# Review of the Doctoral Dissertation
## "Pricing time-capped American options"

by
Paweł Stępniak

The doctoral dissertation under review is devoted to the valuation of American options whose maturity is constrained by a random time. The candidate investigates several classes of such contracts and formulates the associated pricing problem as an optimal stopping problem for the payoff

$$\sup_{\tau \in \mathcal{T},\, \tau \leq T} \mathbb{E}\Big[e^{-r(\tau\wedge\theta)} G(S_{\tau\wedge\theta})\Big],$$

where $S$ denotes the underlying asset price, $G$ is the payoff function, $T$ is the (possibly infinite) maturity horizon, and $\theta$ is the random event triggering forced exercise. The dissertation combines analytical methods from the theory of optimal stopping for spectrally negative Lévy processes with a numerical Least Squares Monte Carlo (LSMC) approach adapted to the capped setting.

The dissertation is composed of five chapters. Chapter 1 presents the necessary financial and probabilistic background. Chapter 2 is devoted to perpetual American put options capped by the first exit time from an interval. Chapter 3 concerns drawdown-capped contracts, first in the Black–Scholes framework and subsequently in a jump-diffusion/Lévy setting. Chapter 4 examines an option linked to the last passage time above a fixed level. Chapter 5 develops a modified LSMC algorithm for finite-maturity time-capped American options.

## 1. Main scientific contribution

In my assessment, the dissertation contains a clear, substantial, and nontrivial scientific contribution. The principal achievements of the thesis may be summarized as follows.

1) The dissertation introduces and systematically investigates a broad class of *time-capped American options*, in which the cap affects the exercise horizon rather than the payoff level. This constitutes a conceptually original and mathematically well-motivated extension of classical capped options.

2) For perpetual American put options capped by the *first exit time* of the underlying asset from a prescribed interval, the thesis derives an explicit candidate value function, identifies the optimal exercise threshold, and establishes optimality by means of a verification argument based on the Hamilton–Jacobi–Bellman system and scale-function techniques.

3) For contracts capped by a *drawdown event*, the analysis demonstrates that the optimal stopping region has a one-sided boundary form depending on the running maximum and yields explicit pricing formulas in two important model classes: the Black–Scholes model and a geometric Lévy model with downward exponential jumps.

4) For the *last-passage cap* problem, which is mathematically delicate because the cap itself is not a stopping time, the dissertation derives explicit formulas for the value function and the optimal exercise level, thereby extending earlier work on cancellable American-style contracts.

5) The dissertation further proposes a *modified Least Squares Monte Carlo method* for finite-maturity time-capped American options. The method is formulated in a reasonably general framework and is supplemented by a convergence argument as well as numerical illustrations for several classes of caps, including caps independent of the asset price.

## 2. Originality and significance

The dissertation addresses a relevant and well-posed topic situated at the intersection of probability theory, stochastic analysis, and mathematical finance. Its originality lies in shifting the capping mechanism from the payoff domain to the time domain and in examining several distinct random mechanisms that may trigger forced exercise.

From the mathematical point of view, the thesis relies on a nontrivial and technically demanding set of tools, including fluctuation theory for spectrally negative Lévy processes, scale functions, change-of-numéraire arguments, Snell envelopes, and verification techniques. The candidate demonstrates the ability to employ these methods in a creative, competent, and problem-oriented manner. In particular, the treatment of drawdown-capped problems and of the last-passage problem requires considerably more than a routine adaptation of existing formulas.

From the perspective of applications, the dissertation is likewise meaningful. Even if some of the contracts considered are more naturally interpreted as mathematically motivated instruments than as immediately market-ready products, the analysis significantly broadens the class of derivative structures that may be priced and understood within a rigorous optimal stopping framework.

The dissertation is also closely connected with the candidate's publication record. As stated in the thesis itself, Chapter 4 is based on the paper *Last-Passage American Cancelable Option in Lévy Models*, published in *Journal of Risk and Financial Management* (2023), whereas Chapter 5 is based on the paper *Pricing time-capped American options using Least Squares Monte Carlo method*, published in *Journal of Computational Finance* (2025). In addition, publicly available records indicate two 2025 preprints devoted to drawdown-capped American options, one in the Black–Scholes model and one in a Lévy market, both closely related to the material presented in Chapter 3. Public records also indicate an earlier co-authored paper by the candidate published in *Mathematica Applicanda* (2020).

The mathematical level of the dissertation is good. The main results are interesting, technically competent, and scientifically relevant. The proofs follow a coherent and recognizable strategy: first formulating the value function, then conjecturing the structure of the stopping region, next deriving a candidate solution by means of fluctuation identities and scale functions, and finally verifying optimality through an HJB-type argument. For the class of problems under consideration, this approach is appropriate and, in the main, successfully executed.

It should be emphasized that the dissertation addresses an original and mathematically meaningful research problem, contains nontrivial new results, and demonstrates command of advanced methods from probability theory and mathematical finance. In particular, the following points deserve to be regarded as decisive:

1) the topic is original, clearly formulated, and scientifically relevant;

2) the dissertation contains results that go well beyond standard textbook material;

3) the thesis demonstrates research independence as well as technical maturity;

4) the work makes a valuable contribution to the literature on optimal stopping and mathematical finance.

## 3. Final conclusion

In my opinion, the doctoral dissertation entitled *"Pricing time-capped American options"* by Paweł Stępniak contains an original solution to a significant scientific problem in the discipline of mathematics.

Accordingly, I conclude that the dissertation satisfies the requirements for a doctoral dissertation laid down in Article 187 of the Act of 20 July 2018 – Law on Higher Education and Science. I therefore recommend that Paweł Stępniak be admitted to the subsequent stages of the doctoral proceedings, including the public defence of the dissertation.

Podpisuję z Cencert

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