\right)^2 \approx 3111.11.$$

It should be noted that, as $\varepsilon \rightarrow 0$ the allocation and price tend to those obtained under market competition. However, this comes at the cost of lengthening the auction.

Under each of these auction procedures, it can be seen that the coalition of both providers obtains a greater profit than the total profit of the two competing providers. Hence, according to each of these auction procedures, the corresponding cooperative game is superadditive. According to the standard Shapley value, the surplus from cooperation is shared equally between the two members of the coalition. It follows that under the first auction procedure, the shares of the two players at the Shapley solution are $v_1 \approx 11666.67, v_2 \approx 1666.67$. Under the second auction procedure $v_1 \approx 7866.67, v_2 \approx 5466.67$. Under the third auction procedure $v_1 \approx 9777.78, v_2 \approx 3555.56$. As expected, the most efficient provider is assigned a larger profit under all of the solutions.

It is clear that the value ascribed to a provider depends on the definition of an auction. In practice, such an auction procedure should be adapted to the complexity of the network, derive an allocation that is similar to the allocation that would evolve under market conditions, while being simple enough to implement and analyse. The first approach is too simplistic for anything but a very simple network, and even for the network considered, it does not give an allocation similar to the market equilibrium allocation. The other two approaches seem to be promising. The following two chapters consider models in which the cost of transmission are linear. It will be argued that the third auction procedure gives solutions that give a good reflection of the market power of the providers. This procedure is also implementable for games with several players on large networks.

This brief example illustrates the model's critical understanding of economics and structure. It demonstrates the interplay between network optimization and coalition theory in achieving both technical and economic efficiency. From an economic perspective, flow optimization minimizes operational costs, and the cooperative game approach ensures that all participants receive equitable compensation for their contributions. Cooperation, justified by the superadditivity of the characteristic function, proves that the service providers' joint performance and profitability exceed that of independent provision. Such principles enable the construction of models of large multi-operator communication systems in which cooperating subnetworks provide benefits that no single operator can obtain on its own.

3.5 Dividing Revenue on the Basis of Measure of Edges' Centrality

It should be noted that the complexity of solving such a cooperative game increases as the size of the network and the number of players increase. While the complexity of solving subgames depends on the assumptions regarding the costs and capacity of links, the complexity of such a solution is always exponential in the number of players. Hence, it would be useful to define a simpler solution to the problem of splitting revenue that, in a realistic situation, gives a similar solution to the cooperative solution. One such concept is the centrality of an edge. We first consider the classical definition of the centrality of an edge.

Let $\sigma_{i,j}$ denote the number of shortest (interpreted as cheapest) paths from node i to node j in a weighted graph, and let $\sigma_{i,j}(e)$ denote the number of those shortest paths that pass through edge e. The classical definition of the centrality of an edge is given by **(Girvan & Newman, 2002).**

$$C(e) = \sum_{i,j \in V} \frac{\sigma_{i,j}(e)}{\sigma_{i,j}}$$

(3.1)

which reflects how often an edge is used in shortest-path connections between node pairs under the assumption that the demand for transmitting between two nodes is the same for each pair of nodes. In order to adapt this classical measure to define economic centrality, it is necessary to define the profit obtained along each path under the optimal solution. For this reason, we will derive the optimal flow in each edge using algorithms that first derive optimal flows along paths **(Ahuja et al., 1993).** Let f_p denote the flow-through path *p* under an optimal solution. It follows that the flow along edge *e* is given by

$$f_e = \sum_{p:e \in p} f_p,$$

(3.2)

The profit obtained by using a given path depends on the cost of each edge along that path, as well as the flow along that path and the revenue obtained for the corresponding route. Given an optimal solution, it may be assumed that the unit cost for using an edge, c_e, is the average cost per unit flow given the total flow through that edge. The average cost for a path *p* can then be defined to be the sum of the average unit costs of the edges along that path.

$$C_p = \sum_{e:e \in p} C_e,$$

(3.3)

The profit made by transferring information along the path *p*, whose ends are the nodes *i* and *j*, is given by w_p, where $w_p = f_p\ [r_{i,j} - c_p]$. Summing the profits over all the paths used, we obtain the total profit under the optimal solution. In order to divide these profits, we consider the following two regimes: a) the profits obtained along a path are split equally between the edges along that path, and b) the profits obtained along a path are split in proportion to the average cost of using an edge. In the first case, the profit from the path *p* ascribed to an edge *e* in that path is defined to be $w_p^U(e)$, where:

$$w_p^U(e) = \frac{f_p[r_{i,j} - c_p]}{n_p},$$

(3.4)

and n_p is defined to be the number of edges in that path. Summing these expressions over all the paths between i and j that pass through the edge e, we obtain the profit from the route (i, j) ascribed to an edge e. Finally, summing over all of the routes, we obtain the economic centrality of the edge e based on the uniform splitting of profits along paths $C^U(e)$. Thus

$$C^U(e) = \sum_{i,j:i<j} \sum_{p \in S_{i,j}; e \in p} \frac{f_p[r_{i,j} - c_p]}{n_p},$$

(3.5)

where $S_{i,j}$ denotes the set of paths whose ends are the nodes i and j.

In the second case, the profit from the path p ascribed to an edge e in that path is defined to be $w_p^R(e)$, where:

$$w_p^R(e) = \frac{f_p[r_{i,j} - c_p]c_e}{c_p},$$

(3.6)

Summing these expressions over all the paths between i and j that pass through the edge e, we obtain the profit from the route (i, j) ascribed to an edge e. Finally, summing over all of the routes, we obtain the economic centrality of the edge e based on the proportional splitting of profits along paths, $C^R(e)$. Thus

$$C^R(e) = \sum_{i,j:i<j} \sum_{p \in S_{i,j}; e \in p} \frac{f_p[r_{i,j} - c_p]c_e}{c_p},$$

(3.7)

It should be noted that this definition is based on the solution of the optimal flow problem faced by the grand coalition and does not require the solution of games played by two coalitions. In general, such a solution is much less complex than the cooperative game solution. However, the division of profits based on centrality does not take into account the possible strategic interactions between the players. Such divisions will be illustrated in Chapters 4 and 5.

3.6 Definition of a Relational Graph

In practice, coalitions tend to expand by incorporating new members that are, in some sense, neighbors of the current members of a coalition. To formalize the interaction structure among players, we represent the network using a relational graph. Such a relational graph is a simple, undirected graph $G_R = (P, F)$ in which each node represents a player and each edge represents a direct bilateral relationship between two players. In this context, a relationship indicates that the two players share a connection that allows the construction of feasible paths when they cooperate **(Myerson, 1977; Madiman, 2008).** In formal terms, there exists an edge between Player i and Player j in the relational graph if and only if there exists a node in the graph describing the network that is the endpoint of at least one edge owned by Player i and at least one edge owned by Player j. In intuitive terms, two players are directly linked in the relational graph when they are neighbors in the communication network. The relational graph, therefore, encodes the basic structure of potential cooperation. Consider the network example shown in Figure 3.4. Each player is a neighbor of the other two players. Hence, the relational graph is a complete graph with three nodes. The set of players (nodes) is $P=\{1,2,3\}$ and the set of edges is $F=\{(1,2),(1,3),(2,3)\}$. Such a relational graph may, for example, specify the structure upon which coalitions can be built. When the set of feasible coalitions is defined on the basis of such a relational graph, a coalition A is said to be feasible when the subgraph of the relational graph containing all of the players in A and all of the direct edges between any pair of players in A is connected. Under this assumption, any coalition of players is feasible in the game corresponding to Figure 3.4. In such a formulation, one way of modeling the growth of a coalition is by initially choosing a player at random and, at each successive stage, extending the coalition to include a player that is a neighbor of one of the players already in the coalition. From the assumptions regarding the communications network (connected with each player owning at least one edge), the grand coalition is, by definition, feasible.

Consider two distinct coalitions, A and $B,$ such that there is no node in the network that is an endpoint of an edge controlled by both a member of A and a member of B. This corresponds to the situation in which no member of coalition A is linked to any member of coalition B in the relational graph. It follows that the coalition $A \cup B$ only serves and controls the routes that are individually served and controlled by one of the two coalitions. Intuitively, there can be no synergy between such coalitions, i.e. $v(A \cup B) = v(A) + v(B)$.

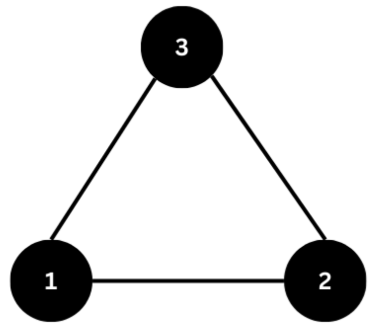


Figure 3.4. The Graph of Relationships Induced by the Network given in Figure 3.2.

3.7 Summary

This chapter has developed a general cooperative game model for communication networks, linking the network's physical structure to the economic interactions among service providers. The physical network is represented as a weighted graph, with nodes corresponding to communication endpoints and edges denoting transmission links with associated traits. Each service provider controls a distinct set of edges, and coalitions are formed by combining these sets of edges. This chapter has also introduced the concepts of served routes and controlled routes. These definitions enable us to construct key sets, such as $S(A)$ and $C(A)$, that capture the operational capabilities of each coalition. The relational graph and coalition-induced subgraphs complement this physical representation by describing which provider groups can feasibly coordinate and how coalitions emerge from the underlying structure of a network.

Building on these structural elements, the chapter has proposed a general approach to defining the characteristic function of a cooperative game $v(A)$, which assigns to each coalition the total profit it can achieve by competing with a coalition made up of the remaining players. This formulation integrates flow, cost, and demand conditions into a coherent game-theoretic framework and establishes the basis for evaluating cooperation. The concept of centrality has also been adapted to the purpose of solving the problem of revenue sharing.

Overall, the chapter has presented an underlying general framework for analyzing cooperation in communications networks using both cooperative game theory and the concept of centrality. This framework will be applied to models that make specific assumptions about the costs and capacities of links, which are presented in Chapters 4 and 5.

Chapter 4

A Model with Linear Costs and No Capacity Constraints

Chapter 4 analyzes a version of the model of a cooperative communications network introduced in Chapter 3. In particular, it builds directly on Section 3.3, which presented the general approach to defining a game's characteristic function. The previous chapter established the framework for analysis, including the representation of the communication system as a weighted graph, the definition of service-provider coalitions, and the economic rules that determine coalition payoffs. In this chapter, we focus on a simplified but analytically important case in which transmission costs are linear, and the network operates without capacity constraints. Under these assumptions, the cost of transmitting information along a path always equals the sum of the weights of the edges in that path, and thus congestion effects are not present. This environment allows us to analyze coalition behavior, control of routes, and profit generation in a clear and tractable setting. Using the definitions of served and controlled routes introduced earlier and introducing the concept of economic control, we develop the cooperative network game corresponding to this linear cost model. The results provide the foundation for the analysis of allocation and stability in the subsequent sections of this chapter.

4.1 Representing a Network with Linear Costs and No Capacity Constraints

This section introduces the basic network structure used to analyze the cooperative communication game. Here, it is assumed that transmission costs are linear and there are no capacity constraints. The model builds on the framework from Chapter 3. It applies this framework to a simplified environment, thereby clarifying the analysis of coalition behavior and route control.

The communication system is represented by a weighted, connected, and simple graph $G = (V, E)$ **(Newman, 2018; Wilson, 2009).** The set $V = \{V_1, V_2, \ldots, V_J\}$ denotes the network's nodes, where each node represents a communication point, such as a router, exchange center, or transmission hub. The set E represents the transmission links that connect these nodes. Each edge is written as a pair of nodes (i, j) with $1 \leq i <$

$j \leq J$, indicating that a direct communication link exists between nodes i and j. The total number of edges in the network is denoted by M. Each edge (i, j) is associated with a weight $w_{i,j}$ that represents the unit transmission cost along that link **(Ahuja et al., 1993).** If no direct connection exists between two nodes, the model assigns $w_{i,j} = \infty$ to indicate that direct transmission between them is not possible. These edge weights determine the cost of transmitting information through the network. Service providers who operate network infrastructure are the players in the cooperative game, each controlling a subset of edges in the network **(Myerson, 1977).** The player set is $P = \{P_1, P_2, \ldots, P_K\}$, where K is the total number of providers. Each player P_k controls a subset of edges, $E_k \subseteq E$, corresponding to the links they operate. The sets $E_1, E_2, \ldots, E_K$ form a partition of E. This means each edge belongs to exactly one player. Players can form a coalition to enable communication along paths whose edges are owned by members of that coalition. Hence, coalitions can serve routes that individual players may not be able to serve on their own. A path is said to be feasible when all its edges are controlled by the members of a single coalition.

In this chapter, the cost of transmitting information is assumed to be linear. The cost of sending one unit of information between two nodes equals the sum of the weights of the edges that form the path used for transmission. This assumption implies that the marginal cost of transmission remains constant regardless of the flow level. In addition, the model assumes that edges have no capacity constraints. Therefore, any coalition can transmit an arbitrary amount of information along the paths it controls without experiencing congestion or saturation. Under these assumptions, the network structure and the distribution of edges among players determine which communication routes each coalition can serve. Intuitively, since there are no capacity constraints, information should always be sent along the cheapest feasible path between two nodes. Control of the cheapest feasible paths between nodes, therefore, becomes the key factor determining the operational capabilities of coalitions and ultimately influencing the value they can generate in the cooperative game. These structural elements form the basis for defining the sets of served and controlled routes (defined in Chapter 3), and the concepts of an economically served and controlled route, which will be introduced in the following sections.

4.2 Revenue and Demand

To fully describe the economics of the network game, two key elements must be defined: the reward from transmitting information between nodes and the demand for each communication route. These two elements determine the potential revenue that coalitions can generate when serving routes in the network.

Let U be the set of all possible routes in the network, where each route is a pair of nodes for potential information transmission (see section 3.2). Each element of U corresponds to a demand for communication between two nodes. Two matrices are used to represent the economics of routing: the maximum revenue for transmitting a unit of information and the required transmission volume. The reward matrix R is a $J \times J$ matrix. The element in row i and column j represents the unit reward $r_{i,j}$ obtained from transmitting one unit of information between nodes i and j when $i \neq j$. The diagonal elements are defined as $r_{i,i} = 0$, because transmission within a single node does not generate revenue. The demand matrix D has the same dimension as the reward matrix. The element $d_{i,j}$ represents the demand for information transmission between nodes i and j. This value indicates the amount of data that must be transmitted between those nodes in the network.

In this model, it is assumed that transmission costs and rewards do not depend on the direction of communication. As a result, transmitting information from node i to node j yields the same reward as transmitting information from node j to node i. Under this assumption, both the reward and demand matrices are symmetric. This means that $r_{i,j} = r_{j,i}$ and $d_{i,j} = d_{j,i}$ for all pairs of nodes. The profit generated from serving a route depends on the difference between the reward and the cost of transmitting information along the path chosen by a coalition **(Ahuja et al., 1993).** If coalition A serves route (i, j), the total revenue obtained from that route equals $r_{i,j}$ multiplied by the demand $d_{i,j}$. The transmission cost can be interpreted as the length of the path connecting the two nodes and is equal to the sum of the edge weights along that path **(Bertsekas, 2021).** These matrices define the economic environment for coalitions. The reward matrix shows revenue opportunities, and the demand matrix shows required communication volume (see Equation 3.4). Combined with the network structure, these parameters determine the potential coalition profit.

To illustrate how routes and paths operate in the network, consider the example introduced earlier in Section 3.2, but with the addition of edge weights representing the cost of transmission (see Figure 4.1). Hence, the cost of transmitting 1 unit of information along edge (1, 4) is 4. In this network, different players or coalitions may serve the same route using different paths. For instance, the route between nodes (1,5) can be served by a coalition of the green and blue players through the path $1 \to 4 \to 5$, or by a coalition of all three players through the path $1 \to 2 \to 4 \to 5$. Each path has a transmission cost equal to the sum of the weights of its edges. This example shows that a single route may have multiple feasible transmission paths and that cooperation among players can create additional routing possibilities **(Floyd, 1962).** The availability of alternative paths, therefore, affects both the cost of transmission and the ability of coalitions to serve communication routes.

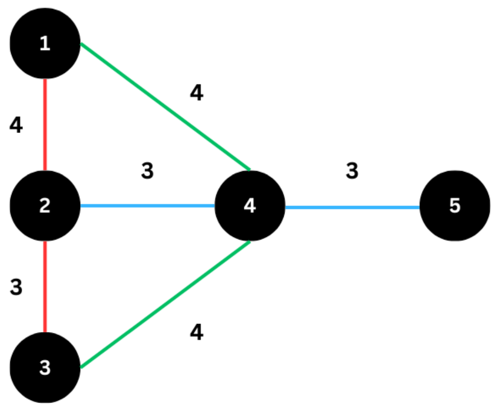


Figure 4.1. A Simple Example of a Weighted Communication Network

For illustrative purposes, we assume that the revenue and demand matrices are given by R and D, respectively, where:

$$R = \begin{pmatrix} 0 & 5 & 8 & 5 & 8 \\ 5 & 0 & 5 & 5 & 8 \\ 8 & 5 & 0 & 5 & 8 \\ 5 & 5 & 5 & 0 & 5 \\ 8 & 8 & 8 & 5 & 0 \end{pmatrix}; \qquad D = \begin{pmatrix} 0 & 10 & 4 & 10 & 4 \\ 10 & 0 & 10 & 10 & 4 \\ 4 & 10 & 0 & 10 & 4 \\ 10 & 10 & 10 & 0 & 10 \\ 4 & 4 & 4 & 10 & 0 \end{pmatrix}; \qquad (4.1)$$

4.3 Route Service and Economic Feasibility

After defining the network structure and the economic parameters of communication routes, the next step is to determine which routes can be served by a coalition of players and under what conditions such service becomes economically feasible. These concepts connect the network's physical connectivity with the economic profitability of providing communication services, a key aspect studied in cooperative network and communication games **(Saad et al., 2009).**

Let A denote a coalition of players such that $A \subseteq P$, where P is the set of all service providers in the network. When players form a coalition, they combine the edges they control and operate them as a unified subnetwork. The coalition can transmit information only through the edges owned by its members. Therefore, the set of edges available to coalition A is the union of all edge sets owned by its members. The coalition is said to serve a route (i, j) when there exists at least one path between nodes i and j that uses only edges belonging to players in coalition A. This condition ensures that the coalition can physically connect the two nodes using its own infrastructure, a principle consistent with cooperative connectivity models where coalitions succeed only if they can connect specific vertices in the network **(Bachrach et al., 2013).** The set of all routes that coalition A can serve is denoted by $S(A)$. If no such path exists, the coalition cannot provide communication service for that route.

In addition to physical connectivity, the coalition must also satisfy economic feasibility conditions. Even if a path exists, the coalition will use it only if the cost of transmitting information along that path does not exceed the reward. Let $c_{i,j}^{A}$ denote the minimum transmission cost between nodes i and j using only the edges controlled by coalition A. This value corresponds to the cost of the shortest path in the subnetwork operated by coalition A. If no path exists in this subnetwork, this cost is set to infinity. The coalition economically serves a route (i, j) if there exists a path between the two nodes whose cost does not exceed the maximum unit reward $r_{i,j}$. In this case, the coalition can provide the communication service without incurring a loss. The set of routes that coalition A can economically serve is denoted by $S_E(A)$. Such cost-based routing decisions are consistent with shortest-path cooperative games, in which coalitions choose the minimum-cost path using their controlled edges to obtain a reward **(Fragnelli et al., 2000; Voorneveld & Grahn, 2002; Baïou & Barahona, 2019).**

These definitions highlight two levels of feasibility in the cooperative network game. The first level concerns structural feasibility: whether the coalition's edges form a connected path between nodes. The second level concerns economic feasibility, which depends on whether the coalition can provide the service at a cost that does not exceed the reward obtained from transmitting information. As coalitions grow larger, they gain access to more edges in the network. This expansion increases the number of routes that can be served and often reduces transmission costs because additional edges may create shorter paths between nodes. Consequently, larger coalitions generally have greater operational capabilities and can economically serve more routes than smaller coalitions. This property plays an important role in determining the value of cooperation in the communication network game, where collaboration among network agents improves connectivity and routing efficiency **(Akkarajitsakul et al., 2013).**

4.4 Route Control by Coalitions

After determining which routes a coalition can serve, the next step is to analyze the degree to which a coalition controls communication routes within the network. Route control plays a central role in determining which coalition can generate profit from serving a particular route. These issues are commonly studied in cooperative network and routing models where coalitions of agents control network links and determine feasible communication paths **(Skorin-Kapov & Skorin-Kapov, 2021).** The following definitions formalize these concepts.

Definition 2. a) *The coalition A is said to economically serve the route (i, j) when there exists a path between nodes i and j whose total cost is less than or equal to* $r_{i,j}$*, and all edges along that path are serviced by firms belonging to coalition A. In this case, coalition A can provide the service without incurring a loss. The set of routes economically served by coalition A is denoted by* $S_E(A)$*.*

b) *The coalition A is said to economically control the route (i, j) when it can economically serve the route (i, j) at a lower unit transmission cost than any coalition disjoint from A. In this case, coalition A provides the most efficient service among competing coalitions, even though other coalitions may also be able to serve the route. The set of routes economically controlled by coalition A is denoted by* $C_E(A)$*.*

c) *The coalition A is said to strongly control a route (i, j) when two conditions are satisfied. First, Coalition A must be able to economically serve the route. Second, no coalition that is disjoint from A can serve the same route. The set of routes strongly controlled by coalition A will be denoted by* $C_S(A)$*. Formally, the set of strongly controlled routes can be expressed as* $C_S(A) = S_E(A) \setminus S(A^c)$*, where* A^c *denotes the complement of coalition A and the notation D \ E represents the elements of set D that do not belong to set E.*

In other words, coalition *A* has strong control over a route (*i, j*) when it has exclusive access to an economically feasible path between the two nodes. When this situation occurs, the coalition faces no competition from other players and therefore captures the entire profit associated with the route. This definition ensures that coalition A can economically serve the route, while its complement cannot even serve that route.

When a coalition economically controls a route (i, j) other coalitions may also serve the route, but coalition A has the lowest transmission cost. Under price competition, the coalition with the lowest cost will capture the entire demand for that route. These definitions establish a hierarchy among the sets of routes associated with each coalition. Every strongly controlled route is also economically controlled, every economically controlled route is economically served, and every economically served route must belong to the set of served routes. Therefore, the following relationship holds: $C_S(A) \subseteq C_E(A) \subseteq S_E(A) \subseteq S(A)$. The definitions of these sets [except for $S(A)$] are specific to the model considered in this chapter, since the cost of a path is fixed and the coalition that controls the shortest path for a route can serve all of the demand for that route.

Coalition size also influences the ability to control routes. If coalition A is a subset of coalition B, then coalition B has access to at least the same edges as coalition A. As a result, coalition B can serve all routes that coalition A can serve and may be able to serve additional routes. This implies that $S(A) \subseteq S(B)$, $S_E(A) \subseteq S_E(B)$, $C_E(A) \subseteq C_E(B)$, $C_S(A) \subseteq C_S(B)$. These properties show that enlarging a coalition cannot reduce its routing capabilities. Larger coalitions typically gain access to more feasible paths and may achieve lower transmission costs. Consequently, cooperation among service providers increases the number of routes that can be served and improves the coalition's ability to control communication services within the network, a phenomenon frequently observed in cooperative network models and communication routing games **(Ergün et al., 2017).** These control relationships will later be used to define the characteristic function that determines each coalition's value. To evaluate whether a coalition can economically serve or control a route, it is necessary to analyze the shortest paths between network nodes.

Table 4.1. The Set of Routes Economically Served and Controlled by Various Coalitions for the Network in Figure 4.1

A	$S_E(A)$	$C_E(A)$
ϕ	ϕ	ϕ
$\{P_1\}$	{(1,3), (1,4), (3,4)}	{(1,4), (3,4)}
$\{P_2\}$	{(1,2), (1,3), (2,3)}	{(1,2), (1,3), (2,3)}

$\{P_3\}$	{(2,4), (2,5), (4,5)}	{(2,4), (2,5), (4,5)}
$\{P_1, P_2\}$	{(1,2), (1,3), (1,4), (2,3), (3,4)}	{(1,2), (1,3), (1,4), (2,3), (3,4)}
$\{P_1, P_3\}$	$U \setminus$ {(1,2), (2,3)}	{(1,4), (1,5), (2,4), (2,5), (3,4), (3,5), (4,5)}
$\{P_2, P_3\}$	$U \setminus$ {(1,4), (3,4)}	{(1,2), (1,3), (2,3), (2,4), (2,5), (4,5)}
$\{P_1, P_2, P_3\} = P$	U	U

In network analysis, several extensions of classical centrality measures have been proposed to better capture the importance of edges within a network. For example, **(De Meo et al., 2012)** extend traditional path-based centrality by considering not only the shortest paths between nodes but also all paths whose lengths do not exceed a specified threshold. In this approach, the contributions of each path are weighted differently in the summation used to compute an edge's centrality. Such extensions build upon classical shortest-path centrality measures originally introduced in the network science literature **(Freeman, 1977).**

In the cooperative communication network considered in this study, coalitions aim to minimize the path lengths used to transmit information. Consequently, shortest-path-based measures indicate the strategic importance of edges in the network **(Freeman, 1977; Kivimäki et al., 2016).** To develop a measure that better captures the economic significance of an edge, the contribution of each route in the centrality calculation should rely on two factors: first, the maximum potential marginal profit derived from transmitting information along that route, denoted by $d_{i,j}(r_{i,j} - c_{i,j}^P)$; and second, the relative importance of an edge within the shortest path serving that route. One way to define the relative importance of an edge within a path is to assume that the contribution of an edge is proportional to its weight in the network graph. Since the total weight of the edges forming a shortest path between nodes *i* and *j* is constant and equal to $c_{i,j}^P$, this leads to the following measure of economic centrality:

$$C^R(e) = \sum_{i,j \in V} \frac{\sigma_{i,j}(e) d_{i,j}(r_{i,j} - c_{i,j}^P)\omega_e}{\sigma_{i,j}\, c_{i,j}^P}, \tag{4.2}$$

Here ω_e is the weight of edge *e*. The term $\sigma_{i,j}(e)$ is the number of shortest paths between nodes *i* and *j* that go through edge *e,* and $\sigma_{i,j}$ is the number of shortest paths between these nodes. We call this the 'proportionally weighted economic centrality'.

Note that it is assumed that when there are several shortest paths for a route, the profit from that route is shared equally among those routes.

An alternative approach assigns equal importance to all edges in a path by dividing the route's value equally among them. Let n_y denote the number of edges in a path y, and let $Y_{i,j}$ represent the set of shortest paths between nodes i and j. When each shortest path is weighted equally, the economic centrality measure can be defined as:

$$C^U(e) = \sum_{i,j \in V} \frac{1}{\sigma_{i,j}} \sum_{y \in Y_{i,j}} \frac{1\,(e \in y)\, d_{i,j}(r_{i,j} - c^P_{i,j})}{n_y}, \tag{4.3}$$

Here, $1\,(e \in y)$ is 1 if edge e is in path y, and 0 if not. This is called the 'uniformly weighted economic centrality.'

By construction, the total sum of the economic centrality measures equals the maximum profit that can be generated by the grand coalition (the set of all players). These measures quantify, for each player, the share of profit attributable to the edges they control. This property ensures that all profit available in the network is distributed across the edges based on the optimal routing solution. The economic centrality of a player is the sum of the centrality values for the edges that player controls. These measures can be interpreted as the share of the total profit that each player contributes to generating when the network operates under the optimal strategy of the grand coalition. Consequently, they provide a natural basis for dividing the total profit among the players. The relative economic power of a player can also be assessed by normalizing the player’s economic centrality by the total profit obtained by the grand coalition. Although these centrality measures are straightforward to compute, they reflect only the behavior of the grand coalition and the structure of the optimal routing solution. They do not explicitly capture the marginal contributions made by individual players when coalitions form incrementally. Therefore, while these measures provide useful insights into the structural importance of edges and players in the network, they do not fully represent the cooperative dynamics captured by solution concepts such as the Shapley value **(Shapley, 1953; Michalak et al., 2013).**

Remark 2. *It is also possible to construct related measures of economic centrality by modifying the set of shortest paths used in the calculations of Equations (4.2) and (4.3). For instance, one may restrict attention to paths that minimize the number of* *colors, or to paths that minimize the number of edges. Such variations allow the analysis to incorporate additional structural constraints on routing within the network.*

4.5 Characteristic Function under Linear Costs

The characteristic function of a cooperative game describes the payoff that each coalition of players can guarantee for itself. In cooperative game theory, the characteristic function assigns a value to every coalition representing the maximum payoff the coalition can secure, independent of the actions of players outside the coalition **(Shapley, 1953; Myerson, 1977).** In the communications network considered in this chapter, the value of a coalition depends on the routes it can serve and the cost advantage it has over competing coalitions. Because several coalitions may serve the same route, it is necessary to specify how competition affects the payoff from each route. Similar issues arise in cooperative network games where coalitions control network links and generate value through connectivity or routing efficiency **(Curiel, 1997; Skorin-Kapov & Skorin-Kapov, 2021).** In this section, two alternative formulations of the characteristic function are introduced.

The first formulation is based on a pessimistic estimate of the payoff that a coalition can secure. Under this approach, a coalition receives profit only from the routes that it strongly controls. Recall that a coalition *A* strongly controls a route (i, j) when it economically serves that route and no coalition disjoint from *A* can serve it. In this situation, the coalition faces no competition and therefore captures the entire profit generated by that route. The payoff obtained from such a route equals the revenue obtained from satisfying the demand minus the transmission cost required to serve that demand. Let $c_{i,j}^{A}$ denote the cost of the shortest path between nodes *i* and *j* using only the edges owned by coalition A. The payoff that coalition A can guarantee under this pessimistic assumption is, therefore,

$$v_1\,(A) = \sum_{(i,j):(i,j)\in C_S(A)} d_{i,j}\left[r_{i,j} - c_{i,j}^{A}\right]. \tag{4.4}$$

This formulation assumes that a coalition can safely rely only on the routes over which it has exclusive economic control. All other routes are ignored because they may be contested by competing coalitions.

The second formulation provides a more optimistic estimate of the payoff that a coalition can guarantee. In this case, the coalition earns profit not only from the routes it strongly controls but also from those it economically controls. Recall that a coalition economically controls a route when it can economically serve that route at a lower cost than any coalition disjoint from it. In this situation, the coalition with economic control over a route can reduce the price per unit of transmission to a level at which the competing coalition cannot make a profit from the route. This idea reflects standard competitive pricing logic in network service markets, where the lowest-cost provider captures demand **(Saad et al., 2009).** Let the effective reward per unit for coalition *A,* which has economic control over route (i, j), be denoted by $\tilde{r}^{A}_{i,j}$. This value is defined as $\tilde{r}^{A}_{i,j} = \min\{ r_{i,j}, c^{A^c}_{i,j} \}$, where $c^{A^c}_{i,j}$ denotes the cost of the cheapest path between nodes i and j using only edges belonging to the complement coalition A^c. This price adjustment ensures that the competing coalition cannot profitably serve the route. Under this assumption, the payoff that coalition *A* can ensure is given by

$$v_2\,(A) = \sum_{(i,j):(i,j) C_S(A)} d_{i,j}\left[r_{i,j} - c^{A}_{i,j}\right] + \sum_{(i,j):(i,j)\in C_E(A)\setminus[C_S(A)]} d_{i,j}\left[\tilde{r}^{A}_{i,j} - c^{A}_{i,j}\right]. \tag{4.5}$$

The first term represents the payoff from routes that coalition *A* strongly controls, while the second term represents the payoff from routes that coalition *A* economically controls through price competition.

It should be noted that when a route (i, j) belongs to $C_S(A)$, the coalition has exclusive control of that route. In this case, the adjusted reward equals the original reward, so $\tilde{r}^{A}_{i,j} = r_{i,j}$. Therefore, the second formulation of the characteristic function can also be written in a simpler form as

$$v_2\,(A) = \sum_{(i,j):(i,j)\in C_E(A)} d_{i,j}\left[\tilde{r}^{A}_{i,j} - c^{A}_{i,j}\right] \tag{4.6}$$

Finally, observe that if coalition *A* is a subset of coalition *B*, then the adjusted reward satisfies the relationship.

$$\tilde{r}^{A}_{i,j} \le \tilde{r}^{B}_{i,j} \le r_{i,j} \tag{4.7}$$

This property reflects the fact that as a coalition becomes larger, the cost incurred by the complementary coalition to serve a route cannot decrease. Similar monotonicity properties are common in cooperative network games where enlarging a coalition increases the set of feasible paths or services it can provide **(Curiel, 1997).** These formulations of the characteristic function provide the basis for evaluating the value generated by each coalition in the cooperative communication network game.

Remark 3. *From an economic perspective, the parameter $r_{i,j}$ can be interpreted as a price cap on the unit revenue obtained from transmitting information between nodes i and j. The model assumes that the demand associated with each route is fixed, at least in the short- to medium-term. When a coalition strongly controls a route, it faces no competition from other coalitions and therefore maximizes its profit by charging the highest feasible unit price, which is equal to the price cap $r_{i,j}$.*

When two opposing coalitions can serve the same route, the formulation of the value v_2 assumes a form of Bertrand competition **(Boone et al., 2012).** The coalition with the lower transmission cost captures the entire demand for that route. In practice, the winning coalition serves the route at a unit price equal to the transmission cost of the competing coalition or the price cap $r_{i,j}$, whichever is smaller. This pricing rule ensures that the competing coalition cannot profitably provide the service. As a result, the coalition with the lowest cost captures the demand, while the rival coalition receives no payoff from that route. This type of price competition is consistent with the logic of cost-based competition in networked markets **(Adler et al., 2020).**

Theorem 1. *The characteristic functions v_i, i=1,2, satisfy the following conditions:*

- **a)** $v_i\ (\phi) = 0$
- **b)** v_i *is superadditive. i.e.,* $v_i\ (A\ \cup B) \geq\ v_i\ (A) +\ v_i\ (B)$ *whenever* $A, B \subset P$ *and* $A\ \cap B = \phi$,
- **c)** v_i *(P) represents the maximal payoff that can be achieved by a single decision maker responsible for satisfying all demand for transmission across the entire network.*

PROOF. Condition a) follows directly from the definition of v_1 and v_2. Since the empty coalition contains no players, it controls no edges and cannot serve any route in the network. Therefore, it cannot obtain any profit, which implies that v_1 (ϕ) = 0 and v_2 (ϕ) = 0.

Condition b), suppose that $A, B \subset P \; and \; A \cap B = \phi$. The coalition ($A \cup B$) has access to all edges of A and all edges of B. Therefore, for any route (i, j), the cost of the shortest path under the merged coalition satisfies $c_{i,j}^{A \cup B} \leq \min\{ c_{i,j}^{A}, c_{i,j}^{B}\}$. This means that cooperation cannot increase the minimum transmission cost for any route. Moreover, any route that is strongly controlled by A or strongly controlled by B must also be strongly controlled by $A \cup B$; hence, $C_S(A) \cup C_S(B) \subseteq C_S(A \cup B)$. By definition, at most one coalition can strongly control a given route. Therefore, $C_S(A) \cap C_S(B) = \phi$. Since all summands in Equation (4.1) are non-negative, it follows that.

$$\begin{aligned} v_1 (A \cup B) &= \sum_{(i,j):(i,j)\in C_S(A\cup B)} d_{i,j}\left[r_{i,j} - c_{i,j}^{A \cup B}\right]. \\ &\geq \sum_{(i,j):(i,j)\in C_S(A)} d_{i,j}\left[r_{i,j} - c_{i,j}^{A \cup B}\right] + \sum_{(i,j):(i,j)\in C_S(B)} d_{i,j}\left[r_{i,j} - c_{i,j}^{A \cup B}\right] \\ &\geq \sum_{(i,j):(i,j)\in C_S(A)} d_{i,j}\left[r_{i,j} - c_{i,j}^{A}\right] + \sum_{(i,j):(i,j)\in C_S(B)} d_{i,j}\left[r_{i,j} - c_{i,j}^{B}\right] \\ &= v_1(A) + v_1(B). \end{aligned} \tag{4.9}$$

Therefore, v_1 is superadditive. A similar argument applies to v_2. When two disjoint coalitions merge, the resulting coalition has access to a larger set of edges, which can only increase the set of weakly controlled routes and can never increase the cost of the shortest path. Hence,

$$\begin{aligned} v_2 (A \cup B) &= \sum_{(i,j):(i,j)\in C_E(A\cup B)} d_{i,j}\left[\tilde{r}_{i,j}^{A \cup B} - c_{i,j}^{A \cup B}\right]. \\ &\geq \sum_{(i,j):(i,j)\in C_E(A)} d_{i,j}\left[\tilde{r}_{i,j}^{A \cup B} - c_{i,j}^{A \cup B}\right] + \sum_{(i,j):(i,j)\in C_E(B)} d_{i,j}\left[\tilde{r}_{i,j}^{A \cup B} - c_{i,j}^{A \cup B}\right] \\ &\geq \sum_{(i,j):(i,j)\in C_E(A)} d_{i,j}\left[\tilde{r}_{i,j}^{A} - c_{i,j}^{A}\right] + \sum_{(i,j):(i,j)\in C_E(B)} d_{i,j}\left[\tilde{r}_{i,j}^{B} - c_{i,j}^{B}\right] \\ &= v_2(A) + v_2(B). \end{aligned} \tag{4.10}$$

Therefore, both of the characteristic functions v_1 and v_2 are superadditive.

Condition c) follows from the fact that the grand coalition P contains all of the players. As a result, it controls all the network edges and can access every feasible path, allowing it to meet all network transmission demands at minimum cost. With no outside coalition to compete with, P captures the entire available network payoff. Therefore, $v_i(P)$ for $i = 1,2$ is the maximal payoff achievable by a single decision maker serving the entire network. This interpretation is standard in cooperative-game formulations of network value **(Ross, 2018; Borkotokey et al., 2014).**

4.5.1 Relation of the Characteristic Function v_2 with Second-Price Auctions

Here, we consider the three second-price auctions considered in Section 3.4. Suppose there are individual type 1 auctions for supplying the entire demand for transmission along each route. Assuming that there is a unique cheapest feasible path for a given route, the coalition that controls this path wins the corresponding auction, and the price set is the minimum of the maximum revenue, $r_{i,j}$, and the second-lowest price bid. Since the unit costs of supply are fixed and the capacities of edges are unlimited, the results of the corresponding type 3 auction would be the same. The winner of such an auction is willing to supply all of the demand for the route (i, j), and the result of an auction for a route has no effect on the bids of a coalition for other routes. It follows that the total profit of a coalition from such an auction is the sum of the profits from the auctions. This corresponds to the value function v_2. Splitting up the demand for each route into batches (a type 2 auction) does not change the overall result of such an auction, since the bids in a round of an auction would not change.

Differences between the value of a coalition according to v_2 and such an auction can only occur when two coalitions make the same price bid for a route. In this case, the value function v_2 assumes that neither coalition makes a profit on this route. This corresponds to the total profit obtained in such auctions when the price increment, ε, tends to zero.

4.6 Example of such Network Games

4.6.1 A Three-Player Cooperative Communication Network Game

This section applies the previously defined characteristic functions to the communication network in Figure 4.1. The five-node network with weighted edges controlled by three players is shown, with the edge weights indicating unit transmission costs. Equation 4.1 defines the reward matrix R and the demand matrix D, which specify transmission revenues and the demand for information flow between nodes.

Building on these matrices and the network's transmission costs, the next step is to determine the shortest paths between nodes for each coalition. The set of strongly controlled routes, $C_S(A)$, consists of routes for which coalition A controls an economically feasible path, while the coalition A^c does not control any feasible path for that route. The set of economically controlled routes, $C_E(A)$, includes all the routes that A can serve economically at a lower cost than the coalition A^c. The characteristic functions $v_1(A)$ and $v_2(A)$ are then computed using Equations (4.4) and (4.6). These are presented in Table 4.2.

Table 4.2 Characteristic Functions for the Communication Network Game

A	ϕ	$\{P_1\}$	$\{P_2\}$	$\{P_3\}$	$\{P_1, P_2\}$	$\{P_1, P_3\}$	$\{P_2, P_3\}$	$\{P\}$
$v_1(A)$	0	0	0	28	54	76	78	110
$v_2(A)$	0	20	30	48	54	76	78	110

The values shown in Table 4.2 represent the maximum payoff that each coalition can guarantee under the pessimistic characteristic function v_1 and the optimistic characteristic function v_2. The difference between the two functions arises from how they treat routes that are weakly controlled by a coalition. Under the pessimistic approach, only routes that are strongly controlled generate profit, whereas under the optimistic approach, coalitions may also obtain profits from routes where they have a cost advantage over competing coalitions. After computing the characteristic functions, the next step is to determine how to distribute the total profit generated by the grand coalition among the players. Let x_i be the payoff given to player i. A distribution is efficient and stable if it meets two conditions.

First, the total payoff distributed among all players must equal the value of the grand coalition.

$$\sum_{i=1}^{K} x_i = v(P), \tag{4.11}$$

Second, the distribution must be such that, for any coalition A of players, the sum of their assigned payoffs is at least as large as the maximum payoff this coalition could guarantee if it acted independently. This disincentivizes any subset of players from leaving the grand coalition.

$$\sum_{i:i \in A} x_i \geq v(A), where\ A \subset \{1,2,\dots,K\}$$

(4.12)

These conditions ensure that the payoff allocation is both efficient and stable. Efficiency guarantees that the total profit generated by the network is fully distributed among the players, while coalition rationality ensures that no subset of players would benefit by leaving the grand coalition and acting independently. The set of allocations that satisfy these conditions forms the core of the cooperative game and represents stable profit distributions among the network providers. The set of all payoff divisions that satisfy the efficiency and coalition-rationality conditions is called the game's core. Each element of the core distributes the payoff from the grand coalition among the players so that no smaller coalition receives less than the value it can guarantee itself. In other words, any allocation belonging to the core ensures that no group of players has an incentive to leave the grand coalition and form a separate coalition.

Several approaches can be used to select a specific payoff allocation that satisfies the efficiency condition. One widely used solution concept is the Shapley value **(Shapley, 1953).** The Shapley value assigns to each player a payoff equal to the expected marginal contribution that player would make if joining a coalition. More precisely, the marginal contribution of player P_i to a coalition A is defined as the increase in the value of a coalition A not including P_i when P_i joins that coalition, i.e., $v(A \cup \{P_i\}) - v(A)$. The Shapley value is the average marginal contribution of a player, calculated by considering all possible orders in which players may join a coalition, each occurring with equal probability ($1/K!$, where K is the number of players). This rule gives an efficient split of the total payoff among players based on what each adds to the group result. Tables 4.3 and 4.4 show how each player's marginal contribution is calculated for all the possible ways in which players can join the grand coalition.

Table 4.3 Marginal Gains from Entry and Derivation of the Shapley Values for Characteristic Function v_1

Order	$v_1(A \cup \{P_1\}) - v_1(A)$	$v_1(A \cup \{P_2\}) - v_1(A)$	$v_1(A \cup \{P_3\}) - v_1(A)$
1, 2, 3	0	54	56
1, 3, 2	0	34	76
2, 1, 3	54	0	56
2, 3, 1	32	0	78
3, 1, 2	48	34	28

3, 2, 1	32	50	28
Shapley value	$\frac{83}{3} \approx 27.67$	$\frac{86}{3} \approx 28.67$	$\frac{161}{3} \approx 53.67$

Table 4.4 Marginal Gains from Entry and Derivation of the Shapley Values for Characteristic Function v_2

Order	$v_2(A \cup \{P_1\}) - v_2(A)$	$v_2(A \cup \{P_2\}) - v_2(A)$	$v_2(A \cup \{P_3\}) - v_2(A)$
1, 2, 3	20	34	56
1, 3, 2	20	34	56
2, 1, 3	24	30	56
2, 3, 1	32	30	48
3, 1, 2	28	34	48
3, 2, 1	32	30	48
Shapley value	$\frac{156}{6} \approx 26$	$\frac{192}{6} \approx 32$	$\frac{312}{6} \approx 52$

The results presented in Tables 4.3 and 4.4 show that the Shapley values obtained under the two characteristic functions are relatively similar. This indicates that the allocation of payoffs among players is not significantly affected by the assumptions used to define the characteristic function. Nevertheless, some differences in the distribution of payoffs remain. The characteristic function v_1 emphasizes strong control over routes. In the network considered, Player 3 is the only provider capable of serving transmissions to node 5. This exclusive access gives Player 3 a strategic advantage within the network. As a result, Player 3 receives a larger share of the total payoff under both Shapley value calculations, although this dominance is more pronounced when the allocation is based on v_1.

In contrast, the characteristic function v_2 gives greater weight to efficiency when coalitions compete to serve the same route. In the example network, Players 1 and 2 compete to serve the route between nodes 1 and 3. The Shapley value based on v_1 allocates nearly equal payoffs to these players. However, the Shapley value based on v_2 assigns a slightly larger share to Player 2. This happens because Player 2 can serve the route between nodes 1 and 3 at a lower transmission cost than Player 1. It is relatively easy to verify that these payoff divisions belong to the core of the appropriately defined cooperative game.

It should be noted that linear programming can be used to determine whether the cooperative game based on the characteristic function v_1 is balanced. Since the coalitions that include just Player 1 or just Player 2 do not achieve a positive payoff, it can be assumed that weights y_1, y_2, y_3, y_4 are assigned to the coalitions $\{P_3\}$, $\{P_1, P_2\}$, $\{P_1, P_3\}$, and $\{P_2, P_3\}$, respectively. It follows that the appropriate linear programming problem used to determine balancedness can be defined as

$$\max 28y_1 + 54y_2 + 76y_3 + 78y_4$$

subject to

$$\begin{gathered} y_2 + y_3 \leq 1 \\ y_2 + y_4 \leq 1 \\ y_1 + y_3 + y_4 \leq 1. \end{gathered}$$

The i-th constraint states that the sum of the weights for the coalitions containing Player i cannot exceed 1. The optimal value of 104 is obtained by setting $y_2 = y_3 = y_4 = 1/2$ and $y_1 = 0$. Since this optimal value does not exceed the value obtained by the grand coalition, the game is balanced, and thus the core is non-empty.

To show that the cooperative game based on the characteristic function v_2 is balanced, a weight must be assigned to every coalition that is a proper subset of the grand coalition $\{P_1, P_2, P_3\}$. Let $y_1, y_2, y_3, y_4, y_5, y_6$ denote the weights assigned to the coalitions $\{P_1\}, \{P_2\}, \{P_3\}, \{P_1, P_2\}, \{P_1, P_3\}, \{P_2, P_3\}$, respectively. If the values of these coalitions v_2 are denoted by

$$v_2(\{P_1\}), v_2(\{P_2\}), v_2(\{P_3\}), v_2(\{P_1, P_2\}), v_2(\{P_1, P_3\}), v_2(\{P_2, P_3\}),$$

then the corresponding linear programming problem can be formulated as

$$\max \quad 20y_1 + 30y_2 + 48y_3 + 54y_4 + 76y_5 + 78y_6$$

subject to

$$\begin{gathered} y_1 + y_4 + y_5 \leq 1 \\ y_2 + y_4 + y_6 \leq 1 \\ y_3 + y_5 + y_6 \leq 1. \end{gathered}$$

and

$$y_i \geq 0, i = 1,2,3,4,5,6$$

The i-th constraint ensures that the total weight assigned to coalitions containing Player i does not exceed 1. The optimal value of 106 is obtained by setting $y_2 = y_5 = 1$ and $y_1 = y_3 = y_4 = y_6 = 0$. Since this optimal value does not exceed the value obtained by the grand coalition, the game is balanced, and thus the core is non-empty.

It follows that the Shapley value is admissible under both of these formulations. To further interpret these results, the Shapley values are compared with the measures of economic centrality defined earlier in Equations (4.2) and (4.3). Table 4.5 presents the calculation of the two economic centrality measures for each edge in the network. In this example, there exists a unique shortest path between every pair of nodes. Under this condition, it is possible to define a binary transmission matrix T of dimension $0.5J(J-1) \times M$. In this matrix, the element in row i and column j takes the value 1 if the edge corresponding to column j lies on the shortest path for the route represented by row i. Otherwise, the entry is 0. The construction of this matrix is illustrated in Table 4.5.

Table 4.5 Shortest Routes between Nodes and Corresponding Marginal Profits: 1 indicates that an Edge belongs to a given Shortest Route.

Route	**Marginal Profit**	**Edge**					
		(1,2)	**(1,4)**	**(2,3)**	**(2,4)**	**(3,4)**	**(4,5)**
(1,2)	10	1	0	0	0	0	0
(1,3)	4	1	0	1	0	0	0
(1,4)	10	0	1	0	0	0	0
(1,5)	4	0	1	0	0	0	1
(2,3)	20	0	0	1	0	0	0
(2,4)	20	0	0	0	1	0	0
(2,5)	8	0	0	0	1	0	1
(3,4)	10	0	0	0	0	1	0
(3,5)	4	0	0	0	0	1	1
(4,5)	20	0	0	0	0	0	1

The economic centrality measures defined in Section 4.4 can be calculated using the transmission matrix shown in Table 4.5. Using this information together with the marginal profit values for each route, the economic centrality of each edge can be determined.

To illustrate the calculation, consider the edge (1,2). The uniform measure of economic centrality is obtained by summing the ratios of the marginal profit to the number of edges in the corresponding shortest path for all routes whose shortest path includes edge (1,2). From Table 4.5, edge (1,2) appears in the shortest paths for routes (1,2) and (1,3). The marginal profit for route (1,2) is 10, and the shortest path contains one edge, while the marginal profit for route (1,3) is 4, and the shortest path contains two edges. Therefore, the uniform measure of economic centrality for edge (1,2) is $C^U(1,2) = 10 + 4/2 = 12$. The proportional measure of economic centrality can be computed similarly. In this case, the marginal profit is multiplied by the edge weight and divided by the total length of the shortest path associated with the route. For edge (1,2), the shortest path for route (1,2) has a total length equal to the weight of edge (1,2), while the shortest path for route (1,3) consists of edges (1,2) and (2,3) with a total length equal to 7. Therefore, the proportional measure of economic centrality for edge (1,2) is $C^R(1,2) = 10 + (4 \times 4)/7 = 86/7 = 12.2857$.

The same procedure can be applied to compute the economic centrality of all edges in the network. The resulting values of both the uniform and proportional economic centrality measures are presented in Table 4.6. These values reflect the relative importance of each edge in generating profit when the network operates under the optimal routing strategy of the grand coalition.

Table 4.6 Measures of the Economic Centrality of Edges

	Edge					
	(1,2)	**(1,4)**	**(2,3)**	**(2,4)**	**(3,4)**	**(4,5)**
Uniform	12	12	22	24	12	28
Proportional	12.2857	12.2857	21.7143	24	12.2857	27.4286

Since each edge in the network corresponds to a particular player, the contribution of a player to the overall network profit is determined by summing the centrality measures of all the edges owned by that player. The resulting economic centrality values for the players are presented in Table 4.7.

According to the construction described above, if a coalition A possesses all the edges that form the unique shortest path between two nodes, then the entire profit associated with that route is attributed to the members of coalition A. This occurs because the coalition fully controls the transmission path that minimizes the cost of serving that route. Consequently, the coalition captures the full marginal profit from that route under the grand coalition's optimal routing strategy.

Table 4.7 Measures of the Economic Centrality of the Players

	Player 1	Player 2	Player 3
Uniform	24	34	52
Proportional	24.571	34	51.4286

The economic centrality measures obtained in this example are very close to the Shapley values corresponding to the characteristic function v_2. This similarity reflects the fact that both approaches capture the relative importance of players in generating profit within the network. The centrality measures assess the structural contributions of edges and players to the optimal routing configuration, whereas the Shapley value assesses the marginal contribution of each player during coalition formation **(Shapley, 1953).**

For the network considered in this example, it is relatively simple to show that the resulting payoff allocations belong to the core of both of the cooperative games defined by the characteristic functions v_1 and v_2. The core is a set of allocations where no subset of players (no coalition) can achieve a better outcome by breaking away from the full group (the grand coalition). Consequently, the profit allocations satisfy both efficiency (all value is allocated) and coalitional rationality (no sub-coalition is better off alone) conditions. This implies that no subset of players can obtain a higher payoff by leaving the grand coalition and forming a separate coalition.

However, this property does not necessarily hold for all communication networks. In some cases, the payoff allocation derived from economic centrality measures or from the Shapley value may lie outside the core (set of stable allocations). This occurs when the allocation violates the coalitional rationality constraints for some coalition of players. Such situations arise when the cooperative game lacks properties such as convexity (where contributions increase as coalitions grow) or balancedness (where each player can invest a certain amount in coalitions), which are needed to guarantee the existence of stable allocations **(Owen, 2013).** To illustrate this point, the next example presents a network in which the payoff allocations derived from centrality measures fall outside the core.

4.6.2 On the admissibility of Centrality-Based Profit Allocations

Consider the game played on the network illustrated in Figure 4.2. The network consists of four nodes and two players. Player 1, represented by the red edges, controls the links (1,2), (1,4), and (4,3), while Player 2, represented by the blue edge, controls the link (2,3). It is assumed that the only demand for data transmission is between nodes 1 and 4, and that the reward for transmitting 1 unit of data along this route is 10.

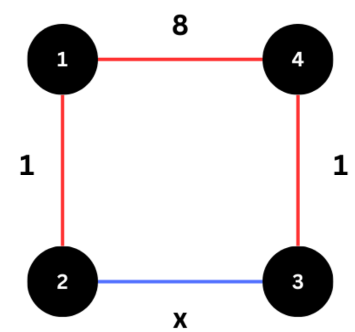


Figure 4.2 Network Game in which the Centrality Measures do not give an Admissible Solution

In this network, Player 1 strictly controls the route from node 1 to node 4. This is the only profitable route. Acting alone, Player 1 can serve this route directly through edge (1,4), which costs 8. So, Player 1's profit when acting alone is $v_1(\{P_1\}) = v_2(\{P_1\}) = 10 - 8 = 2$. Player 2 cannot serve the route between nodes 1 and 4 independently, since it controls only edge (2,3). Therefore, $v_1(\{P_2\}) = v_2(\{P_2\}) = 0$. When the two players cooperate, the grand coalition can use the alternative path $1 \to 2 \to 3 \to 4$. The cost of this route is $1 + x + 1 = x + 2$. If $x < 6$, this path is cheaper than the direct edge (1,4) with cost 8. In that case, cooperation reduces transmission costs and increases total payoffs. The value of the grand coalition is therefore $v_1(P) = v_2(P) = 10 - (x + 2) = 8 - x$. Since the payoff to the grand coalition is greater than the sum of the payoffs to the individual players, it follows that this game is balanced. The Shapley value for this game is obtained by averaging the marginal contributions of the two players over all possible orders of coalition formation. This gives the allocation $(x_1, x_2) = (5 - \frac{x}{2}, 3 - \frac{x}{2})$. This allocation is admissible because it respects each player's individual value and satisfies the game's stability conditions.

Next, consider the payoff allocation based on the uniform measure of economic centrality. In the grand coalition, the shortest path from node 1 to node 4 is $1 \to 2 \to 3$

→ 4, which contains 3 edges. Since Player 1 controls two of these three edges, the uniform measure assigns to Player 1 two-thirds of the total profit. Thus,

$$C^U(P_1) = \frac{2(8-x)}{3}$$

This allocation fails when Player 1 gets less than its solo guarantee of 2. So, for admissibility: $\frac{2(8-x)}{3} \geq 2$ solving gives $x \leq 5$. For $5 < x < 6$, the uniform economic centrality allocation is inadmissible.

Now consider the proportional measure of economic centrality. Here, profit is allocated according to the relative weights of the edges in the shortest path. In the cooperative path 1 → 2 → 3 → 4, Player 1 controls two edges of weight 1 each. Player 2 controls one edge of weight x. Thus, Player 1 receives 2 / (2 + x) of the total profit, which gives,

$$C^R(P_1) = \frac{2(8-x)}{2+\ x}$$

Player 1 gets less as x increases, and the share is inadmissible if Player 1 gets less than 2. For admissibility: $\frac{2(8-x)}{(2+x)} \geq 2$ solving this inequality, $(2/(2 + x)) \times (10 - (x + 2)) \geq 2$, gives $x \leq 4$. Therefore, when $4 < x < 6$, the payoff allocation based on the proportional economic centrality measure is inadmissible.

These results indicate that the economic centrality measures discussed above provide only a partial representation of a player's strategic strength in a communication network game. While these measures capture the structural importance of edges within the optimal routing configuration, they do not fully account for the strategic interactions that arise when players form or dissolve coalitions **(Algaba et al., 2019).** In the present framework, the flow of information through the network is determined endogenously by the players through their cooperative decisions. Consequently, changes in the coalition structure may significantly alter the routing of information across the network. When players form new coalitions, alternative transmission paths may become available, thereby altering both the optimal flow configuration and the associated profits, as illustrated in the previous example.

In contrast, the model considered in **Algaba et al., 2019,** assumes that the flows along network edges are fixed. In that setting, traffic routing is determined externally by users of the transportation or communication system rather than by the network infrastructure managers. As a result, the players are only interested in the allocation of costs or benefits associated with predetermined flows. In the model presented here, a change in the coalition structure can result in a radical change in the underlying flows. This fundamental difference explains why centrality measures play a different role in the two models and why they may fail to fully capture the strategic power of players in the network game analyzed in this chapter.

This example demonstrates that centrality-based allocation rules can fail to produce admissible payoff divisions. For instance, a player who adds little profit to the grand coalition may still get a large payoff because of its strategic position in the routing configuration. This occurs when a player controls a crucial edge but otherwise has a weak role in the network. This example confirms that although economic centrality measures are simple and intuitive, they do not always meet the stability requirements of cooperative game-theoretic solution concepts, even when the underlying game is balanced.

4.6.3 Bridge Network Cooperative Game

Bridges frequently appear in real communication and transportation networks. A bridge is an edge whose removal separates the network into two disconnected components. Such edges often play a strategically important role because they connect otherwise independent subnetworks. For instance, two countries may operate their own internal communication infrastructures, but the networks may be connected through a single transmission link. In such cases, the bridge becomes a critical element of the network structure and may significantly influence the distribution of profits among the players controlling the network.

The presence of bridges also simplifies the analysis of network games. Since a bridge separates the network into two components, it is often possible to study each component independently and then analyze the role of the bridge that connects them. Figure 4.3 illustrates such a network structure. The network consists of two subnetworks connected by a single bridge between nodes 1 and 5. The left subnetwork contains nodes {1,2,3,4}, and the right subnetwork contains nodes {5,6,7,8}. Each subnetwork is controlled by a different player, while the bridge connects them.

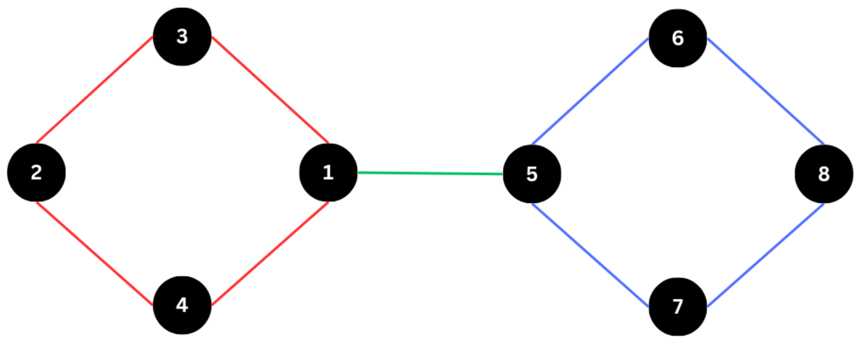


Figure 4.3 Two Subnetworks Connected by a Bridge

To illustrate different strategic implications, we consider three possible ownership structures for the bridge connecting the two subnetworks:

1. The bridge is owned by a third player, separate from the owners of the two subnetworks. This situation corresponds to the case in which an independent provider controls the link connecting the two networks.
2. The bridge is owned by one of the two players that control the subnetworks. In this case, the player who owns the bridge has a strategic advantage because it controls access between the two network components.
3. The bridge is jointly owned by both players controlling the subnetworks. In this situation, the bridge effectively represents shared infrastructure. The two players cooperate in operating this link and therefore form a coalition with respect to the bridge. The additional profit from introducing the bridge is assumed to be divided equally between them, or, more generally, according to their ownership shares.

The first two cases can be analyzed using the standard cooperative game-theoretic framework introduced earlier in this chapter. The third case implicitly assumes that the two players form a coalition because they jointly control the bridge. In this case, the marginal gain generated by adding the bridge is shared between the two players according to the specified ownership structure. In the first two cases, the bridge means that any coalition controlling a route between subnetworks must possess it. This means that any control over economically viable routes between subnetworks must be strong. As a result, the characteristic functions v_1 and v_2 defined earlier become identical in this network. In the rest of this example, this shared characteristic function will simply be called v.

First, consider the case where a third party owns the bridge connecting the subnetworks. **In Figure 4.3,** Player 1 (red) controls all routes within the left subnetwork. Player 2 (blue) controls all routes within the right subnetwork. Player 3 (green) controls the bridge. Let v_L denote the optimal profit obtainable from controlling the left subnetwork, v_R denote the optimal profit from the right subnetwork, and v_B denote the optimal profit from the bridge. Hence, $v(\{P_1\}) = v_L$, $v(\{P_2\}) = v_R$, and $v(\{P_3\}) = v_B$. The coalition $\{P_1, P_2\}$ controls all routes within the two internal subnetworks, so its value is $v(\{P_1, P_2\}) = v_L + v_R$. Coalition $\{P_1, P_3\}$ controls three types of routes:

1. Routes contained within the internal network on the left side of the bridge.
2. The bridge itself.
3. Routes from the node on the right side of the bridge to the nodes within the left internal network, excluding the left endpoint of the bridge, in order to keep the route sets mutually exclusive.

Let the optimal payoff obtained from the third group of routes be denoted by v_{BL}. The value of coalition $\{P_1, P_3\}$ is therefore $v(\{P_1, P_3\}) = v_L + v_B + v_{BL}$. By a similar argument, coalition $\{P_2, P_3\}$ controls routes within the right internal network, the bridge, and routes from the left side of the bridge to nodes in the right internal network. Let v_{BR} denote the optimal payoff for routes connecting the left side of the bridge to the right internal network. Thus, $v(\{P_2, P_3\}) = v_R + v_B + v_{BR}$.

Finally, the grand coalition $\{P_1, P_2, P_3\}$ controls all possible routes in the network. These routes can be divided into the four sets described above, together with an additional set comprising routes between nodes on opposite sides of the bridge that are not directly adjacent to it. Let the optimal payoff obtained from this final set of routes be denoted by v_{LR}. The value of the grand coalition is therefore $v(\{P_1, P_2, P_3\}) = v_L + v_R + v_B + v_{BL} + v_{BR} + v_{LR}$. The Shapley value calculation for this case is presented in Table 4.8.

It should be noted that the Shapley value defined in this way implicitly takes into account the synergy between members of a coalition. Players 1 and 2 are not directly connected in the relational graph describing the relative positions of the players in the overall network. It follows that, in combination, these two players only control the routes in the network that one of them has individual control over. Hence, the payoff of the coalition $\{P_1, P_2\}$ is the sum of the values of the coalitions $\{P_1\}$ and $\{P_2\}$. It follows that within the framework of the games presented in this chapter, the Shapley value corresponds to the Myerson value.

Table 4.8 Marginal Gains from Entry and Derivation of the Shapley Values for the Network Game Based on Figure 4.3

Order	$v(A \cup \{P_1\}) - v(A)$	$v(A \cup \{P_2\}) - v(A)$	$v(A \cup \{P_3\}) - v(A)$
1, 2, 3	v_L	v_R	$v_B + v_{BL} + v_{BR} + v_{LR}$
1, 3, 2	v_L	$v_R + v_{BR} + v_{LR}$	$v_B + v_{BL}$
2, 1, 3	v_L	v_R	$v_B + v_{BL} + v_{BR} + v_{LR}$
2, 3, 1	$v_L + v_{BL} + v_{LR}$	v_R	$v_B + v_{BR}$
3, 1, 2	$v_L + v_{BL}$	$v_R + v_{BR} + v_{LR}$	v_B
3, 2, 1	$v_L + v_{BL} + v_{LR}$	$v_R + v_{BR}$	v_B
Shapley value	$v_L + \frac{v_{BL}}{2} + \frac{v_{LR}}{3}$	$v_R + \frac{v_{BR}}{2} + \frac{v_{LR}}{3}$	$v_B + \frac{v_{BL} + v_{BR}}{2} + \frac{v_{LR}}{3}$

The S-value offers another way to evaluate how payoffs are shared in a network game by taking into account how players are connected when forming coalitions. The S-value, inspired by the Myerson approach, is found by calculating the expected extra value each player brings when the first coalition member is picked at random. After that, new players join by being randomly chosen from those linked to someone already in the coalition. This continues until all players are included. While the Shapley value treats all coalition orders as equally likely, the S-value gives different probabilities to each sequence, depending on how the network is connected.

In the three-player game shown in Figure 4.3, the way players are connected changes how likely different coalition orders are. If Player 1 is picked first, which happens with a 1/3 chance, then Player 3 and Player 2 must join in that order based on the network. Analogously, if Player 2 is chosen first, then Player 3 must be the second to join the coalition. But if Player 3 is picked first, both remaining players are connected to Player 3, so each has a 1/2 chance of being chosen next. To find the S-value, you need to consider each player's marginal contribution and the probability of each possible coalition order. Table 4.9 lists all the possible coalition orders and their probabilities. The S-values are given in Table 4.10.

Table 4.9 Probabilities of Various Sequences in Coalition Formation According to Figure 4.3

Sequence	$(\{P_1, P_3, P_2\})$	$(\{P_2, P_3, P_1\})$	$(\{P_3, P_1, P_2\})$	$(\{P_3, P_2, P_1\})$
Probability	1/3	1/3	1/6	1/6

Table 4.10 Marginal Gains from Entry and Derivation of the S-values

Order	**Probability.**	$\boldsymbol{v(A \cup \{P_1\}) - v(A)}$	$\boldsymbol{v(A \cup \{P_2\}) - v(A)}$	$\boldsymbol{v(A \cup \{P_3\}) - v(A)}$
P_1, P_3, P_2	1/3	v_L	$v_R + v_{BR} + v_{LR}$	$v_B + v_{BL}$
P_2, P_3, P_1	1/3	$v_L + v_{BL} + v_{LR}$	v_R	$v_B + v_{BR}$
P_3, P_1, P_2	1/6	$v_L + v_{BL}$	$v_R + v_{BR} + v_{LR}$	v_B
P_3, P_2, P_1	1/6	$v_L + v_{BL} + v_{LR}$	$v_R + v_{BR}$	v_B
S-value	-	$v_L + \frac{2v_{BL}}{3} + \frac{v_{LR}}{2}$	$v_R + \frac{2v_{BR}}{3} + \frac{v_{LR}}{2}$	$v_B + \frac{v_{BL} + v_{BR}}{3}$

Next, consider the situation in which the bridge is owned by one of the players controlling an internal network. Without loss of generality, assume that Player 1 owns the bridge. In this case, Player 1 effectively acts as a coalition consisting of Players 1 and 3 from the previous scenario. Consequently, $v(\{P_1\}) = v_L + v_B + v_{BL}$, $v(\{P_2\}) = v_R$, and the value of the grand coalition becomes $v(\{P_1, P_2\}) = v_L + v_R + v_B + v_{BL} + v_{BR} + v_{LR}$. The Shapley value for this network configuration is illustrated in Table 4.11.

Table 4.11 Marginal Gains from Entry and Derivation of the Shapley Values for the Network Game Based on Figure 4.3 when Player 1 owns the Bridge.

Order	$\boldsymbol{v(A \cup \{P_1\}) - v(A)}$	$\boldsymbol{v(A \cup \{P_2\}) - v(A)}$
1, 2	$v_L + v_B + v_{BL}$	$v_R + v_{BR} + v_{LR}$
2, 1	$v_L + v_B + v_{BL} + v_{BR} + v_{LR}$	v_R
Shapley value	$v_L + v_B + v_{BL} + \frac{v_{BR} + v_{LR}}{2}$	$v_R + \frac{v_{BR} + v_{LR}}{2}$

Finally, consider the case in which the bridge is jointly and equally owned by the two players controlling the internal subnetworks. In this situation, the bridge represents shared infrastructure between the two players. The additional profit created by introducing the bridge into the network is therefore divided equally between them. The marginal gain from adding the bridge consists of the profit from the bridge itself, plus the profits from the routes that become feasible once the bridge connects the two subnetworks. This marginal gain is equal to $v_B + v_{BL} + v_{BR} + v_{LR}$. Since the bridge is jointly owned, this additional value is split equally between the two players. Consequently, the values assigned to the players in this game are

$$x_1 = v_L + \frac{v_B + v_{BL} + v_{BR} + v_{LR}}{2}\,; \qquad x_2 = v_R + \frac{v_B + v_{BL} + v_{BR} + v_{LR}}{2}\,. \tag{4.13}$$

Thus, each player receives the value generated by the routes within its own internal network, together with half of the additional profit created by the bridge and the routes that connect the two subnetworks. This allocation reflects the shared ownership of the bridge and the equal contribution of both players to maintaining the connection between the two network components.

4.6.4 Analysis of the Bridge Network Game

This section further examines the implications of subsection 4.6.3 by analyzing how different bridge ownership structures influence the distribution of profits in the network. The example illustrates that a bridge can play a crucial strategic role in cooperative communication networks by connecting two otherwise independent subnetworks. By changing the ownership of the bridge, the players' bargaining power and their contributions to the network can change significantly. The results obtained in the previous example show that the allocation of profits depends not only on the structure of the network but also on the way critical infrastructure, such as bridges, is owned and managed. In the following discussion, we analyze the payoff allocations under different ownership scenarios and highlight how the bridge's position affects the relative economic power of the players in the network.

We now consider the network shown in Figure 4.3 under the following assumptions. First, each edge has unit weight 1. Second, the reward for any internal connection, namely a route that does not cross the bridge, is equal to 3, while the reward for any external connection, including the connection between nodes 1 and 5, is equal to 6. Third, the demand for each internal connection and for the route between nodes 1 and 5 is 10, whereas the demand for each external connection is 5.

We first derive the values of the main route groups introduced earlier. To compute v_L, observe that there are six pairs of nodes in the left subnetwork. For four of these pairs, the shortest distance is 1, while for the remaining two pairs, the shortest distance is 2. Hence,

$$v_L = \sum_{1 \le i < j \le 4} d_{i,j}\left(r_{i,j} - c_{i,j}\right) = 4 \times 10(3-1) + 2 \times 10(3-2) = 100.$$

By symmetry, the same argument applies to the right subnetwork, so v_R = 100. Next, the value generated by the bridge itself is obtained from the route between nodes 1 and 5. Since the demand is 10, the reward is 6, and the bridge has a unit cost of 1, we obtain $v_B = 10(6 - 1) = 50$. To calculate v_{BL}, consider the routes from node 5 to nodes 2, 3, and 4 in the left subnetwork. The distance from node 5 to node 2 is 3, while the distances from node 5 to nodes 3 and 4 are both 2. Therefore,

$$v_{BL} = \sum_{i=2}^{4} d_{i,5}\left(r_{i,5} - c_{i,5}\right) = 5(6-3) + 2 \times 5(6-2) = 55.$$

By symmetry, v_{BR} = 55. Finally, we determine v_{LR}, which corresponds to routes between nodes in different subnetworks that are not directly adjacent to the bridge. There are nine such pairs, formed by taking one node from the set $\{N_2, N_3, N_4\}$ and one node from the set $\{N_6, N_7, N_8\}$. The route between nodes 2 and 8 has the shortest distance, 5. The routes (2,6), (2,7), (3,8), and (4,8) each have the shortest distance of 4. The routes (3,6), (3,7), (4,6), and (4,7) each have the shortest distance 3. It follows that

$$v_{LR} = \sum_{i=2}^{4}\sum_{j=6}^{8} d_{i,j}\left(r_{i,j} - c_{i,j}\right) = 5(6-5) + 4 \times 5(6-4) + 4 \times 5(6-3) = 105.$$

Using these values, we now evaluate the Shapley allocations for the alternative bridge ownership structures.

First, suppose the bridge is owned by a third party, namely Player 3. Based on the Shapley value calculation for the three-player bridge game, the resulting allocation is $x_1 = 162.5$, $x_2 = 162.5$, $x_3 = 140$. This result shows that although Player 3 owns only one edge, the edge is strategically critical because it connects the two subnetworks. As a result, Player 3 receives a substantial share of the total payoff.

Second, suppose the bridge is owned by Player 1. In this case, the game reduces to a two-player setting in which Player 1 controls both the left subnetwork and the bridge. The corresponding Shapley values are $x_1 = 285$, $x_2 = 180$. Although Player 1 in this case may be viewed as combining the roles of Players 1 and 3 from the previous three-player game, the payoff received by Player 1 is smaller than the sum of those two earlier Shapley values. This suggests that forcing the bridge owner and the left network owner into a single player reduces their flexibility in coalition formation and slightly strengthens Player 2's relative position.

Finally, consider the case in which the bridge is jointly and equally owned by the two local network operators. Under the equal sharing rule for the bridge's marginal gain, the value assigned to each player is $x_1 = x_2 = 232.5$.

We now compare the Shapley value allocations obtained earlier with those derived from the edges' economic centrality. Because all edges in the network have the same weight, the proportional and uniform measures of economic centrality coincide. This simplifies the analysis, since the relative contribution of each edge depends only on the network's shortest-path structure. From the symmetry of the network, it follows that the economic centrality values for several edges are identical. In particular, $C^R(2,3) = C^R(2,4) = C^R(6,8) = C^R(7,8)$. To illustrate the calculation, consider edge (2,3). This edge appears in several shortest paths between pairs of nodes in the network. Specifically, edge (2,3):

a) lies on one of the two shortest paths between nodes 1 and 2, each of length 2.
b) forms the unique shortest path between nodes 2 and 3, of length 1.
c) lies on one of the two shortest paths between nodes 2 and 5, each of length 3.
d) lies on one of the two shortest paths between nodes 2 and 6, each of length 4.
e) lies on one of the two shortest paths between nodes 2 and 7, each of length 4.
f) lies on two of the four shortest paths between nodes 2 and 8, each of length 5.
g) lies on one of the two shortest paths between nodes 3 and 4, each of length 2.

Using Equation (4.1), the economic centrality of edge (2,3) is obtained by summing the weighted marginal profits associated with each of these routes. Carrying out this calculation yields $C^R(2,3) = 30.5$. By symmetry, the same value applies to edges (2,4), (6,8), and (7,8).

Next, consider edge (1,3). This edge also participates in several shortest paths. In addition to the paths listed above for edge (2,3), edge (1,3) also appears in the following routes:

a) one of the two shortest paths between nodes 1 and 2, each of length 2,
b) the unique shortest path between nodes 1 and 3, of length 1,
c) one of the two shortest paths between nodes 2 and 5, each of length 3,
d) one of the two shortest paths between nodes 2 and 6, each of length 4,
e) one of the two shortest paths between nodes 2 and 7, each of length 4,

f) two of the four shortest paths between nodes 2 and 8, each of length 5,
g) one of the two shortest paths between nodes 3 and 4, each of length 2,
h) the unique shortest path between nodes 3 and 5, of length 2,
i) the unique shortest path between nodes 3 and 6, of length 3,
j) the unique shortest path between nodes 3 and 7, of length 3,
k) one of the two shortest paths between nodes 3 and 8, each of length 4.

Applying Equation (4.1) to these contributions gives $C^R(1,3) = 53$. The economic centrality of edge (1,5), which corresponds to the bridge connecting the two subnetworks, can be calculated in a similar way. Alternatively, we can use the fact that the sum of the economic centrality values over all edges equals the network's total maximum profit, which is the sum of the Shapley values. In this example, the total profit equals 465. Therefore, $C^R(1,5) = 465 - 4 \times 30.5 - 4 \times 53 = 131$.

Using these edge centrality values, we can determine the economic centrality of the players by summing the centrality values of the edges they control. When the bridge is owned by a third party, the resulting player shares are $C^R(P_1) = C^R(P_2) = 2 \times 30.5 + 2 \times 53 = 167$, $C^R(P_3) = 131$.

In this case, the bridge owner receives a slightly smaller share of the total profit than under the Shapley value allocation. However, the resulting allocation remains admissible under the game's characteristic function. If Player 1 owns the bridge, the players' shares become $C^R(P_1) = 298$ and $C^R(P_2) = 167$. The payoff assigned to Player 1 is equal to the combined payoffs of Players 1 and 3 in the previous scenario. This indicates that when the bridge owner joins Player 1's network, Player 1 does not lose bargaining power under the economic centrality allocation. Note that this results directly from the fact that the allocation to a coalition according to a centrality measure is simply the sum of the centrality measures for the edges held by that coalition. In fact, Player 1 receives a slightly larger payoff than under the Shapley value allocation. More complex networks of this type, in which internal subnetworks are connected by bridges, can be analyzed using the same approach. Similar methods can also be applied to networks in which different subnetworks are connected by a small number of edges.

This example highlights the strategic importance of bridge ownership in cooperative communication networks. Even when a bridge consists of only one edge, its position can make it highly valuable because it enables all profitable communication between the two subnetworks. The example also shows that changes in bridge ownership can significantly affect profit allocation, even when the underlying physical network remains unchanged.

4.6.5 European Communication Network Game

This example illustrates the application of the cooperative network model to a simplified representation of a European communication network, the PIONIER Internet Network (PIONIER Consortium, 2025; see Figure 4.4). In this network, each node represents a city and each edge represents a communication link between two cities. The cities are in four different countries, interconnected through three service providers. Each provider operates a "national" communication network, shown in different colors on the graph. The resulting network structure captures the interaction between national infrastructures and international connections. This example demonstrates how the theoretical framework developed in this chapter can be applied to a larger and more realistic communication network.

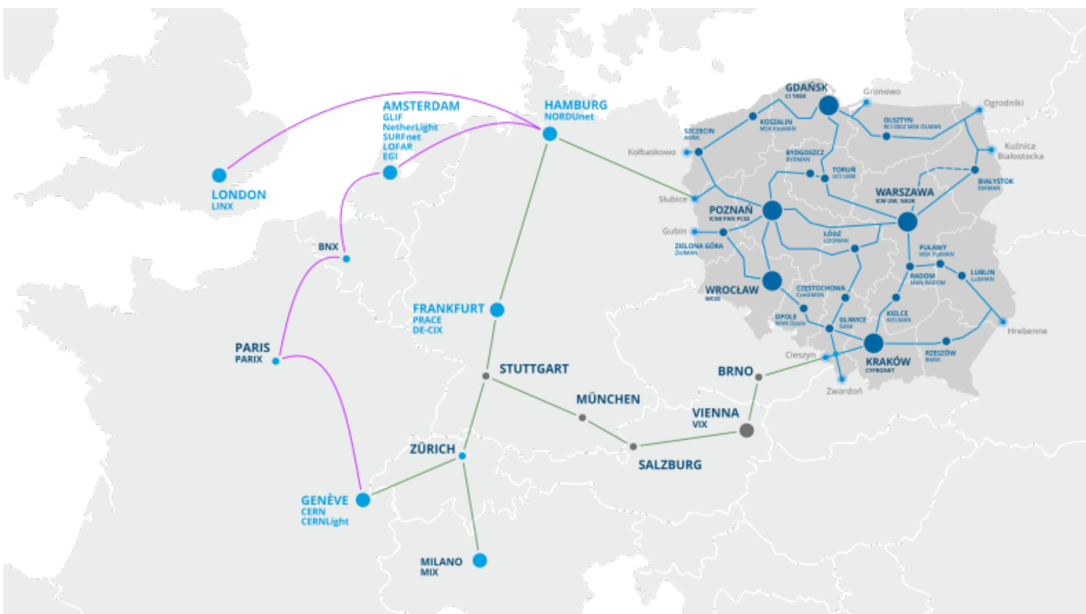


Figure 4.4 The PIONIER Internet Network **(PIONIER Consortium, 2025).**

We now consider a simplified model of this network. In this model, cities are represented by nodes and communication links by edges. These cities are located in four countries, while three service providers operate the infrastructure connecting them. Figure 4.5 illustrates the network's structure, a simplified representation of the PIONIER Internet network.

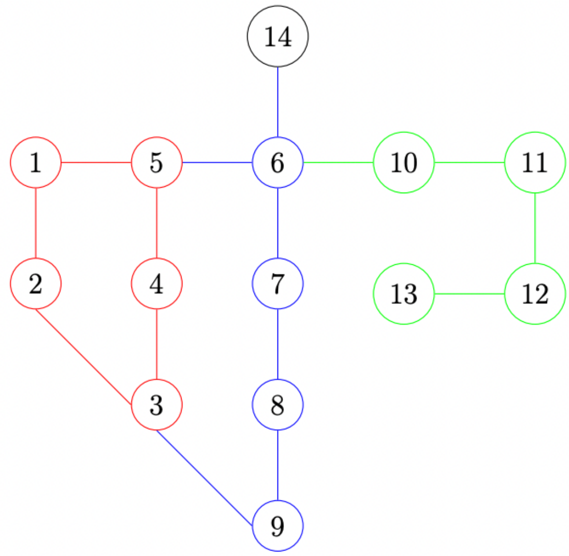


Figure 4.5 Simplified European Communication Network with Three Service Providers

In Figure 4.5, the colors red, blue, and green represent three national communication networks corresponding to Players 1, 2, and 3, respectively. In addition, City 14 belongs to a fourth country and is connected to the network through one of the providers. Each player controls the edges belonging to their own national network. Communication between countries, however, occurs through connections between these subnetworks. To analyze this network, a set of computational procedures was implemented in Python. First, the colored graph representing the network structure is constructed. Next, Dijkstra's algorithm is applied to determine the shortest paths between nodes within the relevant subgraphs corresponding to different player coalitions. Then, the economic centrality values of edges and players are calculated based on the formulas introduced earlier in this chapter. Finally, the cooperative game associated with the network is defined, and the Shapley value for the resulting characteristic function is computed.

The following assumptions are used in the model: First, the cost of transmitting one unit of information along any edge is 1. Second, the revenue from transmitting information between two cities within the same country is assumed to be 4. Finally, when information is transmitted between cities in different countries, the revenue per unit increases to 8. The demand for communication between two cities depends on their populations. Specifically, if both cities belong to the same country, the demand for information transmission is assumed to equal the product of their populations, measured in thousands. Conversely, if the cities are in different countries, the demand is assumed to be equal to one-half of the product of their populations. The population sizes of the cities used in the model are presented in Table 4.12.

Table 4.12: Populations of Cities (in thousands)

City	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Population	486	1862	806	674	541	1906	764	632	2006	883
City	**11**	**12**	**13**	**14**						
Population	2187	204	428	8962						

Using the characteristic function based on strict control, denoted by v_1, the values of the coalitions in this network game are given in Table 4.13:

Table 4.13. Characteristic Function Values Based on Strict Control v_1

Coalition *A*	$v_1(A)$
ϕ	0
{1}	16 692 744
{2}	194 413 843
{3}	31 003 358
{1, 2}	319 151 562
{1, 3}	47 696 102
{2, 3}	338 027 320.5
{1, 2, 3}	482 917 824.5

These values represent the maximum profits the respective coalitions can guarantee when only routes strictly controlled by the coalition yield payoffs. Next, consider the characteristic function under economic control, denoted by v_2. In this case, the coalition values are seen in Table 4.14

Table 4.14. Characteristic Function Values Based on Economic Control v_2

Coalition *A*	$v_2(A)$
ϕ	0
{1}	17 564 836

{2}	194 413 843
{3}	31 003 358
{1, 2}	319 151 562
{1, 3}	48 568 194
{2, 3}	338 027 320.5
{1, 2, 3}	482 917 824.5

The Shapley value solution offers a fair way to divide the total value of the European communication network among three service providers. It does this by averaging each provider's contribution to different groups, using the game's characteristic function (v_1 or v_2). This method considers both the control each provider has over network links and the benefits of working together to connect cities. The final allocation shows how important each provider is for keeping communication efficient. Player 2 gets the largest share because it contributes the most to network connectivity and demand. Player 3 gets the smallest share due to its limited influence, while Player 1 falls in the middle with a moderate contribution. The Shapley value helps ensure that network profits are shared fairly based on each provider's role.

The values based on economic control differ slightly from those obtained under strong control because the economic control approach also accounts for routes where a coalition can serve demand more efficiently than competing coalitions. The Shapley values corresponding to both characteristic functions and the values based on economic centrality are also reported in Table 4.15. Again, the proportional and uniform measures of centrality give the same values, since it is assumed that the edges have the same cost.

Table 4.15 Distributions of Profits According to Various Solutions of the Cooperative Game

Method	Player 1	Player 2	Player 3
Shapley value based on v_1	77 432 826.50	311 458 985.25	94 026 012.75
Shapley value based on v_2	77 868 872.50	311 022 939.25	94 026 012.75
Economic Centrality	84 843 494.74	292 311 130.96	105 763 198.80

The Shapley values obtained from the characteristic functions under strict control and economic control are very similar. They also belong to the core for the corresponding cooperative game. The only difference is that Player 1 (red) has economic control over the route between cities 3 and 5, but not strict control over it.

This occurs because Player 2 can also serve this route, albeit at a higher transmission cost. As a result, Player 1 receives a slightly larger share of the total payoff under the characteristic function v_2 compared with v_1 , while Player 2 receives a slightly smaller share. It should also be noted that the payoff allocation based on economic centrality lies in the core of the cooperative game under both characteristic functions v_1 and v_2. However, the economic centrality solution captures only the flows of information under the optimal network routing configuration. In contrast, the Shapley value accounts for each player's strategic contribution to all possible coalitions within the network. Because of this broader perspective, the Shapley value assigns Player 2 a larger share of the total payoff than the allocation based solely on economic centrality. This result reflects Player 2's strong strategic position within the overall network structure. In particular, Player 2 occupies a central position in the network, so cooperation between Players 1 and 3 alone does not yield additional benefits unless Player 2 also participates. Furthermore, Player 2 controls the communication routes leading to the largest city in the network, with City 14 playing a role analogous to London in the simplified European network.

This example illustrates how the cooperative communication network model developed in this chapter can be applied to a larger and more realistic network structure. Specifically, by combining shortest-path calculations with cooperative game theory, it becomes possible to analyze the distribution of profits among service providers and evaluate the strategic importance of different parts of the communication infrastructure.

4.7 Discussion and Implications

Although the model developed in this chapter shares certain features with classical shortest path games, important differences exist. In classical shortest path games, the value of a coalition is determined by its ability to establish a path between a designated source and destination. Depending on the model, players may own nodes or edges, and coalition value is generally derived from the existence of a feasible path connecting the source and sink. In contrast, the present model considers multiple communication routes, transmission costs, revenues, and route control, thereby providing a broader framework for analyzing cooperation in communication networks **(Fragnelli et al., 2000; Grahn, 2001; Voorneveld & Grahn, 2002).**

This chapter has introduced a cooperative game theoretic framework for analyzing communication networks under the assumptions of linear transmission costs and the absence of capacity constraints. The model represents the physical network as a weighted graph in which players control subsets of edges and cooperate to serve communication routes between nodes. The characteristic function of the game was defined as the profit that coalitions can obtain by transmitting information along the shortest feasible paths **(Ahuja et al., 1993).** The analysis builds on the theory of cooperative games on networks, where the structure of the communication graph determines how players can cooperate and generate value **(Myerson, 1977; Curiel, 1997).** Profit allocation among network operators can then be analyzed using cooperative solution concepts such as the Shapley value **(Shapley, 1953).**

The positions of edges in the network, particularly bridges and other centrally located links, play a critical role in determining the strategic importance of players, as highlighted in centrality-based network analyses **(Freeman, 1977).** Two versions of the characteristic function were considered. The first is based on strict route control, while the second incorporates economic control when competing coalitions can serve the same route at different costs.

Building on this framework, the examples presented in this chapter illustrate how the structure of a communication network influences the distribution of profits among service providers. The results show that the position of edges in the network, particularly bridges and other centrally located links, plays a critical role in determining the strategic importance of players. In several examples, players controlling only a small number of edges obtained a relatively large share of the total payoff because those edges were essential for connecting different parts of the network.

The comparison between the Shapley value and the economic centrality measures highlights important differences between structural and strategic evaluations of network importance. Economic centrality reflects the contribution of edges to the optimal routing of information in the network. These measures are relatively simple to compute and provide useful insights into the structural role of edges and players. However, they capture only the flow of information under the optimal grand-coalition configuration. In contrast, the Shapley value accounts for each player's marginal contribution across all possible coalition formations. This approach reflects the strategic bargaining power of players in the cooperative network game. As demonstrated in the examples, the Shapley value may assign a larger payoff to players occupying structurally central positions in the network, even when their direct contribution to the optimal routing solution appears limited.

In addition to these analytic distinctions, the examples also demonstrate that ownership structures can significantly influence the allocation of profits. In networks containing bridges or other critical connections, the owner of such infrastructure can gain substantial bargaining power. Changes in ownership, such as transferring a bridge from an independent operator to a network provider or sharing ownership among players, may substantially alter the resulting payoff allocations.

Turning to practical applications, the simplified European communication network presented in the final example illustrates how the proposed model can be applied to larger and more realistic network structures. In this case, the Shapley value and the economic centrality measures produced similar results, although the Shapley value highlighted the strategic importance of certain players more clearly. This demonstrates that cooperative game theory can provide valuable insights into profit allocation in complex communication networks. Overall, the framework developed in this chapter provides a foundation for analyzing cooperation among service providers in communication networks. The model can be used to study strategic interactions in infrastructure sharing, interconnection agreements, and revenue allocation in multi-operator networks. These insights may also be relevant to policymakers and regulators seeking to understand how network structure and ownership influence competition and cooperation in communication systems.

To build on these findings, future research can extend this framework by incorporating additional features that arise in real communication networks. One important extension is the introduction of capacity constraints on network links. In practical systems, transmission links cannot carry unlimited amounts of information, and congestion may occur when demand exceeds available capacity. Introducing capacity limits allows the model to represent more realistic operating conditions and enables the analysis of how limited resources affect cooperation among service providers. Accordingly, the next chapter extends the model to incorporate capacity constraints and examines how these constraints influence coalition formation, route feasibility, and the resulting profit allocation in communication networks.

Chapter 5

A Model with Linear Costs and Capacity Constraints

Chapter 5 extends the cooperative communication network model developed in the previous chapters. In Chapter 4, the network was analyzed under linear transmission costs without capacity constraints, allowing coalitions to transmit any amount of information along the paths they controlled. Building upon this foundation, we now consider a more realistic situation in which transmission costs remain linear, but each network link has a limited capacity. As a result, even if a coalition has a path connecting two nodes, it may not be able to serve all transmission demand if the edges' capacities are insufficient. By incorporating these capacity constraints into the model, this chapter first examines how limited network resources influence route feasibility, optimal flow allocation, and the total profit that can be generated when all providers cooperate. Next, building on these insights, the chapter extends the analysis to a full cooperative game-theoretic framework. Here, the value of each possible coalition is determined through a competitive auction mechanism in which coalitions bid for routes based on the cheapest paths under their control. The resulting characteristic function is then used to derive the Shapley value, providing a principled allocation of profit that reflects each provider's marginal strategic contribution to the network.

5.1 Introduction

Communication networks are now vast, complex systems managed by multiple service providers **(Sergiou et al., 2020).** Each provider typically controls a subset of the network's transmission links, which connect devices such as routers, switching centers, and transmission stations. Because these infrastructures are shared, providers must cooperate to ensure the successful transmission of data between locations. The network's overall efficiency depends on how well providers coordinate their resources and operational decisions **(Heranz, 2010).**

Physical and technical limits challenge communication systems. Each transmission link has a finite capacity, setting a maximum information rate. In real systems like Internet backbones, optical fiber networks, and large platforms, these constraints stem from bandwidth, hardware, and network policies **(Habib et al., 2019; Wang et al., 2022).** When demand surpasses capacity, congestion causes untransmitted traffic, leading to lost revenue, higher costs, or inefficiency. Thus, capacity limits shape how network resources are allocated and services delivered **(Dubey & Gupta, 2021).** Managing communication networks with multiple providers poses two primary challenges. The first challenge concerns efficiently routing information through the network to satisfy demand at the lowest possible cost. The second involves determining how revenue should be distributed among providers who deliver the data. These issues are commonly addressed using network flow optimization and cooperative game theory frameworks **(Ahuja et al., 1993; Saad et al., 2009; Han et al., 2012; Bertsekas, 2021).** In such models, providers are represented as players controlling network edges and may form coalitions to jointly deliver information between origin and destination nodes.

Centrality measures play a key role in assessing the importance of nodes or edges within networks. Examples include betweenness centrality, closeness centrality, and eigenvector centrality. These measures are widely used to identify critical network components across social, biological, and communication contexts **(Lü et al., 2016; Newman, 2018; Saxena & Iyengar, 2020).** However, classical centrality mainly reflects structural network properties and generally ignores operational aspects. Factors such as transmission costs, capacity constraints, or the volume of data handled are typically absent **(Barthélemy, 2004; Opsahl et al., 2010; Wang et al., 2022).**

A provider's economic significance in communication networks extends beyond mere network position. Providers can be vital by carrying much of the network traffic or controlling links that form throughput bottlenecks. Analogous examples can be found in transportation and supply chain systems, where route capacity limits affect overall performance and profit **(Nagurney, 2006; Mizgier et al., 2013; Mohanty, 2025).** While several studies offer weighted centrality measures with extra parameters **(Galafassi & Bazzan, 2013; Bloch et al., 2019; Li et al., 2023),** interactions among demand, capacity, and flow optimization are underexplored. In particular, there is little research on how capacity limitations shape coalition value and profit distribution through mechanisms such as the Shapley value.

The previous chapter's model relied on two simplifying assumptions that impact realism. First, it assumed linear edge transmission costs. Second, it assumed unlimited edge capacities. Under these conditions, any provider coalition could transmit any amount of information as long as a feasible, cost-effective path existed. These assumptions facilitate the study of coalition behavior and profit generation but reduce realism, since real networks often operate with capacity constraints **(Habib et al., 2019).**

Introducing capacity constraints on network edges makes the model more realistic. Now, each transmission link has a fixed capacity, limiting information flow per period. Only traffic actually transmitted along a route uses the capacities of its links. Providers, therefore, must judge not just a route's profitability but also whether sufficient capacity exists to meet demand. Capacity constraints introduce further operational considerations, making coalition feasibility and profitability reliant on both routing costs and resource availability **(Wang et al., 2022; Amin & Johari, 2025).**

This chapter develops a cooperative game-theoretic framework for communication networks with linear transmission costs and capacity constraints. The model has two aims. First, it integrates network flow optimization with a revenue-allocation mechanism based on economic centrality. An iterative flow augmentation algorithm identifies cost-efficient flows that satisfy demand while respecting capacity limits. These flow patterns then inform proportional and uniform centrality measures, yielding revenue distribution. Second, the model extends to a full cooperative game formulation, where the value of a coalition is determined by a competitive auction in which providers bid for routes based on the paths they control. The resulting characteristic function is used to compute the Shapley value, offering a strategic, marginal-contribution-based alternative for revenue allocation.

5.2 Network Representation with Linear Costs and Capacity Constraints

Consider a communication network managed by service providers $P = \{P_1, P_2, \dots, P_K\}$. The network is represented by a connected weighted graph $G = (V, E)$. V is the set of nodes. E is the set of transmission links. Each node represents a communication hub such as a router, switching center, or transmission station. Each edge $e \in E$ is characterized by two parameters. The first parameter is the unit transmission cost, which represents the cost of sending one unit of information along the edge. The second parameter is the edge's capacity, which represents the maximum amount of information that can pass through it during a given period. Therefore, each edge e can be described by the pair.

$$(w_{1,(e)}, w_{2,(e)}) \tag{5.1}$$

where $w_{1,(e)}$ denotes the unit transmission cost and $w_{2,(e)}$ denotes the capacity of the edge. Each edge is controlled by exactly one provider. Let E_k denote the set of edges owned by provider P_k. The sets E_1, E_2, …, E_K form a partition of the edge set E. A provider may control several edges across different parts of the network, and its economic power depends on both the number of edges it controls and their positions within the network structure.

Coalitions arise when providers cooperate and combine the edges they control, enabling resource sharing and improving routing performance in communication networks **(Moncecchi et al., 2020; Jia et al., 2021; Kim, 2022).** For any coalition $A \subseteq P$, the set of edges available to that coalition is the union of the edges controlled by its members. By pooling their resources, coalitions can access a wider set of transmission paths, potentially increasing the number of covered communication routes, improving network efficiency, and allowing providers to serve routes that individual members could not serve on their own. As in Chapter 4, to describe the network's economic environment, we define two matrices. The reward matrix R specifies the unit revenue obtained from transmitting one unit of information between two nodes. The demand matrix D specifies the amount of information that must be transmitted between each pair of nodes. Both matrices are symmetric because the revenue and demand are assumed to be independent of the direction of transmission.

In networks with capacity constraints, the existence of an economically viable path between two nodes is no longer sufficient to guarantee that all the demand for that route can be met. The total flow assigned to any edge cannot exceed its capacity. Consequently, routing decisions must satisfy both cost-efficiency and capacity-feasibility conditions, which are typically modeled using cooperative or network flow optimization frameworks **(Babu et al., 2022; Zhao et al., 2023; Andriopoulos et al., 2024).** These structural elements form the basis for analyzing communication networks with capacity constraints and cooperative interactions among service providers **(Wang et al., 2020; Kim, 2022; Zhao et al., 2023).** In the next section, we introduce the optimization problem that determines the feasible flow of information to maximize total profit in the network.

5.3 Information Flow in Capacity Constrained Networks

When capacity constraints are present, the allocation of flows across the network must satisfy both demand requirements and capacity restrictions. The objective is to determine how information should be routed so that the total profit obtained from transmitting information is maximized while respecting these constraints. This problem can be formulated as a multicommodity network flow optimization problem **(Ahuja et al., 1993).** Each route between two nodes corresponds to a commodity that must be transmitted through the network. The flow assigned to each path must satisfy two conditions **(Bertsekas, 2021).** First, the total flow assigned to the paths serving a route cannot exceed the route's demand. Second, the total flow passing through any edge cannot exceed the capacity of that edge.

Let $G = (V, E)$ be a communication network. V is the set of nodes, and E is the set of edges. For each edge e in E, let u_e be its capacity, and c_e be its unit transmission cost. Let $d_{i,j}$ be the communication demand between nodes V_i and V_j. Let $r_{i,j}$ be the unit revenue for transmitting one unit of information between V_i and V_j. Let $Q_{i,j}$ be the set of feasible paths connecting V_i and V_j. For each path q in $Q_{i,j}$, let f_q be the amount of flow on path q. Let k_q be the total unit transmission cost of path q.

With these definitions established, we now turn to the formulation of the multicommodity flow optimization problem as a linear programming problem:

Maximize

$$max \sum_{1 \le i < j \le J} \sum_{q \in Q_{i,j}} f_q \left(r_{i,j} - k_q \right) \tag{5.2}$$

Subject to

$$\sum_{q \in Q_{i,j}} f_q \le d_{i,j}, \qquad \forall \left(V_i, V_j \right) \tag{5.3}$$

and

$$\sum_{e \in q} f_q \le u_e, \qquad \forall e \in E \tag{5.4}$$

with

$$f_q \ge 0, \qquad \forall q \in Q$$

The objective function maximizes the total profit obtained from transmitting information through the network. The first set of constraints ensures that the total flow assigned to the paths serving a route does not exceed the communication demand associated with that route. The second set of constraints guarantees that the total flow transmitted through each edge does not exceed the edge capacity. Finally, the nonnegativity constraints ensure that all path flows are feasible. This formulation represents a standard linear programming model for multicommodity network flow optimization and provides the basis for determining the optimal allocation of information flows in a communication network with capacity constraints **(Ahuja et al., 1993).**

The profit from transmitting information along a path depends on the difference between the revenue from serving the route and the path's transmission cost **(Nagurney, 2006)**. Let f_q denote the flow assigned to path q and let k_q denote the unit cost of that path. The profit generated by transmitting this flow is therefore $f_q\ (r_{i,j} - k_q)$, where $(V_i,\ V_j)$ denotes the route served by path q. The total profit generated by the network is the sum of the profits associated with all paths in the optimal routing solution. If Q denotes the set of all paths used to satisfy demand, the total profit can be expressed as $S - K$, where S represents the total revenue obtained from satisfying demand, and K represents the total transmission cost associated with the flows assigned to the network. Determining the optimal flow allocation is a key step in analyzing capacity-constrained communication networks. Once the optimal flows are identified, the resulting profit can be distributed among the providers based on the economic importance of the edges they control. To further this analysis, the following section introduces a measure of economic centrality with respect to a given flow.

5.4 Economic Centrality with Respect to a Given Flow

In communication networks managed by multiple providers, each provider's contribution depends on the role of the edges they control in information transmission. To evaluate this contribution, it is useful to extend the concept of centrality to reflect not only the structural position of edges in the network but also the economic value of the flows that pass through them. When capacity constraints are present, the flows in the network are determined by solving a multicommodity flow optimization problem rather than solely by the shortest-path structure. As a result, shortest path-based centrality measures may not accurately capture the economic importance of edges. In networks with capacity constraints, information transmission is determined by the optimal solution to the multicommodity flow problem. The resulting flow allocation specifies how much information is transmitted along each path while satisfying the capacity limits of the network edges. These optimal flows provide a natural basis for evaluating the economic importance of network edges. Here, we define measures of the economic centrality of an edge with respect to a given flow. The corresponding splits of profit between the members of the grand coalition are then based on these measures of centrality with respect to the optimal flow.

Let Q denote the set of paths used in the optimal flow. A path in this context is a sequence of edges that connects two nodes without repeating edges. For each path q in Q, let f_q denote the amount of information transmitted along that path. Let k_q denote the unit transmission cost associated with the path, which equals the sum of the unit costs of the edges that form the path. Suppose that path q serves the route between nodes V_i and V_j, where $r_{i,j}$ denotes the unit revenue obtained for transmitting one unit of information between these nodes. The profit generated by the flow transmitted along path q is therefore $f_q\ (r_{i,j} - k_q)$. This value represents the economic contribution of path q to the network's total profit. Since each path consists of a sequence of edges, the profit generated by that path must be allocated among the edges that participate in the transmission.

One possible allocation rule distributes the profit in proportion to the transmission costs of the edges belonging to the path. Let c_e denote the unit cost of edge e and let k_q denote the total transmission cost of path q. The proportional economic centrality of edge e with respect to path q is defined as

$$\tilde{C}_P(e|q) = \frac{f_q\left[r_{i,j} - k_q\right]c_e}{k_q} \tag{5.5}$$

This measure assigns a larger share of the profit to edges with higher transmission costs along the path. The proportional economic centrality of edge e in the entire network is obtained by summing these contributions over all routes and all paths used in a network flow.

$$\tilde{C}_P(e) = \sum_{1\ \leq i<j\leq J}\ \sum_{q\in Q_{i,j}} \frac{f_q\left[r_{i,j} - k_q\right]c_e}{k_q}, \tag{5.6}$$

where $Q_{i,j}$ denotes the set of paths used to transfer information between nodes V_i and V_j. An alternative allocation rule distributes the profit generated by a path equally among all edges that form the path. Let n_q denote the number of edges in path q. The uniform economic centrality of edge e with respect to path q is defined as

$$\tilde{C}_U(e|q) = \frac{f_q\,[r_{i,j} - k_q]}{n_q} \tag{5.7}$$

for each edge that belongs to path q. The total uniform economic centrality of edge e is obtained by summing these contributions across all routes and all paths used to satisfy the demand for communication.

$$\tilde{C}_U(e) = \sum_{1 \le i<j \le J} \sum_{q \in Q_{i,j}} \frac{f_q\,[r_{i,j} - k_q]}{n_q} \tag{5.8}$$

These two measures provide alternative ways of evaluating the economic importance of edges in a communication network with capacity constraints. The proportional measure distributes profit according to the relative transmission costs of the edges along a path, while the uniform measure distributes the profit equally among all edges that participate in the transmission. As in Chapter 4, the economic centrality of a player is assumed to be the sum of the centrality measures of the edges that it controls.

An important property of both measures is that the sum of the economic centrality values of all edges equals the network's total profit (as in Chapter 4, we assume that no costs are incurred for leaving demand unsatisfied, except for the loss of potential profit). This property guarantees that the resulting allocation satisfies the efficiency condition, since the entire profit generated by the network is distributed among the edges that participate in information transmission **(Shapley, 1953).** Providers whose edges are not used in the optimal flow receive no share of the profit. Such providers can therefore be interpreted as null players in the profit allocation process **(Owen, 2013).** This framework allows the contribution of each provider to be evaluated in a manner that reflects both the network's structure and the actual flows of information determined by the capacity-constrained optimization model **(Myerson, 1977; Saad et al., 2009; Ramsey et al., 2025).** The next section considers the derivation of the optimal flow.

5.5. Optimization of The Flow

Note that, in general, we cannot assume that all of the demand is met. The following algorithm maximizes the profit obtained by the firm while satisfying the constraints on the capacities of the edges and ensuring that the transfer of information along a route is not greater than the demand. The profit is defined to be the revenue obtained from the flow of information minus the costs of information transfer. It is assumed that there is no penalty for unsatisfied demand. Within this framework, the initial (feasible) solution assumes that no information is transferred. The corresponding profit is equal to zero. First, using a flow augmentation procedure, we find a good feasible solution. In each step of this procedure, we introduce a flow along a path that maximizes the ratio of the profit per unit flow under a second-price auction for a route (played against other paths corresponding to that route) to the number of edges used. This augmentation ensures that the constraints are satisfied. This augmentation procedure consists of two main steps:

1. **Prioritization:** For each route with positive demand, determine the cheapest and second-cheapest paths. The shortest paths may be obtained using the Floyd-Warshall algorithm, while the second shortest paths may be derived using an appropriate extension **(Floyd, 1962; Gotthilf & Lewenstein, 2009).** Paths that pass through the same edge more than once are excluded. Each route is then assigned a priority equal to the ratio between the profit obtained using that path according to a second-price auction with a maximum price and the number of edges used in the shortest path. If two routes have the same priority, the tie may be broken in various ways: by choosing the route with the cheapest path, selecting a route that does not lead to the removal of any edges (if possible, see point 2), or at random. If there are routes with positive priority, proceed to Step 2; otherwise, stop.
2. **Flow augmentation:** For the route with the highest priority, assign flow along its cheapest path. The amount of flow sent is equal to the minimum of two quantities: the remaining demand for that route and the minimum residual capacity of the edges on that path. After assigning the flow, both the route demand and the capacities of the edges on the path are reduced accordingly. Routes with zero residual demand are removed. Edges with zero residual capacity are also removed. If no route with positive priority remains, the procedure stops. If no edge is removed, which can only occur when the route previously with the highest priority is removed, then repeat step 2. Otherwise, return to Step 1 on the basis of the current residual graph.

The flow augmentation procedure gives a good, feasible solution to the optimization problem. The procedure used to check the optimality of such a solution and improve it when necessary depends on whether the current solution satisfies all of the demands or not.

a) **The current solution satisfies all of the demand:** From the assumptions of the model, the revenue is fixed $\left(\sum_{i<j} r_{i,j} d_{i,j}\right)$. Hence, such a solution must minimize the costs of satisfying this demand. An appropriate adaptation of the Dantzig-Wolfe procedure can be applied to check that these costs are minimized **(Ahuja et al., 1993).** Under this approach, transmission costs are minimized if there exist nonnegative increments to the unit costs (reduced costs) of the fully utilized edges satisfying two conditions. First, whenever more than one path is used to serve the same route, the adjusted costs of these paths must be equal and minimal. Second, for all remaining routes, the solution's path must remain the shortest path under the adjusted costs. When these optimality conditions are satisfied, the only possible way of increasing profits is by reducing the flow on one of the paths used. However, all of these paths are profitable. Hence, the current solution is optimal. When the current solution does not minimize the cost of satisfying the demand, then an improved solution can be found by decreasing the flow in a path associated with a negative sum of reduced costs and increasing the flow in the path corresponding to the same route where there is residual capacity on all of the edges (here, the residual costs are by definition zero).

b) **The current solution does not satisfy all of the demand:** to check whether the flow for a route with unsatisfied demand should be increased at the cost of satisfying demand for another route, we can use the following procedure:
   1. Determine the set of profitable paths for that route.
   2. For each such path p with n edges, we can consider splits of this path into constituent subpaths. Such a split can be defined by a binary code of length $n - 1$ which does not only contain zeroes, i.e., $2^{n-1} - 1$ possible splits. A 1 in the i-th position of this code indicates a split in the original path p after the i-th edge to form the set of constituent paths. Hence, for the path $V_1 \rightarrow V_2 \rightarrow V_3 \rightarrow V_4$ (3 edges), we have 3 possible splits represented by 10, 01, and 11. These splits give the following partitions of p, i) $V_1 \rightarrow V_2$ and $V_2 \rightarrow V_3 \rightarrow V_4$, ii) $V_1 \rightarrow V_2 \rightarrow V_3$ and $V_3 \rightarrow V_4$, and iii) $V_1 \rightarrow V_2$, $V_2 \rightarrow V_3$ and $V_3 \rightarrow V_4$. We restrict our consideration to cases in which there is a flow in each of these constituent paths that is not used to satisfy demand for the route with unsatisfied demand. Increasing flow in the path p can be

achieved by an equal reduction in the flow in these constituent paths. Let $m(p)$ denote the marginal profit from the path *p.* Suppose p is split into constituent parts, and the minimum marginal profit from a path with positive flow containing p_i, but not any other edges in *p,* is $m_0\ (p_i)$. Optimality of the current flow requires that for each possible split.

$$m(p) \leq \sum_{p_i \in P_0} m_0(p_i), \tag{5.9}$$

where P_0 is the set of constituent paths that contain a fully utilized edge. This condition states that the profit from increasing flow along the path p does not outweigh the loss in profit from the resulting decrease in flow along constituent paths.

3. Now, for each such *p,* consider the set of used paths S_p of which p is a subpath. The flow-through path p can be increased by decreasing the flow through a path q, where $q\ \in\ S_p$. Such a decrease in the flow through path q may enable an increase in flows in the constituent paths of $q \backslash p$, which have by unsatisfied demand. Optimality of the current flow requires that for any $q\ \in\ S_p$ partition of the path $q \backslash p$ into $q_1, q_2, \dots, q_n$.

$$m(q) \geq m(p) + \sum_{q_i \in P_1} m(q_i), \tag{5.10}$$

where $m(p)$ denotes the marginal profit from the path *p,* and P_1 is the set of constituent paths of $q \backslash p$ corresponding to a route with unsatisfied demand.

When condition (5.9) is not satisfied, then the flow in the path p should be increased, and the flow in the constituent paths should be appropriately reduced. When condition (5.10) is not satisfied, then the flow in path q should be reduced and balanced by an increase in flow along the path p and constituent subpaths of $q \backslash p$. The optimization procedure will be illustrated by two examples.

5.5.1 Example 1

To illustrate the proposed model, consider the communication network shown in Figure 5.1. The network consists of five nodes, V_1, V_2, V_3, V_4, and V_5, connected by a set of transmission links. Each edge is associated with two parameters. The first parameter represents the unit cost of transmitting one unit of information along that edge, and the second parameter represents the maximum capacity of the edge. These values are shown next to each edge as (cost, capacity).

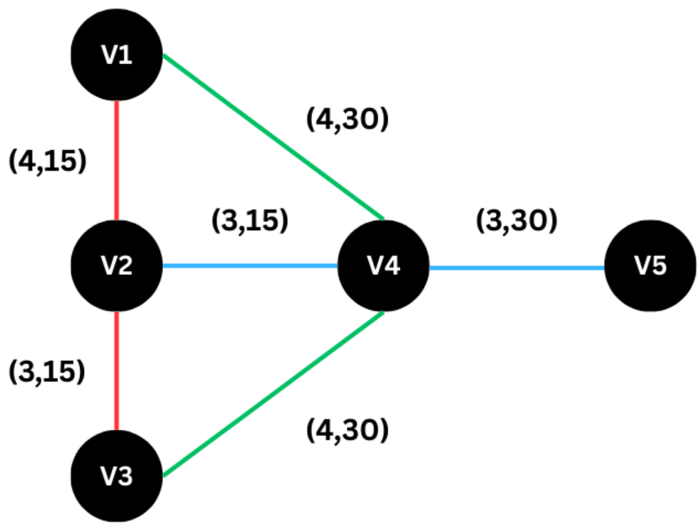


Figure 5.1. A Simple Example of a Weighted Communication Network with Capacity Constraints

In this network, three providers manage the infrastructure. Each provider controls a subset of the network's edges. For example, the edge connecting V_1 and V_4 has a unit transmission cost of 4 and a capacity of 30.

The revenue and demand for transmitting information between nodes are represented by the following two matrices.

$$R = \begin{pmatrix} 0 & 5 & 10 & 5 & 10 \\ 5 & 0 & 5 & 5 & 10 \\ 10 & 5 & 0 & 5 & 10 \\ 5 & 5 & 5 & 0 & 5 \\ 10 & 10 & 10 & 5 & 0 \end{pmatrix}; D = \begin{pmatrix} 0 & 10 & 10 & 10 & 5 \\ 10 & 0 & 10 & 10 & 5 \\ 10 & 10 & 0 & 10 & 5 \\ 10 & 10 & 10 & 0 & 10 \\ 5 & 5 & 5 & 10 & 0 \end{pmatrix}; \tag{5.11}$$

The network's objective is to transmit information to maximize total profit. Each edge has a limited capacity. Thus, routing must satisfy two conditions. The total flow assigned to a route cannot exceed its demand. Also, the total flow through any edge must not exceed its capacity. Flows are assigned to the cheapest available paths using the earlier-described optimization procedure. Capacity constraints are respected at all times. This yields the routing configuration that maximizes profit. The resulting decomposition of the flow can be used to evaluate the centrality of edges and distribute total network profit among providers.

First, the cheapest and second-cheapest paths are identified for each route with positive demand (see Table 5.1). The price resulting from an auction for each route is the minimum of the maximum revenue and the 2nd cheapest cost. The priority ascribed to each cheapest path is the resulting profit for that path divided by the number of edges used. Assume that priority is ascribed to the route (V_4, V_5). Since the demand on this route is 10 and the capacity of the edge $V_4 \leftrightarrow V_5$ is 30, the full demand is assigned to the path that uses just this edge. As a result, the route (V_4, V_5) is eliminated, and the residual capacity of the edge $V_4 \leftrightarrow V_5$ decreases from 30 to 20. Now ascribe priority to the route (V_2, V_3). Its cheapest path is the direct edge $V_2 \rightarrow V_3$ with cost 3. Since this edge has a capacity of 15 and the demand is 10, the full demand is assigned to this path. The route is then eliminated, and the residual capacity of the edge $V_2 \leftrightarrow V_3$ decreases from 15 to 5. After that, priority can be assigned to route (V_2, V_4), which is served through the direct edge $V_2 \rightarrow V_4$. Again, the full demand of 10 is assigned, and the residual capacity of the edge $V_2 \leftrightarrow V_4$ decreases from 15 to 5.

Table 5.1 Initial Priorities of the Routes.

Route	Cheapest Path	Cost	2nd Cheapest Path	Cost	Max. Rev.	Priority
(V_1, V_2)	$V_1 \rightarrow V_2$	4	$V_1 \rightarrow V_4 \rightarrow V_2$	7	5	1
(V_1, V_3)	$V_1 \rightarrow V_2 \rightarrow V_3$	7	$V_1 \rightarrow V_4 \rightarrow V_3$	8	10	½
(V_1, V_4)	$V_1 \rightarrow V_4$	4	$V_1 \rightarrow V_2 \rightarrow V_4$	7	5	1
(V_1, V_5)	$V_1 \rightarrow V_4 \rightarrow V_5$	7	$V_1 \rightarrow V_2 \rightarrow V_4 \rightarrow V_5$	10	10	3/2
(V_2, V_3)	$V_2 \rightarrow V_3$	3	$V_2 \rightarrow V_4 \rightarrow V_3$	7	5	2
(V_2, V_4)	$V_2 \rightarrow V_4$	3	$V_2 \rightarrow V_3 \rightarrow V_4$	7	5	2
(V_2, V_5)	$V_2 \rightarrow V_4 \rightarrow V_5$	6	$V_2 \rightarrow V_3 \rightarrow V_4 \rightarrow V_5$	10	10	2
(V_3, V_4)	$V_3 \rightarrow V_4$	4	$V_3 \rightarrow V_2 \rightarrow V_4$	6	5	1
(V_3, V_5)	$V_3 \rightarrow V_4 \rightarrow V_5$	7	$V_3 \rightarrow V_2 \rightarrow V_4 \rightarrow V_5$	9	10	1
(V_4, V_5)	$V_4 \rightarrow V_5$	3	-	-	5	2

The remaining route with priority 2 is (V_2, V_5). Its cheapest path is $V_2 \rightarrow V_4 \rightarrow V_5$, with a total cost of 6. The demand on this route is 5, while the minimum residual capacity along the path is 5, determined by edge (V_2, V_4). Therefore, the full demand of 5 is assigned to this path. After this assignment, the route (V_2, V_5) is eliminated. The residual capacity of edge $V_4 \leftrightarrow V_5$ decreases from 20 to 15, while the residual capacity of edge $V_2 \leftrightarrow V_4$ decreases from 5 to 0. Hence, edge $V_2 \leftrightarrow V_4$ is eliminated from the residual graph. The graph describing this reduced problem is given in Figure 5.2.

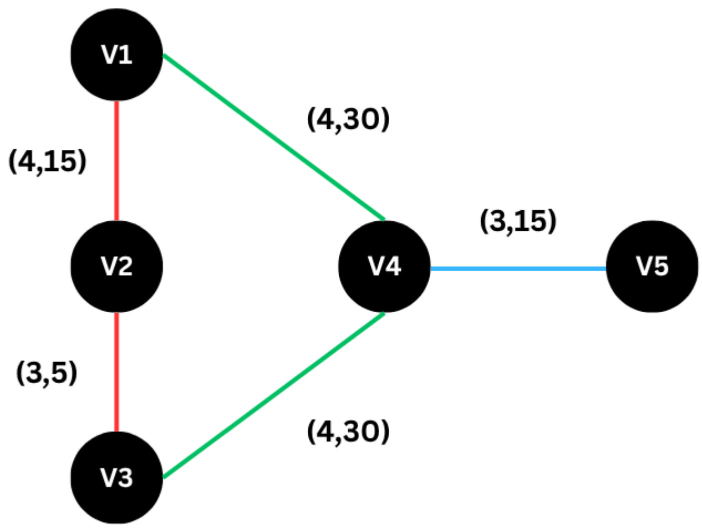


Figure 5.2. The Graph Describing the Reduced Network used to Derive the First Update of the Priorities of Routes with Unsatisfied Demand.

Since an edge has been removed, the priorities must now be updated. In the updated graph, the remaining routes are reconsidered. Table 5.2 gives the updated priorities. Note that if the cheapest path for a route is feasible after updating, then it remains the cheapest. Given that the cheapest path for a route is still feasible, if the 2nd cheapest path is still feasible, then it remains the second cheapest path.

Table 5.2 Updated Priorities of the Remaining Routes

Route	Cheapest Path	Cost	2nd Cheapest Path	Cost	Max. Rev.	Priority
(V_1, V_2)	$V_1 \rightarrow V_2$	4	$V_1 \rightarrow V_4 \rightarrow V_3 \rightarrow V_2$	11	5	1
(V_1, V_3)	$V_1 \rightarrow V_2 \rightarrow V_3$	7	$V_1 \rightarrow V_4 \rightarrow V_3$	8	10	½
(V_1, V_4)	$V_1 \rightarrow V_4$	4	$V_1 \rightarrow V_2 \rightarrow V_3 \rightarrow V_4$	11	5	1

(V_1, V_5)	$V_1 \rightarrow V_4 \rightarrow V_5$	7	$V_1 \rightarrow V_2 \rightarrow V_3 \rightarrow V_4 \rightarrow V_5$	14	10	3/2
(V_3, V_4)	$V_3 \rightarrow V_4$	4	$V_3 \rightarrow V_2 \rightarrow V_1 \rightarrow V_4$	11	5	1
(V_3, V_5)	$V_3 \rightarrow V_4 \rightarrow V_5$	7	$V_3 \rightarrow V_2 \rightarrow V_1 \rightarrow V_4 \rightarrow V_5$	14	10	3/2

Next, the route (V_1, V_5) is served through path $V_1 \rightarrow V_4 \rightarrow V_5$ with flow 5. This reduces the residual capacities of the edges $V_1 \leftrightarrow V_4$ and $V_4 \leftrightarrow V_5$ to 25 and 10, respectively. The route (V_3, V_5) is then served via the path $V_3 \rightarrow V_4 \rightarrow V_5$ with flow 5, reducing the residual capacities of the edges $V_3 \leftrightarrow V_4$ and $V_4 \leftrightarrow V_5$ to 25 and 5, respectively. The route (V_1, V_2) is served directly through edge $V_1 \rightarrow V_2$ with flow 10, reducing its residual capacity from 15 to 5. The route (V_1, V_4) is then served directly through edge $V_1 \rightarrow V_4$ with flow 10, reducing its residual capacity from 25 to 15. Similarly, the route (V_3, V_4) is served directly through edge $V_3 \rightarrow V_4$ with flow 10, reducing its residual capacity from 25 to 15.

At this stage, the only unsatisfied route is (V_1, V_3), with a total demand of 10. Its cheapest path is $V_1 \rightarrow V_2 \rightarrow V_3$, with a total cost of 7. However, after the previous allocations, both edges $V_1 \leftrightarrow V_2$ and $V_2 \leftrightarrow V_3$ have only 5 units of residual capacity. Therefore, only 5 units of flow can be assigned along this cheapest path. This fully exhausts the capacities of edges $V_1 \leftrightarrow V_2$ and $V_2 \leftrightarrow V_3$, so both edges are removed from the network. The remaining demand of 5 units for route (V_1, V_3) is then assigned through the only remaining feasible path, namely $V_1 \rightarrow V_4 \rightarrow V_3$, with a total cost of 8. It should be noted that if the residual graph is a tree and all of the residual demand for the routes can be met economically using the (uniquely defined) corresponding paths, then profit is maximized by satisfying all of the demand.

Table 5.3 presents the resulting flows. The total flow in the network is decomposed into components that indicate how the demand between each pair of vertices is satisfied through specific transmission paths. Each component represents the amount of information transmitted along a particular path and the corresponding route it serves. This flow decomposition is essential for the subsequent analysis because it identifies the edges that actually carry profitable traffic in the network. Based on this information, the network's revenue can be allocated to providers according to the economic importance of the edges they control.

Table 5.3 Decomposition of the Proposed Flow Example 1

Route	Path(s) Used	Flow(s)	Cost(s)	Revenue	Profit
(V_1, V_2)	$V_1 \rightarrow V_2$	10	4	5	10
(V_1, V_3)	$V_1 \rightarrow V_2 \rightarrow V_3$; $V_1 \rightarrow V_4 \rightarrow V_3$	5; 5	7; 8	10	25
(V_1, V_4)	$V_1 \rightarrow V_4$	10	4	5	10
(V_1, V_5)	$V_1 \rightarrow V_4 \rightarrow V_5$	5	7	10	15
(V_2, V_3)	$V_2 \rightarrow V_3$	10	3	5	20
(V_2, V_4)	$V_2 \rightarrow V_4$	10	3	5	20
(V_2, V_5)	$V_2 \rightarrow V_4 \rightarrow V_5$	5	6	10	20
(V_3, V_4)	$V_3 \rightarrow V_4$	10	4	5	10
(V_3, V_5)	$V_3 \rightarrow V_4 \rightarrow V_5$	5	7	10	15
(V_4, V_5)	$V_4 \rightarrow V_5$	10	3	5	20

Using this decomposition, the total profit of the grand coalition is the sum of the profits gained along the routes, i.e., 165. To verify that this flow is optimal, we calculate appropriate reduced costs for the fully utilized edges.

In the present solution, the edges $V_1 \leftrightarrow V_2$, $V_2 \leftrightarrow V_3$, and $V_2 \leftrightarrow V_4$ are fully utilized. Denote the nonnegative increments added to the unit costs of these edges by $\delta_{1,2}$, $\delta_{2,3}$, and $\delta_{2,4}$, respectively. The route (V_1, V_3) is served by two paths: $V_1 \rightarrow V_2 \rightarrow V_3$, and $V_1 \rightarrow V_4 \rightarrow V_3$. According to the first optimality condition, the adjusted costs of these two paths must be equal. This gives $\delta_{1,2} + \delta_{2,3} = 1$. That is, the sum of the increments on the unit costs for edges $V_1 \leftrightarrow V_2$ and $V_2 \leftrightarrow V_3$ must equal 1. An admissible choice is $\delta_{1,2} = 0.5$, $\delta_{2,3} = 0.5$, and $\delta_{2,4} = 0$. That is, the increments on the unit costs for edges $V_1 \leftrightarrow V_2$ and $V_2 \leftrightarrow V_3$ are each set to 0.5, and there is no increment for $V_2 \leftrightarrow V_4$. With these increments, the solution's paths remain the shortest in the graph with adjusted costs. In particular, the two paths used for the route (V_1, V_3) now have equal, minimal adjusted costs. Therefore, the optimality conditions are satisfied, confirming that the flow is optimal. If these conditions were not satisfied, the solution could be improved by shifting some flow from a more expensive path to a cheaper one, based on the adjusted costs. Thus, the optimal capacity-constrained flow generates a total profit of 165. This flow decomposition also serves as the basis for calculating the economic centrality of edges and determining the corresponding provider payoffs in the next subsection.

5.5.2 Example 2

Consider the problem described in Example 1, Section 5.5.1, such that the capacity of the edge between V_1 and V_4 is reduced to 15. The calculation of the initial priorities gives the same results. After assigning flows to the four routes with the highest priority, as in Example 1, we obtained the residual graph given in Figure 5.3.

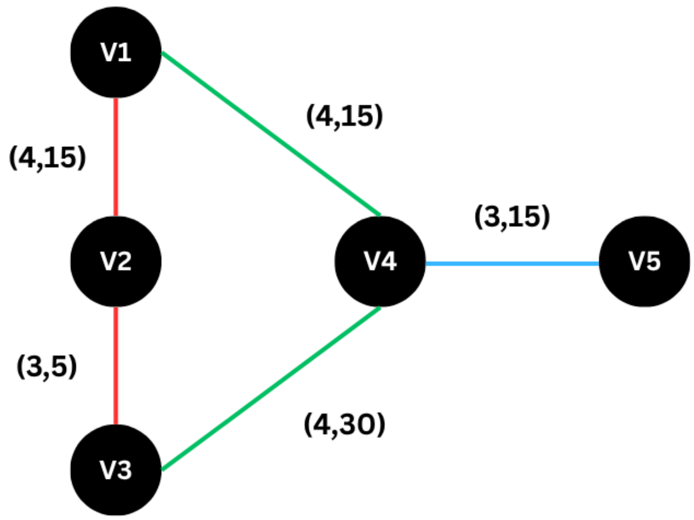


Figure 5.3. The Graph Describing the Reduced Network used to Derive the First Update of the Priorities of Routes with Unsatisfied Demand in Example 2.

Again, the updated priorities of the routes are the same as in Example 1. Assigning priority to the route (V_1, V_5), we send five units along the edges $V_1 \rightarrow V_5$ and $V_4 \rightarrow V_5$. This route is removed, and the capacities of the edges $V_1 \rightarrow V_4$ and $V_4 \rightarrow V_5$ are reduced to 10 units. Next, the route (V_3, V_5) has priority. We send five units along the edges $V_3 \rightarrow V_4$ and $V_4 \rightarrow V_5$. This route is removed, the capacity of the edge $V_3 \rightarrow V_4$ is reduced to 25 units, and the capacity of the edge $V_4 \rightarrow V_5$ is reduced to 5 units. Now assign priority to the route (V_1, V_2). We send ten units along the edge $V_1 \rightarrow V_2$. This route is removed, and the capacity of the edge is reduced to 5 units. Now assign priority to the route (V_3, V_4). We send ten units along the edge $V_3 \rightarrow V_4$. This route is removed, and the capacity of the edge is reduced to 15 units. Now the route (V_1, V_4) has priority. We send ten units along the edge $V_1 \rightarrow V_4$. This edge now has no excess capacity. The residual graph is given in Figure 5.4.

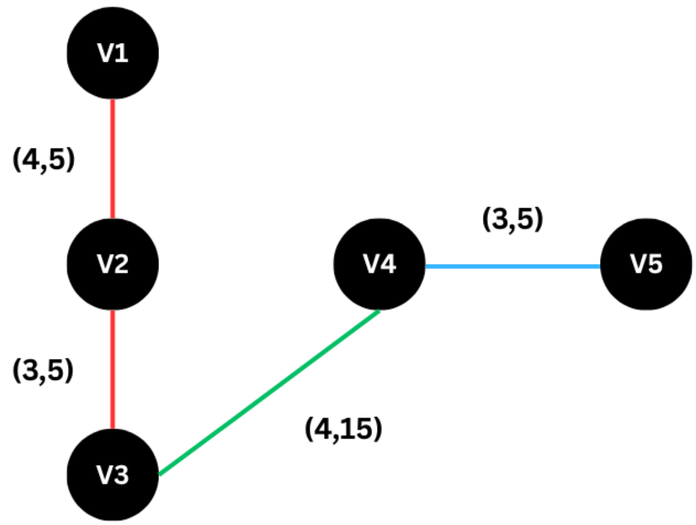


Figure 5.4. The Graph Describing the Reduced Network used to Derive the Second Update of the Priorities of Routes with Unsatisfied Demand in Example 2.

The only remaining route is (V_1, V_3). We can send five units along the path $V_1 \rightarrow V_2 \rightarrow V_3$. This concludes the flow augmentation procedure. The proposed solution is given in Table 5.4,

Table 5.4 Decomposition of the Proposed Flow Example 2

Route	**Path(s) Used**	**Flow(s)**	**Cost(s)**	**Revenue**	**Profit**
(V_1, V_2)	$V_1 \rightarrow V_2$	10	4	5	10
(V_1, V_3)	$V_1 \rightarrow V_2 \rightarrow V_3$	5	7	10	15
(V_1, V_4)	$V_1 \rightarrow V_4$	10	4	5	10
(V_1, V_5)	$V_1 \rightarrow V_4 \rightarrow V_5$	5	7	10	15
(V_2, V_3)	$V_2 \rightarrow V_3$	10	3	5	20
(V_2, V_4)	$V_2 \rightarrow V_4$	10	3	5	20
(V_2, V_5)	$V_2 \rightarrow V_4 \rightarrow V_5$	5	6	10	20
(V_3, V_4)	$V_3 \rightarrow V_4$	10	4	5	10
(V_3, V_5)	$V_3 \rightarrow V_4 \rightarrow V_5$	5	7	10	15
(V_4, V_5)	$V_4 \rightarrow V_5$	10	3	5	20

The total profit is 155. The only route for which the demand is not satisfied is (V_1, V_3). The two profitable paths for this route are $V_1 \rightarrow V_2 \rightarrow V_3$ and $V_1 \rightarrow V_4 \rightarrow V_3$. We first consider increasing the flow along the path $V_1 \rightarrow V_2 \rightarrow V_3$. A unit increase in such a flow gives a marginal profit of 3. Since no flow has been assigned to paths that strictly contain $V_1 \rightarrow V_2 \rightarrow V_3$, we only have to consider solutions that balance such an increase by possibly decreasing flows in the constituent paths (edges). $V_1 \rightarrow V_2$ and $V_2 \rightarrow V_3$. The only flow through these edges that does not serve the route (V_1, V_3) serves the routes (V_1, V_2) and (V_2, V_3). Since there is no residual capacity in either of these edges, increasing the flow in the path $V_1 \rightarrow V_2 \rightarrow V_3$ by a unit has to be accompanied by a unit decrease in both of the flows along the path $V_1 \rightarrow V_2$ and $V_2 \rightarrow V_3$. These decreases correspond to marginal falls in profits 1 and 2, respectively. It follows that such a change in the assignment of the flows does not lead to a change in the profits. It should be noted that such a change in the assignment of the flows does not change the amount flowing in each edge of the network. It can thus be seen that when some demand is not satisfied, various total flows in the path corresponding to a route can correspond to the same overall flow in the network. However, when the demand for each route is satisfied, such shifts in the assignments of flows through paths are impossible, since the supply for some route would exceed the demand for it.

We now consider increasing the flow along the path $V_1 \rightarrow V_4 \rightarrow V_3$. A unit increase in such a flow gives a marginal profit of 2. Since no flow has been assigned to paths that strictly contain $V_1 \rightarrow V_4 \rightarrow V_3$, we only have to consider solutions that balance such an increase by possibly decreasing flows in the constituent paths (edges), $V_1 \rightarrow V_4$ and $V_3 \rightarrow V_4$. Since there is residual capacity of 15 in the edge $V_3 \rightarrow V_4$, it follows that it is only necessary to reduce flow in the edge $V_1 \rightarrow V_4$. This gives a marginal reduction in the profit of one. Hence, it is optimal to increase flow in the path $V_1 \rightarrow V_4 \rightarrow V_3$ by five units and balance this with an analogous reduction in the flow through the path $V_1 \rightarrow V_4$. Hence, the total profit increases to 160.

Since the only route with unsatisfied demand is $V_1 \rightarrow V_4$, and increasing flow in this path would lead to a fall in profit, this is the optimal flow as described in Table 5.5.

Table 5.5 Decomposition of the Proposed Optimal Flow Example 2

Route	**Path(s) Used**	**Flow(s)**	**Cost(s)**	**Revenue**	**Profit**
(V_1, V_2)	$V_1 \rightarrow V_2$	10	4	5	10
(V_1, V_3)	$V_1 \rightarrow V_2 \rightarrow V_3$; $V_1 \rightarrow V_4 \rightarrow V_3$	5; 5	7; 8	10	25
(V_1, V_4)	$V_1 \rightarrow V_4$	5	4	5	5
(V_1, V_5)	$V_1 \rightarrow V_4 \rightarrow V_5$	5	7	10	15
(V_2, V_3)	$V_2 \rightarrow V_3$	10	3	5	20

(V_2, V_4)	$V_2 \rightarrow V_4$	10	3	5	20
(V_2, V_5)	$V_2 \rightarrow V_4 \rightarrow V_5$	5	6	10	20
(V_3, V_4)	$V_3 \rightarrow V_4$	10	4	5	10
(V_3, V_5)	$V_3 \rightarrow V_4 \rightarrow V_5$	5	7	10	15
(V_4, V_5)	$V_4 \rightarrow V_5$	10	3	5	20

5.6 Profit Allocation Based on Centrality

After determining the optimal flow in the network, the next step is to allocate the total profit among the service providers. The allocation presented in this subsection is based on the economic centrality of the edges introduced in Section 5.4. Since each provider controls a subset of edges, the profit assigned to a provider is the sum of the centrality values of the edges it owns. This approach ensures that providers receive a share of the total profit that reflects the economic importance of the infrastructure that they contribute to the network.

Using the flow decomposition obtained in Section 5.5, the economic centrality of each edge can be calculated according to one of two rules: proportional and uniform allocation. Under the proportional rule, the profit from each path is distributed among its edges based on their transmission costs. Under the uniform rule, the profit from a path is divided equally among all its edges. Next, applying the proportional allocation rule and the uniform allocation rule yields the following economic centrality values for the edges, as can be seen in Tables 5.6 and 5.8.

Table 5.6 Calculation of the Proportional Measures of Economic Centrality. The Values in the Columns Corresponding to Edges Give the Appropriate Split of the Profit with Respect to a Given Path.

Path	**Profit**	**Edge (s)**					
		(V_1, V_2)	(V_1, V_4)	(V_2, V_3)	(V_2, V_4)	(V_3, V_4)	(V_4, V_5)
$V_1 \rightarrow V_2$	10	10	0	0	0	0	0
$V_1 \rightarrow V_2 \rightarrow V_3$	15	$\frac{60}{7}$	0	$\frac{45}{7}$	0	0	0
$V_1 \rightarrow V_4 \rightarrow V_3$	10	0	5	0	0	5	0
$V_1 \rightarrow V_4$	10	0	10	0	0	0	0
$V_1 \rightarrow V_4 \rightarrow V_5$	15	0	$\frac{60}{7}$	0	0	0	$\frac{45}{7}$
$V_2 \rightarrow V_3$	20	0	0	20	0	0	0
$V_2 \rightarrow V_4$	20	0	0	0	20	0	0

$V_2 \rightarrow V_4 \rightarrow V_5$	20	0	0	0	10	0	10
$V_3 \rightarrow V_4$	10	0	0	0	0	10	0
$V_3 \rightarrow V_4 \rightarrow V_5$	15	0	0	0	0	$\frac{60}{7}$	$\frac{45}{7}$
$V_4 \rightarrow V_5$	20	0	0	0	0	0	20

Table 5.7 Economic: Centrality of Network Edges Under Proportional Measure

Edge (s)	$\tilde{\mathbf{C}}_{\boldsymbol{P}}(\boldsymbol{e})$
(V_1, V_2)	$\frac{130}{7} = 18{,}571$
(V_1, V_4)	$\frac{165}{7} = 23{,}571$
(V_2, V_3)	$\frac{185}{7} = 26{,}428$
(V_2, V_4)	30
(V_3, V_4)	$\frac{165}{7} = 23{,}571$
(V_4, V_5)	$\frac{300}{7} = 42{,}857$
Σ	**165**

Table 5.8 Calculation of the Uniform Measures of Economic Centrality. The Values in the Columns Corresponding to Edges Give the Appropriate Split of the Profit with Respect to a Given Path.

Path	**Profit**	**Edge (s)**					
		(V_1, V_2)	**(V_1, V_4)**	**(V_2, V_3)**	**(V_2, V_4)**	**(V_3, V_4)**	**(V_4, V_5)**
$V_1 \rightarrow V_2$	10	10	0	0	0	0	0
$V_1 \rightarrow V_2 \rightarrow V_3$	15	$\frac{15}{2}$	0	$\frac{15}{2}$	0	0	0
$V_1 \rightarrow V_4 \rightarrow V_3$	10	0	5	0	0	5	0
$V_1 \rightarrow V_4$	10	0	10	0	0	0	0
$V_1 \rightarrow V_4 \rightarrow V_5$	15	0	$\frac{15}{2}$	0	0	0	$\frac{15}{2}$
$V_2 \rightarrow V_3$	20	0	0	20	0	0	0
$V_2 \rightarrow V_4$	20	0	0	0	20	0	0
$V_2 \rightarrow V_4 \rightarrow V_5$	20	0	0	0	10	0	10
$V_3 \rightarrow V_4$	10	0	0	0	0	10	0

$V_3 \rightarrow V_4 \rightarrow V_5$	15	0	0	0	0	$\frac{15}{2}$	$\frac{15}{2}$
$V_4 \rightarrow V_5$	20	0	0	0	0	0	20

Table 5.9 Economic: Centrality of Network Edges Under Uniform Measures

Edge (s)	$\tilde{\mathbf{C}}_{U}(\boldsymbol{e})$
(V_1, V_2)	$\frac{35}{2} = 17{,}5$
(V_1, V_4)	$\frac{45}{2} = 22{,}5$
(V_2, V_3)	$\frac{55}{2} = 27{,}5$
(V_2, V_4)	30
(V_3, V_4)	$\frac{45}{2} = 22{,}5$
(V_4, V_5)	45
Σ	**165**

To determine the providers' payoffs, the centrality values of each provider's edges are summed. In this example, the red provider controls edges (V_1, V_2) and (V_2, V_3), the blue provider controls edges (V_2, V_4) and (V_4, V_5) and the green provider controls edges (V_1, V_4) and (V_3, V_4). Under the proportional allocation rule and uniform allocation rule, the providers' payoffs can be seen in Tables 5.10 and 5.11, respectively:

Table 5.10 Payoffs Under the Proportional Allocation Rule

Player	Payoffs
Player 1 (Red)	$\frac{130}{7} + \frac{185}{7} = 45$
Player 2 (Blue)	$30 + \frac{300}{7} = \frac{510}{7} \approx 72{,}86$
Player 3 (Green)	$\frac{165}{7} + \frac{165}{7} = \frac{330}{7} \approx 47{,}14$

Table 5.11 Payoffs Under the Uniform Allocation Rule

Player	Payoffs
Player 1 (Red)	$\frac{35}{2} + \frac{55}{2} = 45$
Player 2 (Blue)	$30 + 45 = 75$

Player 3 (Green)	$\frac{45}{2} + \frac{45}{2} = 45$

These results show that the blue provider receives the largest share of the profit under both allocation rules. This occurs because the edges controlled by this provider, particularly (V_4, V_5), carry a significant portion of the network's profitable flow. Since all routes to node V_5 must pass through this edge, it plays a crucial role in satisfying demand and generating revenue. Consequently, the provider controlling this infrastructure obtains the largest payoff in the cooperative network game. The green player is assigned a larger payoff according to the proportional allocation rule than according to the uniform allocation rule. This is due to the fact that the green player has higher transmission costs. When such costs are justifiable, the proportional allocation rule could be argued as being fairer. However, when cost reductions are possible, the uniform allocation gives a greater incentive to reduce costs.

5.7 Formulation of Such Problems as a Cooperative Game

The analysis of flow optimization and profit allocation presented in Sections 5.5 and 5.6 assumes that all providers cooperate within the grand coalition. To extend the model to a cooperative game-theoretic framework, it is necessary to evaluate the profit that each coalition of providers can obtain when competing against the complementary coalition. This yields a characteristic function that captures the strategic value of each possible coalition.

The competitive interaction between coalitions is modeled through an iterated second-price auction mechanism. In each round of the auction, both coalitions announce the lowest price at which they are willing to supply a given route, corresponding to the cost of the cheapest, currently active, path under its control. The current price for route (i, j) is then set to the second-lowest bid with a ceiling price $r_{i,j}$. If a route is not economically viable for a coalition, i.e., the cost for serving that route is $\geq r_{i,j}$, that coalition does not submit a bid. When both coalitions make the same bid for a route $p_{i,j}$, then the current price is set to be $p_{i,j} + \varepsilon$, where ε is a positive constant, assumed to be small enough that supply using any path of cost $> p_{i,j}$ is not profitable. The coalitions making the lowest price bid for a route are invited to supply that route. Once these invitations are given, each coalition announces how much it is willing to supply on each route at the given price (an amount not more than the demand for a route). These volumes are chosen to maximize a coalition's profit at the current prices, subject to capacity constraints on that coalition and independently of the opposing coalition. This optimization is carried out using the same procedure as described in Section 5.5. When both coalitions announce the same price for a route and the total volume announced by them is greater than demand, then it is assumed that the demand for that route is split between the coalitions in proportion to the announced supplies. Both coalitions are then obliged to supply the assigned amounts at the final auction price, which is guaranteed not to be less than the price set in a given round.

If unsatisfied demand remains and reserve capacity exists to economically supply such demand after a round of bidding, the auction proceeds to a subsequent round with both coalitions bidding for the routes with unsatisfied demand according to the residual capacities of their networks. The auction terminates when all demand has been served, or no coalition has remaining capacity to economically serve a route with residual demand.

Suppose that a coalition has been invited to supply a route in a round of the auction. Under the assumptions of the bidding system, it is clearly profitable for this coalition to serve that route using the least-cost paths in the residual network that it controls. Hence, under the optimal responses to the invitations to supply a route in a round, either all the demand for a route will be satisfied, or the coalitions will be unable to supply any more flow along the corresponding least-cost paths. It follows that the lowest price bids for the active routes in the next round of bidding must be higher than the analogous bid in the current round. Thus, any such auction must end after a finite number of rounds.

The auction procedure is illustrated based on Example 1 given in Section 5.5.1. Three subgames are defined, one for each possible coalition of two providers competing against the remaining singleton provider. Using the network introduced in Section 5.5.1, the three providers are referred to as Red (R), Green (G), and Blue (B). For each subgame, the profit of the two-provider coalition and that of the singleton provider are determined by the auction mechanism described above. These results, together with the grand coalition profit of 165 established in Section 5.5.2, yield the characteristic function of the game.

5.7.1 Subgame 1: Coalition {*R*, *G*} versus *B*

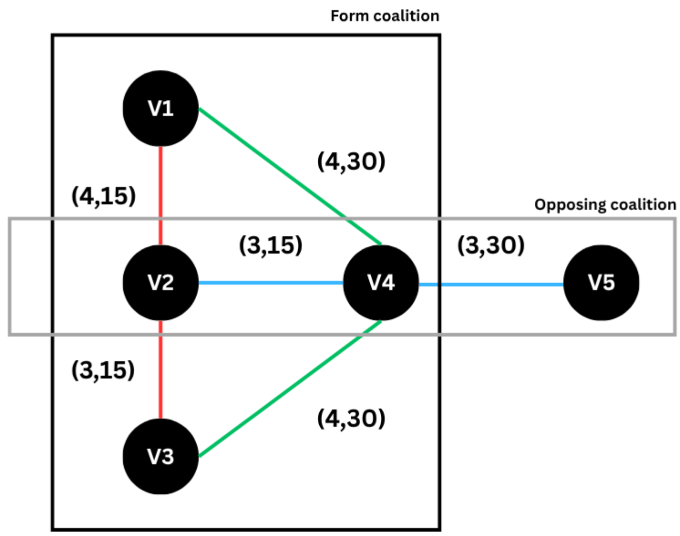


Figure 5.5 Coalition Formation and Competition Structure in the Cooperative Network Subgame 1

First, consider the game in which the red and green providers form a coalition. The blue provider serves as the opposing coalition, as shown in Figure 5.5. The analysis begins by identifying the set of routes that can be served under this coalition structure. Note that the routes (V_1, V_5) and (V_3, V_5) cannot be served by either coalition. These routes are therefore excluded from the analysis, and both coalitions obtain zero profit from them. Furthermore, both coalitions are technically able to serve the route (V_2, V_4). However, the red and blue coalition cannot serve this route economically, since the cost of the corresponding cheapest path exceeds the maximum revenue. As a result, this coalition does not submit a bid for this route, leaving it to the green provider.

In the first round of bidding, the red and green coalition submits bids for the routes (V_1, V_2), (V_1, V_3), (V_1, V_4), (V_2, V_3), and (V_3, V_4). The announced prices correspond to the costs of the cheapest paths controlled by the coalition and are equal to 4, 7, 4, 3, and 4, respectively. Simultaneously, the blue provider submits bids for the routes (V_2, V_4), (V_2, V_5), and (V_4, V_5), with announced prices of 3, 6, and 3, respectively. Because no route is contested by both coalitions, there is no effective competition. Therefore, the final price for each route is equal to the ceiling price, $r_{i,j}$. Consequently, the first round of the auction procedure decomposes into two independent optimization problems, one for each coalition.

Focusing first on the blue provider, the routes assigned to this coalition each have a priority of two. Prioritizing these routes in any order, the provider can satisfy all demand without conflicts between paths. Since the network controlled by the blue provider is a tree, this is the only set of flows that matches the demand. Hence, the optimal flow allocation is as follows:

- A flow of 5 units along the path $V_2 \rightarrow V_4 \rightarrow V_5$
- A flow of 10 units along the path $V_2 \rightarrow V_4$
- A flow of 10 units along the path $V_4 \rightarrow V_5$

All demand on these routes is satisfied, and the resulting profit is $5 \times (10 - 6) + 20 \times (5 - 3) = 60$.

Turning to the red and green coalition, they are invited to serve five routes. The priorities of these routes are described in Table 5.12.

Table 5.12 Initial Priorities of the Routes in the Red and Green Coalition.

Route	**Cheapest Path**	**Cost**	**2nd Cheapest Path**	**Cost**	**Max. Rev.**	**Priority**
(V_1, V_2)	$V_1 \rightarrow V_2$	4	$V_1 \rightarrow V_4 \rightarrow V_3 \rightarrow V_2$	11	5	1
(V_1, V_3)	$V_1 \rightarrow V_2 \rightarrow V_3$	7	$V_1 \rightarrow V_4 \rightarrow V_3$	8	10	½
(V_1, V_4)	$V_1 \rightarrow V_4$	4	$V_1 \rightarrow V_2 \rightarrow V_3 \rightarrow V_4$	11	5	1
(V_2, V_3)	$V_2 \rightarrow V_3$	3	$V_2 \rightarrow V_1 \rightarrow V_4 \rightarrow V_3$	12	5	2
(V_3, V_4)	$V_3 \rightarrow V_4$	4	$V_3 \rightarrow V_2 \rightarrow V_1 \rightarrow V_4$	11	5	1

Hence, initially 10 units of flow are assigned to the route (V_2, V_3) along the path $V_2 \rightarrow V_3$. Following this, 10 units of the flow can be assigned to each of the routes (V_1, V_2), (V_1, V_4), and (V_3, V_4) along the corresponding shortest path. The residual capacities of the edges $V_1 \rightarrow V_2$ and $V_2 \rightarrow V_3$ are both equal to five. Hence, 5 units of flow are assigned to the route (V_1, V_3) along path $V_1 \rightarrow V_2 \rightarrow V_3$. The remaining demand for this route is satisfied by assigning 5 units of flow to the path $V_1 \rightarrow V_4 \rightarrow V_3$.

Considering the reduced costs of flows along edges at such a solution, only edges $V_1 \rightarrow V_2$ and $V_2 \rightarrow V_3$ are fully utilized. Since the paths $V_1 \rightarrow V_2 \rightarrow V_3$ and $V_1 \rightarrow V_4 \rightarrow V_3$ of cost 7 and 8, respectively, are utilized at the given solution, it follows that $\delta_{1,2} + \delta_{2,3} = 1$. Setting $\delta_{1,2} = \delta_{2,3} = 0.5$, all of the paths used are the shortest route in the problem, where these reduced costs are added to the original cost. Hence, this solution is optimal. The resulting profit of the red and green coalition from each route is outlined in Table 5.13:

Table 5.13 The Resulting Profit of the Routes in the Red and Green Coalition.

Route	Path Used	Volume(s)	Cost(s)	Revenue	Profit
(V_1, V_2)	$V_1 \rightarrow V_2$	10	4	5	10
(V_1, V_3)	$V_1 \rightarrow V_2 \rightarrow V_3$; $V_1 \rightarrow V_4 \rightarrow V_3$	5; 5	7; 8	10	25
(V_1, V_4)	$V_1 \rightarrow V_4$	10	4	5	10
(V_2, V_3)	$V_2 \rightarrow V_3$	10	3	5	20
(V_3, V_4)	$V_3 \rightarrow V_4$	10	4	5	10

Since all of the demand for each of the routes that can be served is satisfied by one of the coalitions, the auction procedure terminates after the first round. Hence, the values of the coalitions in this subgame are given by $v(\{R, G\}) = 75$ and $v(\{B\}) = 60$. This subgame shows that when no direct competition arises, coalition values depend on cost efficiency and the ability to allocate flows within capacity constraints.

5.7.2 Subgame 2: Coalition {*G*, *B*} versus *R*

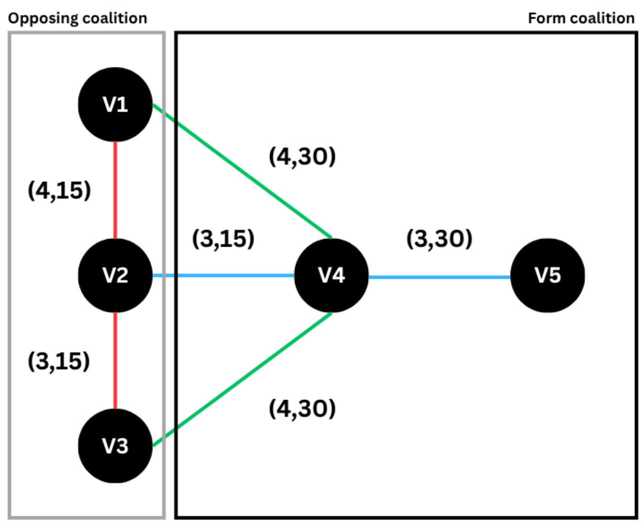


Figure 5.6 Coalition Formation and Competition Structure in the Cooperative Network Subgame 2

We now consider the subgame in which the green and blue providers form a coalition, while the red provider acts as the opposing coalition, as shown in Figure 5.6. In this case, every route in the network can be served by at least one of the two coalitions. However, the ability to serve a route does not automatically imply that it is economically rational to bid for that route. The value of each coalition, therefore, depends not only on connectivity but also on the relationships among transmission costs, available capacities, and the auction mechanism.

Both coalitions can serve the routes (V_1, V_2), (V_1, V_3), and (V_2, V_3). These are also the only routes served by the red provider. However, the green and blue coalition cannot economically serve the routes (V_1, V_2) and (V_2, V_3), because the costs of the corresponding cheapest feasible paths are too high relative to the available revenues. For this reason, the green and blue coalition does not bid for these two routes.

In the first round of the auction, the green and blue coalition submits bids for the routes (V_1, V_3), (V_1, V_4), (V_1, V_5), (V_2, V_4), (V_2, V_5), (V_3, V_4), (V_3, V_5), and (V_4, V_5). The announced prices are 8, 4, 7, 3, 6, 4, 7, and 3, respectively. The red provider submits bids for the routes (V_1, V_2), (V_1, V_3), and (V_2, V_3), with announced prices of 4, 7, and 3, respectively. As a result of this bidding process, the red provider is assigned the routes (V_1, V_2), (V_1, V_3), and (V_2, V_3). Since both coalitions bid for route (V_1, V_3), the current unit revenue is determined by the second price bid and is therefore equal to 8. For the routes (V_1, V_2) and (V_2, V_3), there is no effective competition, so the final revenue is equal to the ceiling price, that is, 5 in each case.

The red provider must then determine the optimal flow allocation across these three routes. Due to the limited capacities of its edges, it is impossible to satisfy the full demand on all three routes simultaneously. Consequently, the problem is not one of finding the cheapest feasible flow. Instead, it is strictly a problem of maximizing profit subject to capacity constraints. Since this is a relatively simple problem, it can be solved analytically. Let $x_{1,3}$ denote the amount of flow assigned to route (V_1, V_3) along the path $V_1 \rightarrow V_2 \rightarrow V_3$, where $0 \leq x_{1,3} \leq 10$. The maximum flows that can be allocated to the direct routes (V_1, V_2) and (V_2, V_3) are each equal to $15 - x_{1,3}$. Since each of these route flows cannot exceed the corresponding demand of 10, it follows that $x_{1,3}$ must be at least 5. The current profit margins for the routes (V_1, V_2), (V_2, V_3), and (V_1, V_3) are 1, 2, and 1, respectively. Under this allocation, the total profit of the red provider is given by

$$\left(15 - x_{1,3}\right) + 2\left(15 - x_{1,3}\right) + 4x_{1,3} = 45 - 2x_{1,3}$$

This expression is decreasing in $x_{1,3}$. Therefore, profit is maximized when $x_{1,3}$ takes the smallest feasible value, namely $x_{1,3} = 5$. It follows that the optimal allocation for the red provider is to assign a flow of 10 to route (V_1, V_2), a flow of 10 to route (V_2, V_3), and a flow of 5 to route (V_1, V_3). It should be noted that this is the solution found by the flow augmentation algorithm. The priorities of the routes (V_1, V_2), (V_2, V_3), and

(V_1, V_3) are 1, 2, and 0.5, respectively. After assigning 10 units of flow to the routes (V_2, V_3) and (V_1, V_2), the red provider can only assign 5 units of flow to the route (V_1, V_3).

In this initial round, the green and blue coalition is assigned the routes (V_1, V_4), (V_1, V_5), (V_2, V_4), (V_2, V_5), (V_3, V_4), (V_3, V_5), and (V_4, V_5). The subnetwork controlled by this coalition forms a tree. Moreover, the capacities of its edges are sufficient to satisfy all assigned demand, and each of these routes generates positive profit. Hence, the coalition can maximize its payoff simply by satisfying the full demand on each assigned route. The optimal flows are as follows:

- For the route (V_1, V_4), a flow of 10 is sent along $V_1 \rightarrow V_4$.
- For route (V_1, V_5), a flow of 5 is sent along $V_1 \rightarrow V_4 \rightarrow V_5$.
- For route (V_2, V_4), a flow of 10 is sent along $V_2 \rightarrow V_4$.
- For route (V_2, V_5), a flow of 5 is sent along $V_2 \rightarrow V_4 \rightarrow V_5$.
- For the route (V_3, V_4), a flow of 10 is sent along $V_3 \rightarrow V_4$.
- For route (V_3, V_5), a flow of 5 is sent along $V_3 \rightarrow V_4 \rightarrow V_5$.
- For the route (V_4, V_5), a flow of 10 is sent along $V_4 \rightarrow V_5$.

After the first round of the auction, five units of demand for the route (V_1, V_3) remain unsatisfied. At this stage, the red provider has no residual capacity and therefore cannot compete for this remaining demand. In contrast, the green and blue coalition still has available capacity on the edges $V_1 \leftrightarrow V_4$ and $V_3 \leftrightarrow V_4$, each with 15 units of residual capacity. Consequently, in the second round of the auction, the green and blue coalition submits a bid to serve the remaining demand on route (V_1, V_3) at a cost of 8. Since the red provider cannot contest this bid, the coalition is assigned the remaining demand. The final unit revenue for this route is therefore equal to the ceiling price, $r_{1,3} = 10$. This final price is also paid to the red provider. The profits gained by the two coalitions are given in Tables 5.14 and 5.15.

Table 5.14 The Resulting Profit of the Routes in the Red Coalition.

Route	**Path Used**	**Volume(s)**	**Cost(s)**	**Revenue**	**Profit**
(V_1, V_2)	$V_1 \rightarrow V_2$	10	4	5	10
(V_1, V_3)	$V_1 \rightarrow V_2 \rightarrow V_3$	5	7	10	15
(V_2, V_3)	$V_2 \rightarrow V_3$	10	3	5	20

Table 5.15 The Resulting Profit of the Routes in the Blue and Green Coalition

Route	Path Used	Volume(s)	Cost(s)	Revenue	Profit
(V_1, V_3)	$V_1 \rightarrow V_4 \rightarrow V_3$	5	8	10	10
(V_1, V_4)	$V_1 \rightarrow V_4$	10	4	5	10
(V_1, V_5)	$V_1 \rightarrow V_4 \rightarrow V_5$	5	7	10	15
(V_2, V_4)	$V_2 \rightarrow V_4$	10	3	5	20
(V_2, V_5)	$V_2 \rightarrow V_4 \rightarrow V_5$	5	6	10	20
(V_3, V_4)	$V_3 \rightarrow V_4$	10	4	5	10
(V_3, V_5)	$V_3 \rightarrow V_4 \rightarrow V_5$	5	7	10	15
(V_4, V_5)	$V_4 \rightarrow V_5$	10	3	5	20

The total value of the auction to the green and blue coalition is therefore 120. The total value of the auction to the red provider is 45. Thus $v(\{G, B\}) = 120$ and $v(\{R\}) = 45$. This subgame illustrates clearly that, under capacity constraints, the value of a coalition depends not only on the routes it can initially win in the auction but also on the residual capacities it retains after the first allocation. The green and blue coalition obtains a substantial advantage because its tree structure allows it to efficiently satisfy all assigned demand while still preserving enough spare capacity to capture additional demand in a later auction round.

5.7.3 Subgame 3: Coalition $\{R, B\}$ versus G

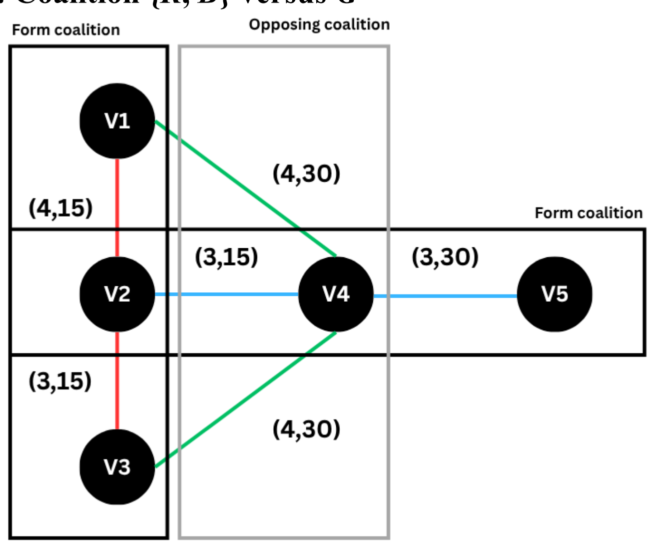


Figure 5.7 Coalition Formation and Competition Structure in the Cooperative Network Subgame 3

We now consider the subgame in which the red and blue providers form a coalition, while the green provider acts as the opposing coalition, as shown in Figure 5.7. In this case, every route can be served by at least one of the two coalitions. However, the route (V_1, V_5) is not profitable for the red-blue coalition. Since the green provider cannot serve this route, no bids for this route are made in the auction, and neither side generates any profit from it. Both coalitions can serve the routes (V_1, V_3), (V_1, V_4), and (V_3, V_4). These are also the only routes served by the green provider. However, the red and blue coalition cannot economically serve the routes (V_1, V_4) and (V_3, V_4). Therefore, it does not submit bids for these two routes.

In the first round of the auction, the red and blue coalition bids for the routes (V_1, V_2), (V_1, V_3), (V_2, V_3), (V_2, V_4), (V_2, V_5), (V_3, V_5), and (V_4, V_5), announcing prices equal to 4, 7, 3, 3, 6, 9, and 3, respectively. The green provider bids for the routes (V_1, V_3), (V_1, V_4), and (V_3, V_4), announcing prices of 8, 4, and 4, respectively. As a result of the auction, the green provider is assigned the routes (V_1, V_4) and (V_3, V_4). Since there is no effective competition on these routes, the unit revenue on each route equals the ceiling price of 5. The network controlled by the green provider forms a tree, and the capacities of its edges are sufficient to satisfy all demand on these routes. Therefore, the green provider maximizes its profit by assigning a flow of 10 units along $V_1 \rightarrow V_4$ and a flow of 10 units along $V_3 \rightarrow V_4$.

In the first round, the red and blue coalition is assigned the routes (V_1, V_2), (V_1, V_3), (V_2, V_3), (V_2, V_4), (V_2, V_5), (V_3, V_5), and (V_4, V_5). For the route (V_1, V_3), the current unit revenue is determined by the second price bid and is therefore equal to 8. For all remaining assigned routes, the revenue equals the ceiling price. The subnetwork controlled by the red and blue coalition is a tree, but its capacities are insufficient to meet the full demand on all assigned routes simultaneously. Consequently, the coalition must solve a profit-maximization problem subject to capacity constraints rather than a simple shortest-path allocation problem. The priorities ascribed to the routes are given in Table 5.16. Since the network controlled by this coalition is a tree, we do not have to consider the second-cheapest path.

Table 5.16 Initial Priorities of the Routes in the Red and Blue Coalition.

Route	**Cheapest Path**	**Cost**	**Revenue**	**Priority**
(V_1, V_2)	$V_1 \rightarrow V_2$	4	5	1
(V_1, V_3)	$V_1 \rightarrow V_2 \rightarrow V_3$	7	8	½
(V_2, V_3)	$V_2 \rightarrow V_3$	3	5	2

(V_2, V_4)	$V_2 \rightarrow V_4$	3	5	2
(V_2, V_5)	$V_2 \rightarrow V_4 \rightarrow V_5$	6	10	2
(V_3, V_5)	$V_3 \rightarrow V_2 \rightarrow V_4 \rightarrow V_5$	9	10	1/3
(V_4, V_5)	$V_4 \rightarrow V_5$	3	5	2

The demand for the routes with priority 2 can be supplied. This leads to a reduced graph in which there is no residual capacity for the edge $V_2 \rightarrow V_4$. The demand for route (V_1, V_2) is then satisfied. At this point, there is a residual capacity of 5 for both of the edges $V_1 \rightarrow V_2$ and $V_2 \rightarrow V_3$. This can be used to supply five units of demand for the route (V_1, V_3).

Under the current solution, the red and blue coalition cannot satisfy the remaining demand from the routes (V_1, V_5) and (V_3, V_5). The route (V_1, V_5) is not profitable and thus should not be served. Reassigning flows to satisfy demand on the route (V_3, V_5) requires a corresponding reduction of the flow on the edge $V_2 \rightarrow V_3$ (as well as other reductions) to satisfy demand for route (V_2, V_3). Since the profit margin on the route (V_2, V_3) is greater than the profit on the route (V_3, V_5) such a reassignment reduces profit. Hence, the flows announced in the first round by the red and blue coalition for each of the routes are given in Table 5.17.

Table 5.17 The Flows Announced by the Red and Blue Coalition after the First Round of the Auction.

Route	**Flow**
(V_1, V_2)	10
(V_1, V_3)	5
(V_2, V_3)	10
(V_2, V_4)	10
(V_2, V_5)	5
(V_3, V_5)	0
(V_4, V_5)	10

After the first round of the auction, there are 5 unsatisfied units of demand for each of the routes (V_1, V_3), (V_1, V_5), (V_3, V_5). The red and blue coalition has no residual capacity to serve this remaining demand. In contrast, the green provider has sufficient residual capacity on the edge $V_1 \rightarrow V_4$ and $V_4 \rightarrow V_3$ to submit a bid to serve the route (V_1, V_3) at a cost of 8. Since there is no competing bid, the final unit revenue for this route equals the ceiling price, that is, $r_{1,3} = 10$. The final allocations and profits from each route are described in Tables 5.18 and 5.19.

Table 5.18 The Resulting Profit of the Routes in the Green Coalition

Route	Path Used	Volume(s)	Cost(s)	Revenue	Profit
(V_1, V_4)	$V_1 \rightarrow V_4$	10	4	5	10
(V_1, V_3)	$V_1 \rightarrow V_2 \rightarrow V_3$	5	8	10	10
(V_3, V_4)	$V_2 \rightarrow V_3$	10	4	5	10

Table 5.19 The Resulting Profit of the Routes in the Blue and Red Coalition.

Route	Path Used	Volume(s)	Cost(s)	Revenue	Profit
(V_1, V_2)	$V_1 \rightarrow V_2$	10	4	5	10
(V_1, V_3)	$V_1 \rightarrow V_4 \rightarrow V_3$	5	7	10	15
(V_2, V_3)	$V_2 \rightarrow V_3$	10	3	5	20
(V_2, V_4)	$V_2 \rightarrow V_4$	10	3	5	20
(V_2, V_5)	$V_2 \rightarrow V_4 \rightarrow V_5$	5	6	10	20
(V_4, V_5)	$V_4 \rightarrow V_5$	10	3	5	20

Hence, the values of the coalitions in this subgame are $v(\{R, B\}) = 105$ and $v(\{G\}) = 30$. This subgame shows that a larger coalition does not necessarily obtain all the possible benefits from cooperation. Although the red and blue coalition controls a larger share of the network, its payoff is constrained by capacity limits. At the same time, the green provider benefits from its residual capacity and its ability to capture unmet demand in the second round of the auction.

5.7.4 Characteristic Function and Shapley Value

From the results of the three subgames, the characteristic function of the cooperative game can now be defined. The value of the empty coalition is zero, and the values of the coalitions that are smaller than the grand coalition are obtained from the outcomes of the games describing auctions between two coalitions. Hence, the characteristic form of the game described by the network from Example 1 is given by Table 5.20.

Table 5.20 Characteristic Function of the Cooperative Game

Coalition	Value $v(S)$
$v(\emptyset)$	0

$v(\{R\})$	45
$v(\{B\})$	60
$v(\{G\})$	30
$v(\{R, G\})$	75
$v(\{G, B\})$	120
$v(\{R, B\})$	105
$v(\{R, G, B\})$	**165**

These values summarize the economic strength of each coalition in the capacity-constrained communication network. The grand coalition achieves a total payoff of 165, which cannot be exceeded by any partition of players. The values of the two-player coalitions show that the efficacy of cooperation varies according to the structure of coalition. The coalition formed by the green and blue providers has the largest two-player value. Although the green provider is less efficient than the red provider, its central position and relatively large capacity makes it a more useful ally to the blue provider than the red player can be. This is due to the fact that the coalition of blue and green providers can economically serve the routes (V_1, V_5) and (V_3, V_5), unlike the coalition of the red and blue players. This difference reflects the unequal strategic roles of providers in the network.

Based on this characteristic function, the Shapley value can be used to determine a fair allocation of the total profit among the three providers. The Shapley value evaluates each provider through its marginal contribution across all possible orders in which the grand coalition may be formed. Since there are three providers, there are six possible orders to consider. For each order, the marginal contribution of a provider is determined by comparing the coalition's value before and after that provider joins.

Table 5.21 Marginal Inputs and Derivation of the Shapley Value for the Game

Order	Input of *G*	Input of *B*	Input of *R*
R, G, B	30	90	45
R, B, G	60	60	45
G, R, B	30	60	45
G, B, R	30	60	45
B, R, G	60	60	45
B, G, R	60	60	45
Average	**45**	**75**	**45**

Table 5.21 presents the marginal contributions for all possible orders of coalition formation. Averaging these marginal contributions gives the Shapley value allocation. The resulting payoffs are 45 for the green provider, 75 for the blue provider, and 45 for the red provider. Thus, the Shapley value vector for this game is $\phi(v) = (45, 75, 45)$. This allocation shows that the blue provider has the strongest strategic position in the game. The reason is that the blue provider combines relatively low transmission costs with exclusive access to routes connected to node V_5. In particular, the blue provider has a monopoly over profitable transmission to V_5 in several coalition structures. This increases its marginal contribution and leads to the highest Shapley value.

The green and red providers receive equal final payoffs, although their positions in the network differ. The red provider benefits from relatively low costs on some key edges, but its role is constrained by the network's structure and competition from other coalitions. The green provider does not control the cheapest links, but it occupies a more central position in the network and can capture residual demand due to the relatively large capacity of the edges it possesses. These effects cancel each other out, resulting in equal Shapley allocations.

It should be noted that this is the payoff split determined according to the uniform measure of economic centrality. Since this cooperative game is balanced, the corresponding Shapley value is admissible. It is also relatively easy to check that the payoff division according to the proportional measure of centrality is admissible, since the sum of payoffs to the players in a coalition is always at least as great as the value of a coalition.

The Shapley value, therefore, provides an allocation that reflects both economic efficiency and strategic importance. Unlike the edge-based centrality measures discussed earlier, this approach evaluates providers at the coalition level. It takes into account not only the routes that a provider can serve directly, but also how the provider changes the value of cooperation when joining different coalitions. In this sense, the Shapley value complements centrality-based allocation rules by providing a cooperative game-theoretic interpretation of profit sharing in capacity-constrained communication networks.

This example also reveals several broader observations. First, capacity constraints play a central role in determining coalition values because they limit the amount of demand that can be served, even when routes are available. Second, monopoly control over specific parts of the network can strongly increase a provider's marginal contribution and, consequently, its Shapley payoff.

5.7.5 Remarks on the Auction Mechanism

The auction-based procedure used to determine coalition values raises several important considerations regarding the assumptions of the model, computational aspects, and economic interpretation. These remarks clarify the limitations of the approach and provide directions for further analysis.

Firstly, coalitions make allocations in each round based on the current prices. Hence, the final allocations may not necessarily give equilibrium allocations at the final prices. Although the auction procedure is artificial, it reflects the behavior and evolution of coalitions that are likely to first concentrate on satisfying demand in local networks before competing in more global networks where competition is more likely. In this way, this procedure can be seen as giving a reasonable assessment of the power of various coalitions. In practice, one may interpret the outcome of an auction as the result of implicit competition between providers. Under such an interpretation, the announced prices and flow allocations represent stable outcomes that providers are willing to implement. Nevertheless, further research is needed to establish the formal equilibrium properties of the mechanism.

Secondly, the computational complexity of the auction mechanism may become significant in large networks. In each round, coalitions must identify feasible paths, compute costs, and solve flow optimization problems under updated capacity constraints. In networks with many alternative paths and limited capacities, the number of possible routing configurations can be large. This may lead to high computational effort, especially when repeated recalculations are required after each update of residual capacities. Efficient implementations may therefore require advanced optimization techniques or heuristic methods to reduce computational burden.

Finally, the auction procedure shares important similarities with classical algorithms for minimum-cost flow problems. In particular, the mechanism tends to prioritize routes for which a coalition has a clear cost advantage over its competitors. As the auction progresses, the effective revenue associated with contested routes increases due to competitive bidding. This creates an incentive for coalitions to first serve routes where they have a natural monopoly, that is, routes for which their transmission costs are significantly lower than those of competing coalitions. Only after these routes have been satisfied do coalitions compete for the remaining demand.

This behavior aligns with economic intuition and reinforces the interpretation of the auction as a mechanism that balances efficiency and competition.

These observations highlight that the proposed auction mechanism provides a realistic and flexible framework for evaluating coalition values in capacity-constrained communication networks, while also pointing to several directions for further theoretical and computational refinement.

5.8 Discussion and Implication

The approach presented in this chapter consists of two main stages. In the first stage, the multicommodity flow problem is solved to obtain an optimal decomposition of network flows **(Ahuja et al., 1993).** This step determines how the demand for communication between pairs of nodes is routed through specific transmission paths while satisfying capacity constraints on each edge. The optimization procedure identifies feasible flow assignments that maximize the total network profit subject to physical constraints **(Bertsekas, 2021).** In the second stage, the total profit is allocated among providers using economic centrality measures that evaluate each provider's edges based on realized flows.

The examples presented in Sections 5.5 – 5.7 provide concrete insight into how this framework operates in practice. When the capacity of edges is limited, it is possible that not all traffic on a route will be directed through the shortest path. Instead, flows are split across multiple paths when necessary. This reflects the impact of capacity constraints, which force the system to use alternative routes even when they are more expensive. As a result, network efficiency depends not only on shortest paths but also on the availability of residual capacity.

The allocations based on economic centrality reveal that providers controlling critical edges receive higher payoffs. In the example, the edge connected to node V_5 carries a significant portion of profitable flow. This explains why the provider controlling this edge obtains a large share of the total profit. The results confirm that economic importance is determined by actual flow usage rather than purely by network structure.

The cooperative game formulation further extends this analysis by incorporating strategic interaction between providers. The subgame analysis shows that coalition values depend on both the network structure and the availability of capacity. For example, in Subgame 2, the blue and green coalition achieves a high value because the geometry and capacity of its network enable it to economically serve a large number of routes. In contrast, the red provider is limited by capacity and cannot fully exploit the few routes that it can serve. On the other hand, the red and blue coalition spans the entire set of routes, but still fails to satisfy demand due to binding capacity constraints and the less favorable geometry of the combined network.

The Shapley value results provide additional insight into the strategic importance of providers. The blue provider receives the highest payoff because it combines low transmission costs with a strong position in accessing node V_5. The red and blue providers receive equal payoffs, but for different reasons. The green provider benefits from a central position in the network and the ability to capture residual demand, while the red provider benefits from relatively low costs on certain edges. These findings highlight that both cost efficiency and network position influence the distribution of profit. Second, not all routes are economically viable. Some routes may generate costs that exceed their revenue **(Nagurney, 2006).** Extending the model to include penalties for unsatisfied demand would allow the system to decide whether serving a route is justified. Another important issue concerns the existence of multiple optimal flow solutions. Different flow decompositions may yield the same total profit while assigning flows differently across edges. Since economic centrality depends on the distribution of flows, this may lead to different allocations. A possible solution is to define centrality as an average over all optimal solutions.

Finally, the centrality-based allocation assumes full cooperation among providers without any consideration of how such a grand coalition might form. The cooperative game formulation introduced in Section 5.7 relaxes this assumption by explicitly modeling coalition formation and competition. The example shows that coalition values are shaped by auction mechanisms, capacity constraints, and residual demand **(Ramsey et al., 2025).** Approaches using, for example, the Myerson value or S-value would be relatively easy to implement on the basis on the characteristic function defined here and a graph describing the relationships between the players. Extending models of cooperative games to capacity-constrained networks with more complex cost functions remains an important direction for future research, as it provides a more complete representation of both operational and strategic aspects of communication networks.

Chapter 6

Conclusions and Future Studies

Chapter 6 presents the dissertation's conclusion, specifying that the cooperative game-theoretic framework advances understanding of communication network management by unifying network structure, flow optimization, and fair allocation mechanisms. The main findings include the demonstration that coalition behavior and network topology interact to influence both system efficiency and fairness in allocation. The Shapley value allocation and subgame analysis quantitatively reveal how different coalition structures lead to distinct payoffs, highlighting the importance of marginal contributions for both cost efficiency and strategic positioning within the network. The example in Section 5.7 illustrates these findings by showing how coalition values are shaped by the interaction of capacity constraints, auction-based competition, and network topology. The chapter also notes the study's limitations, including the assumption of a static network structure and linear costs. For future research, it suggests extending the model to dynamic networks, adopting more realistic models of cost involving congestion, and using advanced computational methods.

6.1 Conclusions

The thesis has addressed the problem of fair and efficient profit allocation in cooperative communication networks, arising from multiple service providers controlling different parts of the network infrastructure. The complexity of network structure, flow interactions, and economic dependencies creates challenges in evaluating each provider's contribution and in distributing network benefits appropriately. Addressing these challenges is essential for ensuring stability and cooperation in multi-provider communication networks. The contributions of this can be summarized as follows:

First, we have analyzed allocation mechanisms based on cooperative game theory and centrality measures to distribute the network's total profit among service providers. We have applied solution concepts such as the Shapley value to evaluate each provider's marginal contribution. This approach ensures that each provider is rewarded based on its role in enabling communication across the network. The allocation is computed by considering all possible coalition formations, which provides a fair and consistent distribution of payoffs. Numerical examples have illustrated how different coalition structures lead to distinct payoff distributions and how marginal contributions determine the final allocation.

Secondly, this analysis was extended by incorporating the network structure into the process of coalition formation. Using graph-based cooperative game models, we have applied concepts such as the Myerson value and S-value to reflect the fact that coalitions between providers who occupy "neighboring" positions in a network are more likely to form than coalitions composed of providers who occupy distant positions from each other. This allows the allocation mechanism to reflect not only providers' contributions but also the limitations imposed by the network topology. This highlights how connectivity and network position influence coalition values and allocation outcomes.

Thirdly, we have introduced measures of economic centrality to evaluate the importance of providers based on actual network usage. Unlike traditional structural centrality measures, the proposed approach considers the flow, cost, and revenue associated with each edge. This enables a more accurate assessment of the strategic importance of providers within the network and provides insight into how network resources contribute to overall value generation. Numerical examples have demonstrated that providers controlling edges with high traffic and critical routing roles receive a higher share of total profit.

Fourthly, we have analyzed the stability of allocation outcomes using cooperative game-theoretic concepts such as superadditivity and the core. We have examined whether the proposed allocation rules ensure that no subset of providers has an incentive to deviate and form separate coalitions. The characteristic function based on the model presented in Chapter 4 is superadditive. The author has not found any revenue distributions based on the Shapley that are inadmissible. However, it remains to be shown whether games formulated in this way are necessarily balanced. On the other hand, an example was given in Chapter 4 to show that revenue distribution based on centrality measures may be inadmissible, while the corresponding Shapley values were admissible. This illustrates the advantage of game-theoretic distributions of the revenue over measures of centrality. Centrality measures only take into account the optimal flow under the grand coalition, while game theoretic models take into account the bargaining power of providers in various situations. The advantage of centrality measures lies in the lower level of computational complexity, especially when the two approaches give very similar solutions for networks that are similar to real-world networks.

The calculations for the model presented in Chapter 4 have been fully automated. On the other hand, the solution of games in which the edges have capacity constraints has not yet been fully automated. It also remains to investigate whether the characteristic function for such games is superadditive. This thesis has established a solid framework for the analysis of routing and economic centrality in communication networks and provides a foundation for understanding fairness and stability in cooperative environments.

6.2 Limitations

Although the proposed cooperative game theoretic framework provides a structured approach for analyzing communication networks and allocating profits among service providers, several limitations remain:

1. Limited scalability of the model: The framework has been analyzed mainly using theoretical models and illustrative examples. It has not been tested on large-scale real networks with many nodes, edges, and providers. This limits the evaluation of computational performance and scalability in practical settings. The model presented in Chapter 4 requires the derivation of the shortest (cheapest) path for routes. While there are efficient algorithms for these calculations, the complexity of the formulation of a cooperative game will be exponential in the number of players. Hence, game-theoretic analysis of large networks is only practical when the number of providers is relatively small. When the number of providers is large, the calculation of economic centrality will be much less complex. Hence, it would be good to investigate the conditions under which divisions based on economic centrality differ significantly from divisions based on cooperative game theory.
2. Simplistic assumptions regarding the network: The study assumes a static network structure with fixed nodes and edges. In real communication systems, network topology evolves over time due to failures, upgrades, and infrastructure changes. This reduces the model's applicability in dynamic environments.
3. Assumption of linear transmission costs: The model assumes linear transmission costs to maintain analytical tractability. In practice, costs are often

non-linear due to congestion effects, pricing mechanisms, and service-level requirements. This simplification may limit the realism of the results.

4. Limited treatment of capacity constraints: Capacity constraints have been incorporated in the extended model (presented in Chapter 5) and illustrated through numerical examples. However, the analysis is restricted to specific scenarios and does not fully capture complex congestion patterns or time-varying capacity conditions observed in real networks.
5. Focus on cooperative behavior only: The framework is based on cooperative game theory and assumes that providers are willing to collaborate. It does not model competitive or non-cooperative behavior, which is common in real communication markets. This limits the ability to analyze strategic conflicts and market competition.
6. Limited scope of allocation methods: The study focuses on selected solution concepts, including the Shapley value, Myerson value, and S-value. Other allocation rules and mechanism design approaches are not considered. These alternatives may provide different perspectives on fairness and efficiency.
7. Absence of empirical validation: The framework remains primarily theoretical, illustrated by examples. It has not been validated using real datasets or implemented in operational networks. This limits the assessment of its practical effectiveness.
8. Static demand and flow: The model assumes fixed demand and deterministic flows. In real systems, demand fluctuates and may be uncertain. This can affect routing decisions, coalition formation, and the outcome of profit allocation.

These limitations indicate a number of directions for improving cooperative game-theoretic models for communication network management, particularly by enhancing realism, scalability, and empirical validation.

6.3 Future Studies

Future research should focus on addressing the identified limitations and on extending the cooperative game-theoretic framework for communication network management. Several directions could be explored to enhance the applicability, scalability, and realism of the proposed model:

1. Scalability: Future work should evaluate the proposed framework on large-scale communication networks with many nodes (network points), edges (connections), and service providers (organizations managing parts of the network). This will

help assess computational performance and the feasibility of applying cooperative game solutions in real-world network environments.

2. Applicability across diverse types of networks: The framework could be adapted to various types of networks, including wireless, mobile, Internet of Things, and software-defined networks. This will improve the model's generalizability across different communication infrastructures.
3. Advanced computational platforms: Implementing the model on high-performance computing environments, such as parallel systems, distributed computing platforms, and cloud infrastructures, can significantly improve computational efficiency, especially in the case of calculations for large-scale cooperative games.
4. Distributed and parallel computation: Future studies can develop distributed or parallel algorithms (methods that split calculations across multiple computers or processors) to compute allocation solutions, such as the Shapley value (a mathematical method for fairly distributing resources or costs among participants). This is important because exact computation becomes expensive as the number of providers increases.
5. Extension of cost models: The current model assumes linear transmission costs. Future research should explore non-linear costs, congestion pricing, and quality-of-service constraints to better reflect real network behavior.
6. Integration of non-cooperative behavior: The framework can be extended to include non-cooperative (in which participants act independently for their own benefit) and hybrid game models (combining cooperative and non-cooperative strategies), enabling the analysis of competition, bargaining, and strategic interactions among service providers.
7. Dynamic and time-varying networks: Future work should consider dynamic networks (networks that change over time), where the topology (the arrangement of network nodes and connections), demand, and resource availability evolve. This will enable the study of adaptive coalition formation and real-time decision-making.
8. Expansion of allocation mechanisms: Further research can investigate alternative allocation rules (methods for distributing resources) and mechanism design approaches (strategies for designing incentives and protocols) to improve fairness, efficiency, and incentive compatibility in network resource allocation.

9. Real-world data validation: The framework should be tested on real network datasets to assess its practical effectiveness and compare it with existing methods.
10. Integration with artificial intelligence: Future research can combine cooperative game theory with machine learning and AI to improve prediction, optimization, and decision-making in networks.

By pursuing these directions, the framework can address more complex and realistic network scenarios and better support efficient, fair management.

SCIENTIFIC ACHIEVEMENTS

During my PhD studies, several scientific achievements were accomplished through active participation in research, publications, and academic dissemination. The research results were presented at international conferences on group decision-making, dynamic games and applications, and knowledge-based and intelligent information and engineering systems, providing opportunities to discuss the findings with the scientific community and to receive valuable feedback. In addition, parts of this research have been published in peer-reviewed journals and conference proceedings, advancing knowledge in network management and game-theoretic modeling. The PhD work also involved collaboration with researchers and participation in academic workshops and seminars, which strengthened both theoretical and practical understanding of the subject. These activities demonstrate continuous scientific development and contribute to the visibility and impact of the research at the international level.

Conferences

1) **International Workshop on Multiple Criteria Decision Making** on March 26-28, 2023, Ustron, Poland, with the topic "Development of a Computer System Implementing the SMART, AHP, and ANP Methods of Multi-criteria Analysis", **Rajiansyah**, Grzegorz Filcek, and David M Ramsey.
2) **Quantitative Methods of Group Decision Making** on November 24, 2023, Wroclaw, Poland, with the topic "Routing Traffic Across Networks: A Game Theory Approach", **Rajiansyah**, David M Ramsey, and Dariusz Gąsior.
3) **International Symposium on Dynamic Games and Applications** on July 9-12, 2024, Valladolid, Spain, with the topic "Flow Control in a Computer Network: A Cooperative Game Approach", **Rajiansyah**, David M Ramsey, and Dariusz Gąsior.
4) **Quantitative Methods of Group Decision Making** on November 29, 2024, Wroclaw, Poland, with the topic "A Game Theoretic Model of Cooperative Management of a Communication Network", David M Ramsey, **Rajiansyah**, and Dariusz Gąsior.
5) **International Conference on Knowledge-Based and Intelligent Information and Engineering Systems** on September 10-12, 2025, Osaka, Japan, with a topic "A Cooperative Game Theoretic Model of a Multi-Agent

Information Network Based on the Myerson Value”, **Rajiansyah**, David M Ramsey, and Dariusz Gąsior.

6) **Quantitative Methods of Group Decision Making** on November 28, 2025, Wroclaw, Poland, with the topic “Management of Communications Networks: A Cooperative Game Approach Using Myerson's approach”, **Rajiansyah**, David M Ramsey, and Dariusz Gąsior.
7) **International Conference on Group Decision and Negotiation** on June 28 – July 1, 2026, Katowice, Poland, with a topic “Revenue Sharing in a Network via a Centrality Measure Incorporating Capacity Constraint”, Dariusz Gąsior, **Rajiansyah**, and David M Ramsey.

Publications

1) Ramsey, D., Gąsior, D., & **Rajiansyah, R.** (2024). Management of a communications network involving a number of service providers: A cooperative game approach. *SSRN Electronic Journal*. https://doi.org/10.2139/ssrn.5202728.
2) Ramsey, D., Gąsior, D., & **Rajiansyah, R.** (2025). A cooperative game theoretic model of a multi-agent information network based on the Myerson value. *Procedia Computer Science, 270*, 1669–1678. https://doi.org/10.1016/j.procs.2025.09.287.
3) Gąsior, D., **Rajiansyah, R., &** Ramsey, D. (2026). Revenue Sharing in a Network via a Centrality Measure Incorporating Capacity Constraints.

Publications Not Related to the Thesis

1) **Rajiansyah, R.,** Filcek, G., & Ramsey, D. (2023). Evaluating a Computer Application that Aids Multi-Criteria Decision Making, *Multiple-Criteria Decision Making Journal (18)*, pp. 47-76, https://mcdm.ue.katowice.pl/files/papers/mcdm23(18)_3.pdf
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Other Achievements

- **East Kalimantan Scholarship (Beasiswa Kaltim),** November 13, 2023: Stimulants Scholarship for Ph.D. students from East Kalimantan Province, Indonesia, studying abroad.
- **East Kalimantan Scholarship (Beasiswa Kaltim),** December 22, 2024: Stimulants Scholarship for Ph.D. students from East Kalimantan Province, Indonesia, studying abroad.
- **Mini-grants,** from April 16, 2024, to November 15, 2024, are intended to support the scientific activities of doctoral students related to the subject of their doctoral dissertations.
- **Scientific Cooperation** with Mulawarman University, Indonesia, as a teaching assistant.
- **Gratispol Scholarship**, January 09, 2026: Stimulants Scholarship for Ph.D. students from East Kalimantan Province, Indonesia, studying abroad.

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